Mathics3

A free, open-source alternative to Mathematica

Mathics3 Core Version 9.0.0

The Mathics3 Team

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Part I.

Manual

1. Introduction

Mathics3 is a computer algebra system. It is a free, open-source alternative to *Mathematica*® or the *Wolfram* Language. However, *Mathics3* builds around and on top of the Python ecosystem of libraries and tools. So in a sense, you can think of it as a WMA front-end to the Python ecosystem of tools.

Mathics3 is free both as in "free beer", but more importantly, as in "freedom". *Mathics3* can be run locally. To facilitate installation of the vast amount of software needed to run this, there is a docker image available on dockerhub.

The programming language and built-in functions of *Mathics3* try to match the *Wolfram* Language, which is continually evolving.

Mathics3 is in no way affiliated or supported by *Wolfram*. *Mathics3* will probably never have the power to compete with *Mathematica*® in industrial applications; it is a free alternative, though. It also invites community development at all levels.

See the installation instructions for the most recent instructions for installing from PyPI, or the source.

For implementation details, please refer to the Developers Guide.

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1.1. Why recreate Wolfram Language?

Mathematica® is great, but it might have some disadvantages, depending on your point of view.

- It is not open source.
- Its development is tightly controlled and centralized, and as such
- It can't hook into different kinds of open-source packages that have independently developed algorithms and methods

However, even if you are willing to pay hundreds of dollars for the software, you would not be able to see whatś going on "inside" the program if that is your interest. Thatś what free, open-source, and community-supported software is for!

Mathics3 combines the beauty of *Mathematica*® implemented in an open-source environment written in Python. The Python ecosystem includes libraries and tools like:

- mpmath for floating-point arithmetic with arbitrary precision,

- NumPy for numeric computation,
- SymPy for symbolic mathematics, and
- SciPy for Scientific calculations.

Performance of *Mathics3* is not, right now, fast in large-scale projects and calculations. However, it can be used as a tool for exploration and education and non-computationally intensive use. And *Mathics3* provides better debugging and tracing, since we can be completely transparent about every aspect of its operation.

1.2. What does *Mathics3* offer?

Because *Mathics3* is compatible with the Wolfram-Language kernel within the confines of the Python ecosystem, it is a powerful functional programming language, driven by pattern matching and rule application.

Primitive types include rationals, complex numbers, and arbitrary-precision numbers. Other primitive types, such as images or graphs, or NLP, come from the various Python libraries that *Mathics3* uses.

Outside of the "core" *Mathics3* kernel (which has only a primitive command-line interface), in separate GitHub projects, as add-ons, there are:

- a command-line interface using either prompt-toolkit, or GNU Readline
- a Django-based web server
- a A browser-based no-install online front-end
- a Mathics3 module for Graphs (via NetworkX),
- a Mathics3 module for NLP (via nltk, spacy, and others)
- a Mathics3 Debugger Module (experimental)
- a A docker container which bundles all of the above

In the future, we might provide better interaction with other Open-source projects and tools. For example, we might be able someday to read and write Jupyter notebooks, or translate into pure SymPy and *Mathics3*.

A couple of the notable Mathematica packages that work on *Mathics3*3 are Combinatorica ,and Rubi.

1.3. History

The first alpha versions of *Mathics3* were done in 2011 by Jan Pöschko. He worked on it for a couple of years until the v0.5 release in 2012. By then, it had 386 built-in symbols. Currently, there are over 1,000, and even more when *Mathics3* modules are included.

After that, Angus Griffith took over primary leadership and rewrote the parser to pretty much the stage it is in now. He and, later, Ben Jones worked on it from 2013 to about 2017 for the v1.0 release. Towards the end of this period, Bernhard Liebl worked on this, mostly focusing on graphics.

A docker image of the v.9 release can be found on dockerhub.

Around 2017, the project was largely abandoned in its largely Python 2.7 state, with some support for Python 3.2-3.5 via six.

Subsequently, around mid-2020, it was picked up by the current developers. A list of authors and contributors can be found in the AUTHORS.txt file.

1.4. What is missing?

There are lots of ways in which *Mathics3* could still be improved. FUTURE.rst has the current roadmap.

However, many WMA Built-in functions have not been implemented. To get a list run:

```
git grep -n "\# TODO: " mathics/builtin
```

from the Mathics3 core github repository.

We have a tutorial for adding new Mathics3 functions.

Many of the existing Mathics3 Built-in functions are incomplete or work differently, such as checking for different error conditions.

Although we can always use help in Python programming, there are other ways to help. For example:

- Ensure this document is complete and accurate. We could use help to ensure all of the Built-in functions are described properly and fully, and that they have a link to the corresponding Wiki, SymPy, WMA, and/or mpath links.
- Work on making a comprehensive list of missing or incomplete built-in functions.

  Make sure the built-in summaries and examples are clear and useful.
- We could use help in LaTeX styling, and going over this document to remove overfull boxes and things of that nature. We could also use help and our use of Asymptote. The are some graphics primitives, such as for polyhedra, that haven't been implemented. Similar graphics options are sometimes missing in Asymptote that we have available in other graphics backends.
- Add another graphics backend: it could be a JavaScript library like jsfiddle or some other mechanism.

2. Language Tutorial

The following sections are introductions to the basic principles of the language of *Mathics3*. A few examples and functions are presented. Only their most common usages are listed; for a full description of a Symbol's possible arguments, options, etc., see its entry in the Reference of Built-in Symbols.

If you google for "Mathematica Tutorials", you will find dozens of other tutorials.

