University of Amsterdam
Informatics Institute

Machine Learning 1

Lecture 1.5 - Probability Theory: An Example

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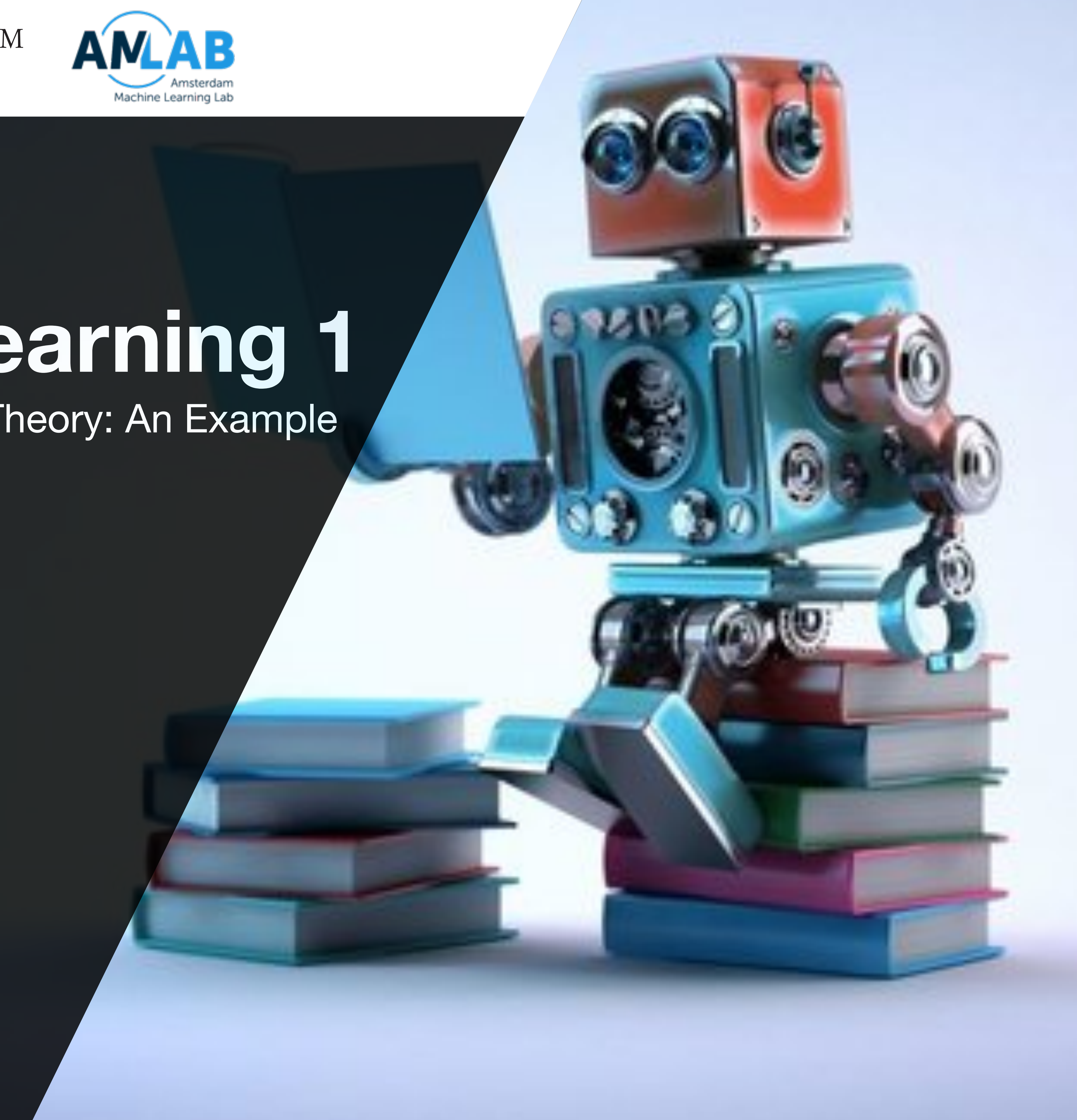
(Bishop 1.2.0 - 1.2.1)



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Example: Fruit in Boxes

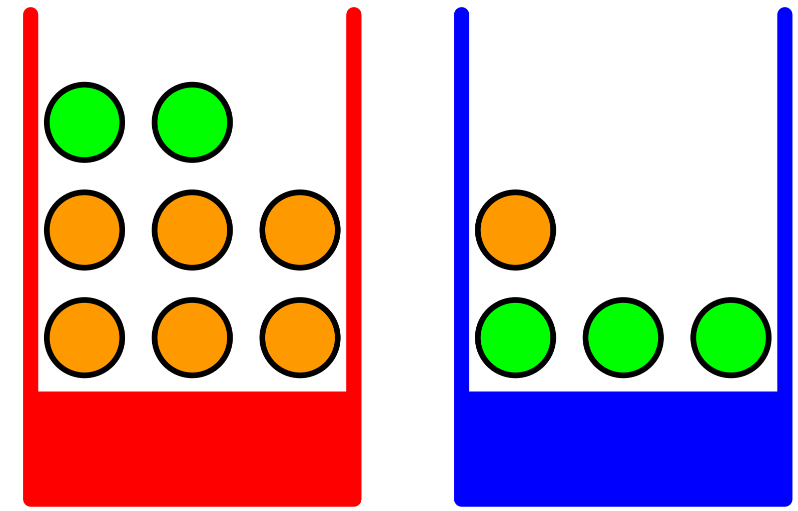

Figure: coloured boxes containing apples and oranges (Bishop 1.9)

