University of Amsterdam
Informatics Institute

AMLAB Amsterdam Machine Learning Lab

Machine Learning 1

Lecture 6.5 - Supervised Learning Classification - Discriminative Models - The Perceptron

Erik Bekkers

(Bishop 4.1.7)

Slide credits: Patrick Forré and Rianne van den Berg

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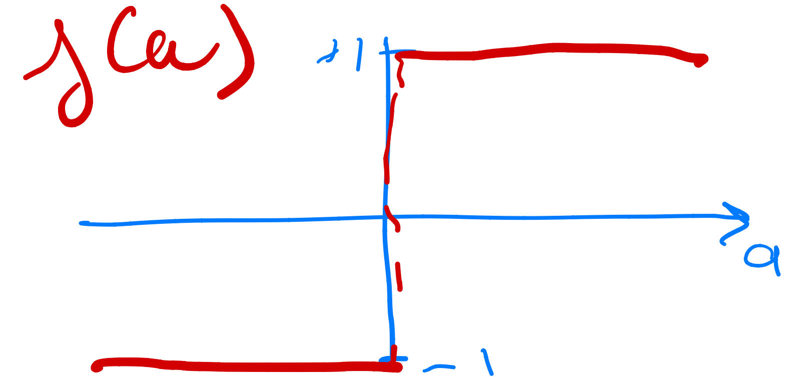
The Perceptron Algorithm



- Input: $x \in \mathbb{R}^D$
- targets: $t \in \{C_1, C_2\}, \rightarrow t \in \{-1, 1\}$ 2 classes
- Prediction: $y(\mathbf{x}) = f(\mathbf{w}^T\boldsymbol{\phi}(\mathbf{x}))$ $\quad f(a) = \begin{cases} 1 & , a \geq 0 \\ -1 & , a < 0 \end{cases}$
- Class decisions: assign **x** to class C_1 if ……… $w^T\phi_n > 0$ (and C_{-1} if ………. $w^T\phi_n < 0$)
- For correct classification: find **w** such that for all (x_n, t_n):

  $w^T\phi_n t_n \geq 0$
- Perceptron criterion: $E_P(\mathbf{w}) = -\sum_{n\in\mathcal{M}} w^T\boldsymbol{\phi}(\mathbf{x}_n)t_n$ $\leftarrow$ all misclassified

$\mathcal{M} : \{n : w^T\phi_n t_n < 0\}$

Perceptron: Stochastic Gradient Descent

• $t = -1$ • $t = 1$

✦ $$E_P(\mathbf{w}) = -\sum_{n \in \mathcal{M}} w^T \phi(\mathbf{x}_n) t_n$$

$$= \sum_{n \in \mathcal{M}} E_n(\mathbf{w})$$

" $E_n(w) = -w^T \phi_n t_n$

✦ Stochastic Gradient Descent (SGD).
For each misclassified $\mathbf{x}_n$:

$$\mathbf{w}^{(\tau+1)} = \mathbf{w}^{(\tau)} - \eta \nabla E_n(\mathbf{w})$$

$$= w^{(\tau)} + \eta (\phi_n t_n)$$

✦If **X** is linearly separable, then perceptron SGD will converge

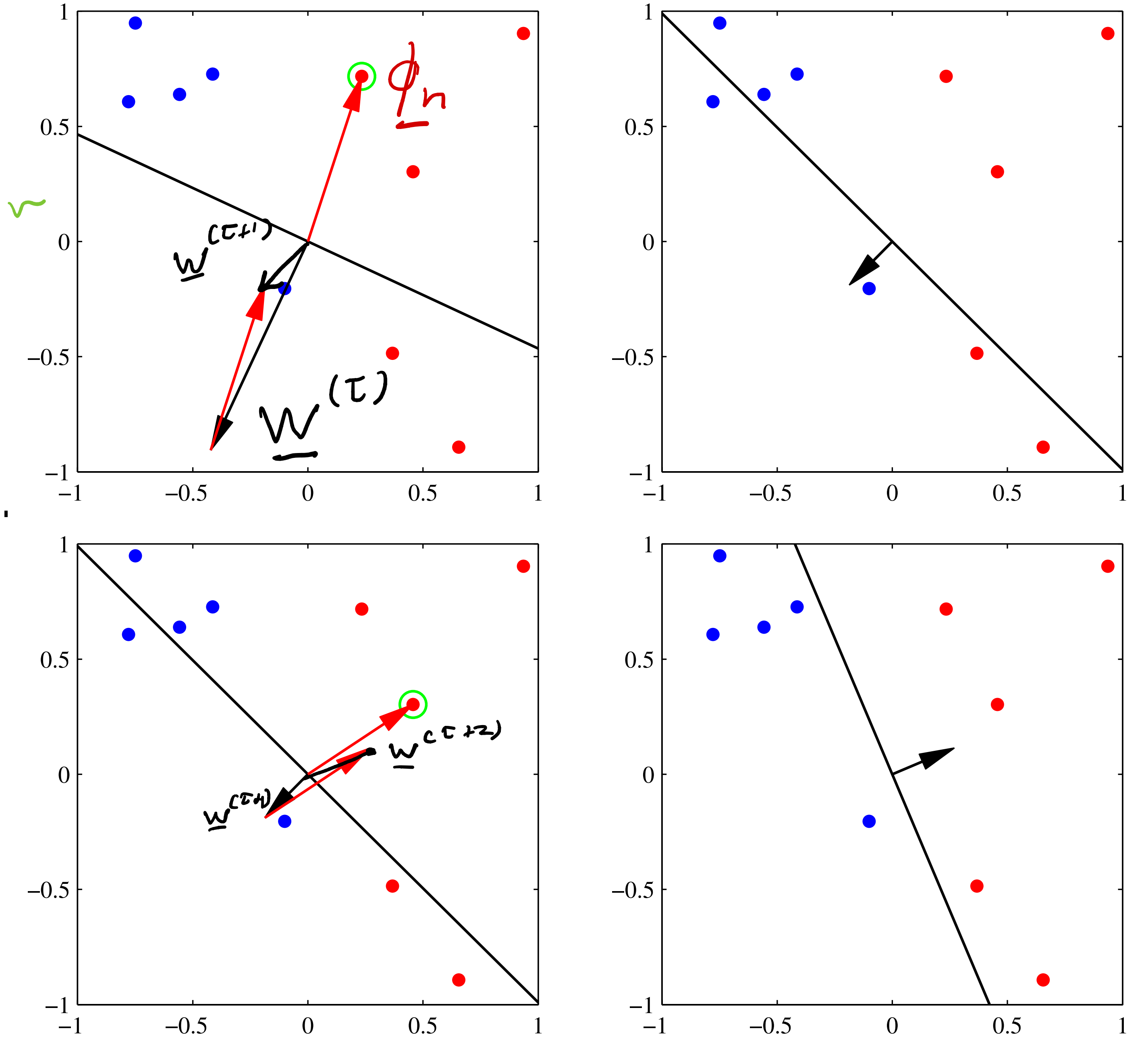


Figure: for $\mathbf{x_n}$ in C_1: add $\phi(\mathbf{x}_n)$ to **w,** for $\mathbf{x_n}$ in C_2: subtract $\phi(\mathbf{x}_n)$ from **w.** SGD for perceptron criterion (Bishop 4.7)

Problems: Perceptron

- Perceptron only works for 2 classes
- There might be many solutions depending on the initialization of **w** and on the order in which data is presented in SGD
- If dataset is not linearly separable, the perceptron algorithm will not converge.
- Based on linear combination of fixed basis functions.