I (Rocky Bernstein) like *"Power Programming with the* Mathematica®*: the Kernel"* by David Wagner. Even though this is old, and the book covers *Mathematica*® 3, it has a lot of useful information that is still relevant. A PDF of this is available here. Wolfram's "The *Mathematica*® Book, Fifth Edition" is useful. If you google for this book, you can probably find a PDF of it for free.

Online, there is An Elementary Introduction to the Wolfram Language. In the docker image that we supply, you can load "workspaces" containing the examples described in the chapters of this introduction.

Be warned, though, that *Mathics3* does not yet offer the full range and features and capabilities of *Mathematica*®.

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2.1. Basic calculations

Mathics3 can be used to calculate basic stuff:

```
>> 1 + 2
   3
```

To submit a command to *Mathics3*, press `Shift+Return` in the Web interface or `Return` in the console interface. The result will be printed in a new line below your query.

The result of the previous query to *Mathics3* can be accessed by `%`:

```
>> % ^ 2
   9
```

Mathics3 understands all basic arithmetic operators and applies the usual operator precedence. Use parentheses when needed:

```
>> 1 - 2 * (3 + 5)/ 4
```
-3

The multiplication can be omitted:

```
>> 1 - 2 (3 + 5)/ 4
```
-3

```
>> 2 4
```
8

Powers can be entered using `^`:

```
>> 3 ^ 4
```
81

Integer divisions yield rational numbers:

```
>> 6 / 4
```
$\frac{3}{2}$

To convert the result to a floating-point number, apply the function `N`:

```
>> N[6 / 4]
```
1.5

As you can see, functions are applied using square braces `[` and `]`, in contrast to the common notation of `(` and `)`. At first hand, this might seem strange, but this distinction between function application and precedence change is necessary to allow some general syntax structures, as you will see later.

Mathics3 provides many common mathematical functions and constants, e.g.:

```
>> Log[E]
```
1

```
>> Sin[Pi]
```
0

```
>> Cos[0.5]
```
0.877583

When entering floating point numbers in your query, *Mathics3* will perform a numerical evaluation and present a numerical result, pretty much like if you had applied `N`.

Of course, *Mathics3* has complex numbers:

```
>> Sqrt[-4]
```
$2I$

```
>> I ^ 2
```
-1

```
>> (3 + 2 I)^ 4
```
$-119 + 120I$

```
>> (3 + 2 I)^ (2.5 - I)
```
$43.663 + 8.28556I$

```
>> Tan[I + 0.5]
```
$0.195577 + 0.842966I$

`Abs` calculates absolute values:

```
>> Abs[-3]
```
3

```
>> Abs[3 + 4 I]
```
5

Mathics3 can operate with pretty huge numbers:

```
>> 55! (* Also known as Factorial[55] *)
```
12696403353658275925965100847566516959580321051449436762275840000000000000

We could easily use a number larger than 55, but the digits will just run off the page.

The precision of numerical evaluation can be set:

```
>> N[Pi, 30]
```
3.14159265358979323846264338328

Division by zero gives an error:

```
>> 1 / 0
Infinite expression 1 / 0 encountered.
```
ComplexInfinity

But zero division returns value 'ComplexInfinity' 31.8.2 and that can be used as a value:

```
>> Cos[ComplexInfinity]
```
Indeterminate

`ComplexInfinity` is a shorthand though for `DirectedInfinity[]` .

Similarly, expressions using 'Infinity' 31.8.2 as a value are allowed and are evaluated:

```
>> Infinity + 2 Infinity
```
∞

There is also the value, 'Indeterminate' 31.8.8:

```
>> 0 ^ 0
Indeterminate expression 0 ^ 0 encountered.
```
Indeterminate

2.2. Precision and Accuracy

Mathics3 handles relative and absolute uncertainty in numerical quantities. The *precision* or relative accuracy is set by adding a RawBackquote character (`` ` ``) and the number of digits of precision in the mantissa. For example:

```
>> 3.1416`3
   3.14
```

Above, two decimal places are shown in the output after the decimal point, but three places of precision are stored.

The relative uncertainty of `` 3.1416`3 `` is 10^-3. It is numerically equivalent, in three places after the decimal point, to 3.1413‘4:

```
>> 3.1416`3 == 3.1413`4
   True
```

We can get the precision of the number by using the *Mathics3* Built-in function 'Precision' 7.2.10:

```
>> Precision[3.1413`4]
   4.
```

While 3.1419 is not the closest approximation to Pi in 4 digits after the decimal point (or with precision 4), for 3 digits of precision it is:

```
>> Pi == 3.141987654321`3
   True
```

The absolute accuracy of a number is set by adding two RawBackquotes ``` `` ``` and the number digits.

For example:

```
>> 13.1416``4
   13.142
```

is a number having an absolute uncertainty of 10¯4.

This number is numerically equivalent to ``` 13.1413``4 ```:

```
>> 13.1416``4 == 13.1413``4
   True
```

The absolute accuracy for the value 0 is a fixed-precision Real number:

```
>> 0``4
   0.0000
```

See also Accuracy and precision.