- Random variables:

fruit : $F = \{\text{apple}(a), \text{orange}(o)\}$

box : $B = \{\text{red}(r), \text{blue}(b)\}$

- Prior Box distribution:

$$p(B = r) = 4/10$$
$$p(B = b) = 6/10$$

- Conditional probabilities of Fruit given Box

$p(F=o \mid B=b) = \frac{1}{4}$ $\quad$ $p(F=o \mid B=r) = \frac{6}{8} = \frac{3}{4}$

$p(F=a \mid B=b) = \frac{3}{4}$ $\quad$ $p(F=a \mid B=r) = \frac{1}{4}$

- Marginal Fruit distributions:

$$p(F = a) = \sum_B p(F=a, B) = \sum_B p(F=a \mid B)\cdot p(B) = \frac{3}{4}\cdot\frac{6}{10} + \frac{1}{4}\cdot\frac{4}{10} = \frac{11}{20}$$

$$p(F = o) = p(F=o \mid B=r)\,p(B=r) + p(F=o \mid B=b)\,p(B=b)$$
$$= \frac{3}{4}\cdot\frac{4}{10} + \frac{1}{4}\cdot\frac{6}{10} = \frac{9}{20} = \left(1 - \frac{11}{20}\right) = \frac{9}{20}$$

Example: Fruit in Boxes

- Prior: p(B = r) = 4/10 & p(B = b) = 6/10
- Marginal: p(F=a) = 11/20 & p(F = o) = 9/20
- Posterior probability of Box color given observed fruit

$$p(B = r|F = o) = \frac{P(F=o \mid B=r) \cdot P(B=r)}{P(F=o)}$$

$$= \frac{6}{\cancel{8}} \cdot \frac{4}{\cancel{10}} \cdot \frac{\cancel{20}}{9} = \frac{6}{9} = \frac{2}{3} \approx 66\%$$

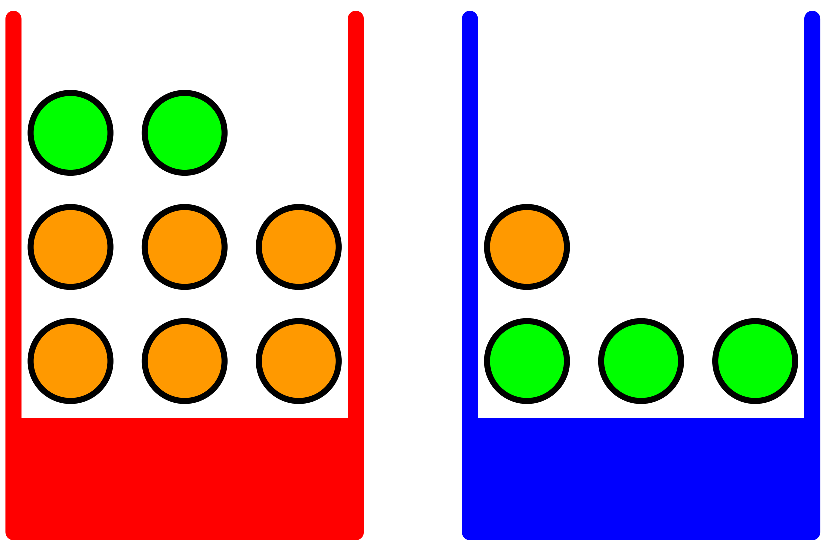

Figure: coloured boxes containing apples and oranges (Bishop 1.9)

- prior probability of red box:

  $p(B = r) = 4/10 < P(B=r \mid F=o)$

- After observing an orange the probability of observing a red box is now larger than observing a blue box!

Independent Random Variables

Two random variables *X* and *Y* are *independent* iff measuring *X* gives no information on *Y*, and vice versa.

- Formally: *X* and *Y* are called independent if

$$= P(x,y) = p(x)\,p(y) \qquad \text{for all} \quad x \in \mathcal{X},\ y \in \mathcal{Y}$$

$p(x|y) \cdot p(y)$

- Equivalent to

$$p(x|y) = \frac{p(x,y)}{p(y)} = \frac{p(x)\,p(y)}{p(y)} = p(x)$$

- Example:

$P(F|B) = P(F) = \frac{1}{2}$

$P(B|F) = P(B)$

$\frac{a}{o} = 1$ $\frac{a}{o} = 1$