2.3. Symbols and Assignments

Symbols need not be declared in *Mathics3*, they can just be entered and remain variable:

```
>> x
```
x

Basic simplifications are performed:

```
>> x + 2 x
```
$3x$

Symbols can have any name that consists of characters and digits:

```
>> iAm1Symbol ^ 2
```
iAm1Symbol^2

You can assign values to symbols:

```
>> a = 2
```
2

```
>> a ^ 3
```
8

```
>> a = 4
```
4

```
>> a ^ 3
```
64

Assigning a value returns that value. If you want to suppress the output of any result, add a `;` to the end of your query:

```
>> a = 4;
```

Values can be copied from one variable to another:

```
>> b = a;
```

Now changing `a` does not affect `b`:

```
>> a = 3;
```

```
>> b
```
4

Such a dependency can be achieved by using "delayed assignment" with the `:=` operator (which does not return anything, as the right side is not even evaluated):

```
>> b := a ^ 2
```

```
>> b
9
>> a = 5;
>> b
25
```

2.4. Comparisons and Boolean Logic

Values can be compared for equality using the operator `==`:

```
>> 3 == 3
True
>> 3 == 4
False
```

The special symbols `True` and `False` are used to denote truth values. Naturally, there are inequality comparisons as well:

```
>> 3 > 4
False
```

Inequalities can be chained:

```
>> 3 < 4 >= 2 != 1
True
```

Truth values can be negated using `!` (logical *not*) and combined using `&&` (logical *and*) and `||` (logical *or*):

```
>> !True
False
>> !False
True
>> 3 < 4 && 6 > 5
True
```

`&&` has higher precedence than `||`, i.e. it binds stronger:

```
>> True && True || False && False
True
>> True && (True || False)&& False
False
```

2.5. Strings

Strings can be entered with `"` as delimiters:

```
>> "Hello world!"
   Hello world!
```

As you can see, quotation marks are not printed in the output by default. This can be changed by using `InputForm`:

```
>> InputForm["Hello world!"]
   "Hello world!"
```

Strings can be joined using `<>`:

```
>> "Hello" <> " " <> "world!"
   Hello world!
```

Numbers cannot be joined to strings:

```
>> "Debian" <> 6
   String expected.
   Debian<>6
```

They have to be converted to strings using `ToString` first:

```
>> "Debian" <> ToString[6]
   Debian6
```

2.6. Working with Lists

Lists can be entered in *Mathics3* with curly braces `{` and `}`:

```
>> mylist = {a, b, c, d}
```

$\{a, b, c, d\}$

There are various functions for constructing lists:

```
>> Range[5]
```

$\{1, 2, 3, 4, 5\}$

```
>> Array[f, 4]
```

$\{f[1], f[2], f[3], f[4]\}$

```
>> ConstantArray[x, 4]
```

$\{x, x, x, x\}$

```
>> Table[n ^ 2, {n, 2, 5}]
```

$\{4, 9, 16, 25\}$

The number of elements of a list can be determined with `Length`:

```
>> Length[mylist]
```
4

Elements can be extracted using double square braces:

```
>> mylist[[3]]
```
c

Negative indices count from the end:

```
>> mylist[[-3]]
```
b

Lists can be nested:

```
>> mymatrix = {{1, 2}, {3, 4}, {5, 6}};
```

There are alternate forms to display lists:

```
>> TableForm[mymatrix]
```
1	2
3	4
5	6

```
>> MatrixForm[mymatrix]
```
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

There are various ways of extracting elements from a list:

```
>> mymatrix[[2, 1]]
```
3

```
>> mymatrix[[;;, 2]]
```
$\{2, 4, 6\}$

```
>> Take[mylist, 3]
```
$\{a, b, c\}$

```
>> Take[mylist, -2]
```
$\{c, d\}$

```
>> Drop[mylist, 2]
```
$\{c, d\}$

```
>> First[mymatrix]
```
$\{1, 2\}$

```
>> Last[mylist]
```
d

```
>> Most[mylist]
```
$\{a, b, c\}$

```
>> Rest[mylist]
```
$\{b, c, d\}$

Lists can be used to assign values to multiple variables at once:

```
>> {a, b} = {1, 2};
```

```
>> a
```
1

```
>> b
```
2

Operations like addition and multiplication, "thread" over lists; lists are combined element-wise:

```
>> {1, 2, 3} + {4, 5, 6}
```
$\{5, 7, 9\}$

```
>> {1, 2, 3} * {4, 5, 6}
```
$\{4, 10, 18\}$

It is an error to combine lists with unequal lengths:

```
>> {1, 2} + {4, 5, 6}
Objects of unequal length cannot be combined.
```
$\{1, 2\} + \{4, 5, 6\}$

2.7. The Structure of *Mathics3* Objects

Every expression in *Mathics3* is built upon the same principle: it consists of a *head* and an arbitrary number of *children*, unless it is an *atom*, i.e., it can not be subdivided any further. To put it another way: everything is a function call. This can be best seen when displaying expressions in their "full form":

```
>> FullForm[a + b + c]
```
Plus $[a, b, c]$

Nested calculations are nested function calls:

```
>> FullForm[a + b * (c + d)]
```
Plus $[a, \text{Times}[b, \text{Plus}[c, d]]]$

Even lists are function calls of the function `List`:

```
>> Head[{1, 2, 3}]
```
List

However, its full form is presented with `{...}`

```
>> FullForm[{1, 2, 3}]
```
$\{1,2,3\}$

The head of an expression can be determined with `Head`:

```
>> Head[a + b + c]
```
Plus

The children of an expression can be accessed like list elements:

```
>> (a + b + c)[[2]]
```
b

The head is the 0th element:

```
>> (a + b + c)[[0]]
```
Plus

The head of an expression can be exchanged using the function `Apply`:

```
>> Apply[g, f[x, y]]
```
$g\left[x,y\right]$

```
>> Apply[Plus, a * b * c]
```
$a+b+c$

`Apply` can be written using the operator `@@`:

```
>> Times @@ {1, 2, 3, 4}
```
24

(This exchanges the head `List` of `{1, 2, 3, 4}` with `Times`, and then the expression `Times[1, 2, 3, 4]` is evaluated, yielding 24.) `Apply` can also be applied on a certain *level* of an expression:

```
>> Apply[f, {{1, 2}, {3, 4}}, {1}]
```
$\{f[1,2],f[3,4]\}$

Or even on a range of levels:

```
>> Apply[f, {{1, 2}, {3, 4}}, {0, 2}]
```
$f\left[f[1,2],f[3,4]\right]$

`Apply` is similar to `Map` (`/@`):

```
>> Map[f, {1, 2, 3, 4}]
```
$\{f[1],f[2],f[3],f[4]\}$

```
>> f /@ {{1, 2}, {3, 4}}
```
$\{f[\{1,2\}], f[\{3,4\}]\}$

The atoms of *Mathics3* are numbers, symbols, and strings. `AtomQ` tests whether an expression is an atom:

```
>> AtomQ[5]
```
True

```
>> AtomQ[a + b]
```
False

The full form of rational and complex numbers looks like they were compound expressions:

```
>> FullForm[3 / 5]
```
Rational [3, 5]

```
>> FullForm[3 + 4 I]
```
Complex [3, 4]

However, they are still atoms, thus unaffected by applying functions, for instance:

```
>> f @@ Complex[3, 4]
```
$3 + 4I$

Nevertheless, every atom has a head:

```
>> Head /@ {1, 1/2, 2.0, I, "a string", x}
```
{Integer, Rational, Real, Complex, String, Symbol}

The operator === tests whether two expressions are the same on a structural level:

```
>> 3 === 3
```
True

```
>> 3 == 3.0
```
True

But:

```
>> 3 === 3.0
```
False

because `3` (an `Integer`) and `3.0` (a `Real`) are structurally different.

2.8. Functions and Patterns

Functions can be defined in the following way:

```
>> f[x_] := x ^ 2
```

This tells *Mathics3* to replace every occurrence of `f` with one (arbitrary) parameter `x` with `x ^ 2`.

```
>> f[3]
```
9

```
>> f[a]
```
a^2

The definition of `f` does not specify anything for two parameters, so any such call will stay unevaluated:

```
>> f[1, 2]
```
$f[1,2]$

In fact, *functions* in *Mathics3* are just one aspect of *patterns*: `f[x_]` is a pattern that *matches* expressions like `f[3]` and `f[a]`. The following patterns are available:

`_` or `Blank[]`
: matches one expression.

`Pattern[`*x*, *p*`]`
: matches the pattern *p* and stores the matching sub-expression into *x*.

x`_` or `Pattern[`*x*`, Blank[]]`
: matches one expression and stores it in *x*.

`__` or `BlankSequence[]`
: matches a sequence of one or more expressions.

`___` or `BlankNullSequence[]`
: matches a sequence of zero or more expressions.

`_`*h* or `Blank[`*h*`]`
: matches one expression with head *h*.

x`_`*h* or `Pattern[`*x*`, Blank[`*h*`]]`
: matches one expression with head *h* and stores it in *x*.

p `|` *q* or `Alternatives[`*p*, *q*`]`
: matches either pattern *p* or *q*.

p `?` *t* or `PatternTest[`*p*, *t*`]`
: matches *p* if the test *t*[*p*] yields `True`.

p `/;` *c* or `Condition[`*p*, *c*`]`
: matches *p* if condition *c* holds.

`Verbatim[`*p*`]`
: matches an expression that equals *p*, without regarding patterns inside *p*.

As before, patterns can be used to define functions:

```
>> g[s___] := Plus[s] ^ 2
```

```
>> g[1, 2, 3]
```
36

`MatchQ[$e$, $p$]` tests whether *e* matches *p*:

```
>> MatchQ[a + b, x_ + y_]
```
True

```
>> MatchQ[6, _Integer]
   True
```

`ReplaceAll` (`/.`) replaces all occurrences of a pattern in an expression using a `Rule` given by `->`:

```
>> {2, "a", 3, 2.5, "b", c} /. x_Integer -> x ^ 2
```

$\{4, \mathrm{a}, 9, 2.5, \mathrm{b}, c\}$

You can also specify a list of rules:

```
>> {2, "a", 3, 2.5, "b", c} /. {x_Integer -> x ^ 2.0, y_String -> 10}
```

$\{4., 10, 9., 2.5, 10, c\}$

`ReplaceRepeated` (`//.`) applies a set of rules repeatedly, until the expression doesnt change anymore:

```
>> {2, "a", 3, 2.5, "b", c} //. {x_Integer -> x ^ 2.0, y_String -> 10}
```

$\{4., 100., 9., 2.5, 100., c\}$

There is a "delayed" version of `Rule` which can be specified by `:>` (similar to the relation of `:=` to `=`):

```
>> a :> 1 + 2
```

a:>$1 + 2$

```
>> a -> 1 + 2
```

$a- > 3$

This is useful when the right side of a rule should not be evaluated immediately (before matching):

```
>> {1, 2} /. x_Integer -> N[x]
```

$\{1, 2\}$

Here, `N` is applied to `x` before the actual matching, simply yielding `x`. With a delayed rule, this can be avoided:

```
>> {1, 2} /. x_Integer :> N[x]
```

$\{1., 2.\}$

`ReplaceAll` and `ReplaceRepeated` take the first possible match. However, `ReplaceList` returns a list of all possible matches. This can be used to get all subsequences of a list, for instance:

```
>> ReplaceList[{a, b, c}, {___, x__, ___} -> {x}]
```

$\{\{a\}, \{a, b\}, \{a, b, c\}, \{b\}, \{b, c\}, \{c\}\}$

`ReplaceAll` would just return the first expression:

```
>> ReplaceAll[{a, b, c}, {___, x__, ___} -> {x}]
```

$\{a\}$

In addition to defining functions as rules for certain patterns, there are *pure* functions that can be defined using the & postfix operator, where everything before it is treated as the function body, and # can be used as an argument placeholder:

```
>> h = # ^ 2 &;
```

```
>> h[3]
```
9

Multiple arguments can simply be indexed:

```
>> sum = #1 + #2 &;
```

```
>> sum[4, 6]
```
10

It is also possible to name arguments using `Function`:

```
>> prod = Function[{x, y}, x * y];
```

```
>> prod[4, 6]
```
24

Pure functions are very handy when functions are used only locally, e.g., when combined with operators like `Map`:

```
>> # ^ 2 & /@ Range[5]
```
$\{1, 4, 9, 16, 25\}$

Sort using the second element of a list as a key:

```
>> Sort[{{x, 10}, {y, 2}, {z, 5}}, #1[[2]] < #2[[2]] &]
```
$\{\{y, 2\}, \{z, 5\}, \{x, 10\}\}$

Functions can be applied using prefix or postfix notation, in addition to using `[]`:

```
>> h @ 3
```
9

```
>> 3 // h
```
9

2.9. Program-Flow Control Statements

Like most programming languages, *Mathics3* has common program-flow control statements for conditions, loops, etc.:

`If`[*cond*, *pos*, *neg*]
: returns *pos* if *cond* evaluates to `True`, and *neg* if it evaluates to `False`.

`Which`[$cond_1$, $expr_1$, $cond_2$, $expr_2$, ...]
: yields $expr_1$ if $cond_1$ evaluates to `True`, $expr_2$ if $cond_2$ evaluates to `True`, etc.

`Do`[*expr*, {*i*, *max*}]
: evaluates *expr max* times, substituting *i* in *expr* with values from 1 to *max*.

`For`[*start*, *test*, *incr*, *body*]
: evaluates *start*, and then iteratively *body* and *incr* as long as *test* evaluates to `True`.

`While`[*test*, *body*]
: evaluates *body* as long as *test* evaluates to `True`.

`Nest`[*f*, *expr*, *n*]
: returns an expression with *f* applied *n* times to *expr*.

`NestWhile`[*f*, *expr*, *test*]
: applies a function *f* repeatedly on an expression *expr*, until applying *test* on the result no longer yields `True`.

`FixedPoint`[*f*, *expr*]
: starting with *expr*, repeatedly applies *f* until the result no longer changes.

```
>> If[2 < 3, a, b]
   a

>> x = 3; Which[x < 2, a, x > 4, b, x < 5, c]
   c
```

Compound statements can be entered with `;`. The result of a compound expression is its last part or `Null` if it ends with a `;`.

```
>> 1; 2; 3
   3

>> 1; 2; 3;
```

Inside `For`, `While`, and `Do` loops, `Break[]` exits the loop, and `Continue[]` continues to the next iteration.

```
>> For[i = 1, i <= 5, i++, If[i == 4, Break[]]; Print[i]]
   1
   2
   3
```

2.10. Scoping

By default, all symbols are "global" in *Mathics3*, i.e., they can be read and written in any part of your program. However, sometimes "local" variables are needed in order not to disturb the global namespace. *Mathics3* provides two ways to support this:

- *lexicalscoping* by `Module`, and
- *dynamicscoping* by `Block`.

`Module[{vars}, expr]`
: localizes variables by giving them a temporary name of the form `name$number`, where number is the current value of `$ModuleNumber`. Each time a module is evaluated, `$ModuleNumber` is incremented.

`Block[{vars}, expr]`
: temporarily stores the definitions of certain variables, evaluates *expr* with reset values, and restores the original definitions afterward.

Both scoping constructs shield inner variables from affecting outer ones:

```
>> t = 3;

>> Module[{t}, t = 2]
   2

>> Block[{t}, t = 2]
   2

>> t
   3
```

`Module` creates new variables:

```
>> y = x ^ 3;

>> Module[{x = 2}, x * y]
```

$2x^3$

`Block` does not:

```
>> Block[{x = 2}, x * y]
   16
```

Thus, `Block` can be used to temporarily assign a value to a variable:

```
>> expr = x ^ 2 + x;

>> Block[{x = 3}, expr]
   12

>> x
   x
```

`Block` can also be used to temporarily change the value of system parameters:

```
>> Block[{$RecursionLimit = 30}, x = 2 x]
   Recursion depth of 30 exceeded.
   $Aborted

>> f[x_] := f[x + 1]; Block[{$IterationLimit = 30}, f[1]]
   Iteration limit of 30 exceeded.
   $Aborted
```

It is common to use scoping constructs for function definitions with local variables:

```
>> fac[n_] := Module[{k, p}, p = 1; For[k = 1, k <= n, ++k, p *= k]; p]
```

```
>> fac[10]
```
3628800

```
>> 10!
```
3628800

2.11. Formatting Output

The way results are formatted for output in *Mathics3* is rather sophisticated; compatibility with *Mathematica®* is one of the design goals. It can be summed up in the following procedure:

1. The result of the query is calculated.
2. The result is stored in `Out` (which `%` is a shortcut for).
3. Any `Format` rules for the desired output form are applied to the result. In the console version of *Mathics3*, the result is formatted as `OutputForm`; `MathMLForm` for the `StandardForm` is used in the interactive Web version; and `TeXForm` for the `StandardForm` is used to generate the LaTeX version of this documentation.
4. `MakeBoxes` is applied to the formatted result, again given either `OutputForm`, `MathMLForm`, or `TeXForm` depending on the execution context of *Mathics3*. This yields a new expression consisting of "box constructs".
5. The boxes are turned into an ordinary string and displayed in the console, sent to the browser, or written to the documentation LaTeX file.

As a consequence, there are various ways to implement your own formatting strategy for custom objects.

You can specify how a symbol shall be formatted by assigning values to `Format`:

```
>> Format[x] = "y";
```

```
>> x
```
y

This will apply to `MathMLForm`, `OutputForm`, `StandardForm`, `TeXForm`, and `TraditionalForm`.

```
>> x // InputForm
```
x

You can specify a specific form in the assignment to `Format`:

```
>> Format[x, TeXForm] = "z";
```

```
>> x // TeXForm
```
\text{z}

Special formats might not be very relevant for individual symbols, but rather for custom functions (objects):

```
>> Format[r[args___]] = "<an r object>";
```

```
>> r[1, 2, 3]
```
<an r object>

You can use several helper functions to format expressions:

`Infix`[*expr*, *op*]
: formats the arguments of *expr* with infix operator *op*.

`Prefix`[*expr*, *op*]
: formats the argument of *expr* with prefix operator *op*.

`Postfix`[*expr*, *op*]
: formats the argument of *expr* with postfix operator *op*.

`StringForm`[*form*, *arg*1, *arg*2, ...]
: formats arguments using a format string.

```
>> Format[r[args___]] = Infix[{args}, "~"];
```

```
>> r[1, 2, 3]
```
$1 \sim 2 \sim 3$

```
>> StringForm["`1` and `2`", n, m]
```
n and m

There are several methods to display expressions in 2-D:

`Row[{...}]`
: displays expressions in a row.

`Grid[{{...}}]`
: displays a matrix in two-dimensional form.

`Subscript`[*expr*, i_1, i_2, ...]
: displays *expr* with subscript indices i_1, i_2, ...

`Superscript`[*expr*, *exp*]
: displays *expr* with superscript (exponent) *exp*.

```
>> Grid[{{a, b}, {c, d}}]
```
$\begin{matrix} a & b \\ c & d \end{matrix}$

```
>> Subscript[a, 1, 2] // TeXForm
```
a_{1,2}

If you want even more low-level control over expression display, override `MakeBoxes`:

```
>> MakeBoxes[b, StandardForm] = "c";
```

```
>> b
```
c

This will even apply to `TeXForm`, because `TeXForm` implies `StandardForm`:

```
>> b // TeXForm
   c
```

Except some other form is applied first:

```
>> b // OutputForm // TeXForm
   b
```

`MakeBoxes` for another form:

```
>> MakeBoxes[b, TeXForm] = "d";
```

```
>> b // TeXForm
   d
```

You can cause a much bigger mess by overriding `MakeBoxes` than by sticking to `Format`, e.g. generate invalid XML:

```
>> MakeBoxes[c, MathMLForm] = "<not closed";
```

```
>> c // MathMLForm
   <not closed
```

However, this will not affect formatting of expressions involving `c`:

```
>> c + 1 // MathMLForm
   <math display="block"><mrow><mn>1</mn>
     <mo>+</mo> <mi>c</mi></mrow></math>
```

Thatś because `MathMLForm` will, when not overridden for a special case, call `StandardForm` first. `Format` will produce escaped output:

```
>> Format[d, MathMLForm] = "<not closed";
```

```
>> d // MathMLForm
   <math display="block"><mtext>&lt;not closed</mtext></math>
```

```
>> d + 1 // MathMLForm
   <math display="block"><mrow><mn>1</mn> <mo>+</mo>
     <mtext>&lt;not closed</mtext></mrow></math>
```

For instance, you can override `MakeBoxes` to format lists in a different way:

```
>> MakeBoxes[{items___}, StandardForm] := RowBox[{"[", Sequence @@
   Riffle[MakeBoxes /@ {items}, " "], "]"}]
```

```
>> {1, 2, 3}
   [123]
```

However, this will not be accepted as input to *Mathics3* anymore:

```
>> [1 2 3]
   Expression cannot begin with "[1 2 3]" (line 1 of ).
```

```
>> Clear[MakeBoxes]
```

By the way, MakeBoxes is the only built-in symbol that is not protected by default:

```
>> Attributes[MakeBoxes]
```

{HoldAllComplete}

MakeBoxes must return a valid box construct:

```
>> MakeBoxes[squared[args___], StandardForm] := squared[args] ^ 2
```

```
>> squared[1, 2]
```

```
>> squared[1, 2] // TeXForm
```

=

The desired effect can be achieved in the following way:

```
>> MakeBoxes[squared[args___], StandardForm] := SuperscriptBox[RowBox[{
   MakeBoxes[squared], "[", RowBox[Riffle[MakeBoxes[#]& /@ {args},
   ","]], "]"}], 2]
```

```
>> squared[1, 2]
```

$\text{squared}[1,2]^2$

You can view the box structure of a formatted expression using ToBoxes:

```
>> ToBoxes[m + n]
```

RowBox [{m, +, n}]

The list elements in this RowBox are strings, though string delimiters are not shown in the default output form:

```
>> InputForm[%]
```

RowBox [{“m”, “+”, “n”}]

2.12. Graphics Introduction Examples

Two-dimensional graphics can be created using the function Graphics and a list of graphics primitives. For three-dimensional graphics see the following section. The following primitives are available:

`Circle[{`x, y`}, `r`]`
: draws a circle.

`Disk[{`x, y`}, `r`]`
: draws a filled disk.

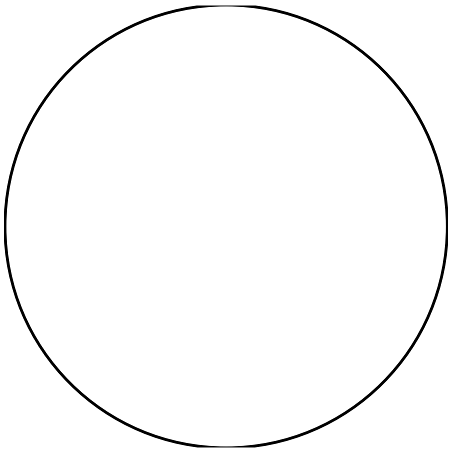
`Rectangle[{`x_1, y_1`}, {`x_2, y_2`}]`
: draws a filled rectangle.

`Polygon[{{`x_1, y_1`}, {`x_2, y_2`}, ...}]`
: draws a filled polygon.

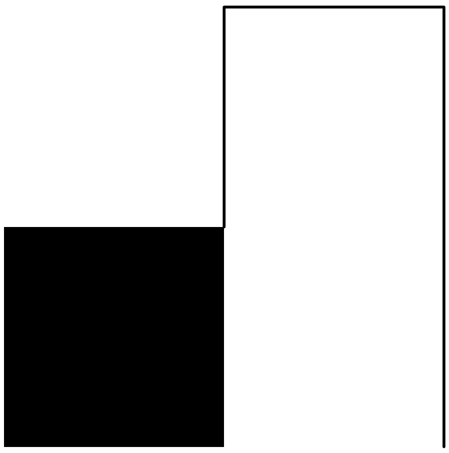
`Line[{{`x_1, y_1`}, {`x_2, y_2`}, ...}]`
: draws a line.

`Text[`*text*`, {`x, y`}]`
: draws text in a graphics.

```
>> Graphics[{Circle[{0, 0}, 1]}]
```



```
>> Graphics[{Line[{{0, 0}, {0, 1}, {1, 1}, {1, -1}}], Rectangle[{0, 0},
   {-1, -1}]}]
```



Colors can be added in the list of graphics primitives to change the drawing color. The following ways to specify colors are supported:

`RGBColor`[r, g, b]
  specifies a color using red, green, and blue.
`CMYKColor`[c, m, y, k]
  specifies a color using cyan, magenta, yellow, and black.
`Hue`[h, s, b]
  specifies a color using hue, saturation, and brightness.
`GrayLevel`[l]
  specifies a color using a gray level.

All components range from 0 to 1. Each color function can be supplied with an additional argument specifying the desired opacity (“alpha”) of the color. There are many predefined colors, such as `Black`, `White`, `Red`, `Green`, `Blue`, etc.

```
>> Graphics[{Red, Disk[]}]
```

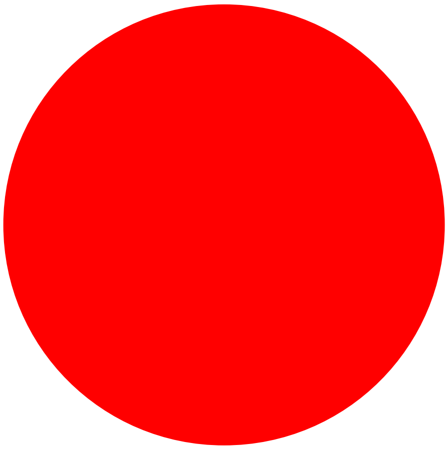
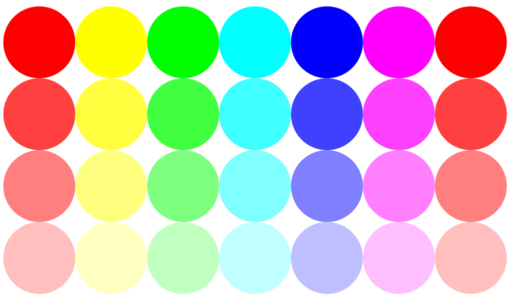


Table of hues:

```
>> Graphics[Table[{Hue[h, s], Disk[{12h, 8s}]}, {h, 0, 1, 1/6}, {s, 0,
   1, 1/4}]]
```



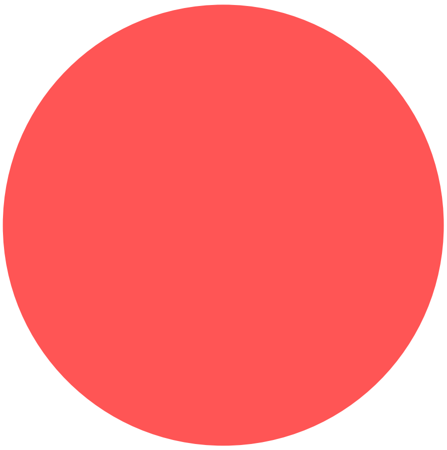
Colors can be mixed and altered using the following functions:

Blend[{*color*1, *color*2}, *ratio*]
: mixes *color*1 and *color*2 with *ratio*, where a ratio of 0 returns *color*1 and a ratio of 1 returns *color*2.

Lighter[*color*]
: makes *color* lighter (mixes it with White).

Darker[*color*]
: makes *color* darker (mixes it with Black).

```
>> Graphics[{Lighter[Red], Disk[]}]
```



Graphics produces a GraphicsBox:

```
>> Head[ToBoxes[Graphics[{Circle[]}]]]
```

GraphicsBox

2.13. 3D Graphics

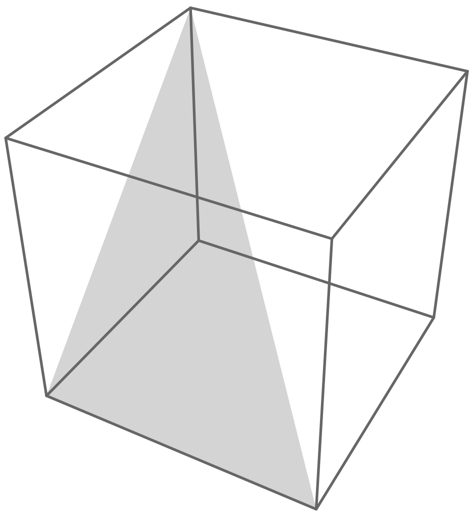
Three-dimensional graphics are created using the function Graphics3D and a list of 3D primitives. The following primitives are supported so far:

Polygon[{{x_1, y_1, z_1}, {x_2, y_2, z_3}, ...}]
: draws a filled polygon.

Line[{{x_1, y_1, z_1}, {x_2, y_2, z_3}, ...}]
: draws a line.

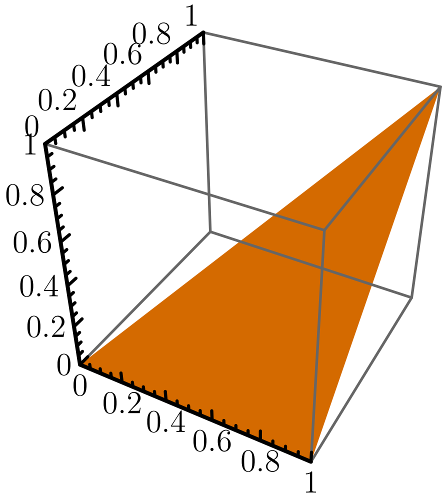
Point[{x_1, y_1, z_1}]
: draws a point.

```
>> Graphics3D[Polygon[{{0,0,0}, {0,1,1}, {1,0,0}}]]
```



Colors can also be added to three-dimensional primitives.

```
>> Graphics3D[{Orange, Polygon[{{0,0,0}, {1,1,1}, {1,0,0}}]}, Axes->True
]
```



`Graphics3D` produces a `Graphics3DBox`:

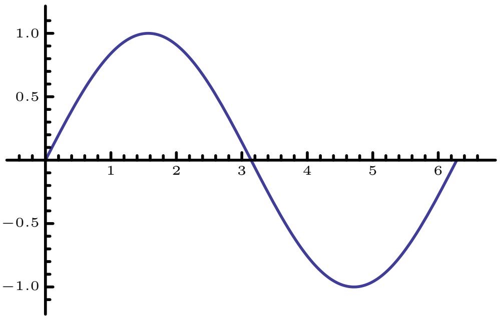
```
>> Head[ToBoxes[Graphics3D[{Polygon[]}]]]
```

Graphics3DBox

2.14. Plotting Introduction Examples

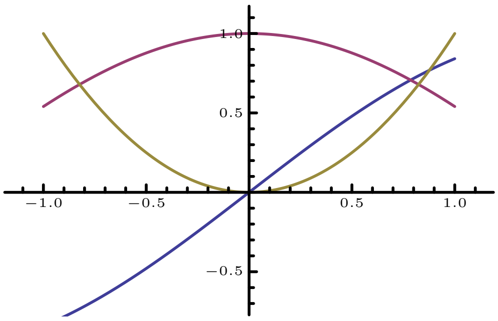
Mathics3 can plot functions:

```
>>  Plot[Sin[x], {x, 0, 2 Pi}]
```



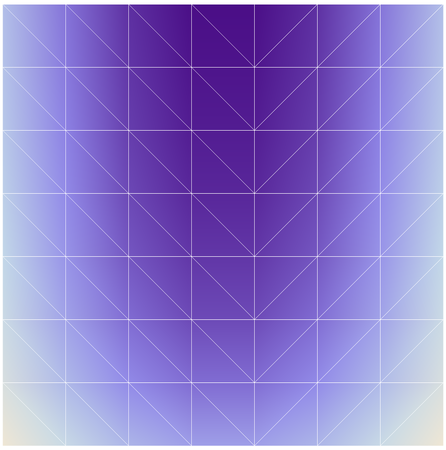
You can also plot multiple functions at once:

```
>>  Plot[{Sin[x], Cos[x], x ^ 2}, {x, -1, 1}]
```



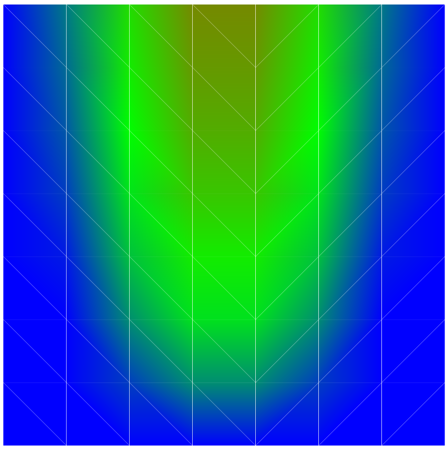
Two-dimensional functions can be plotted using `DensityPlot`:

```
>>  DensityPlot[x ^ 2 + 1 / y, {x, -1, 1}, {y, 1, 4}]
```



You can use a custom coloring function:

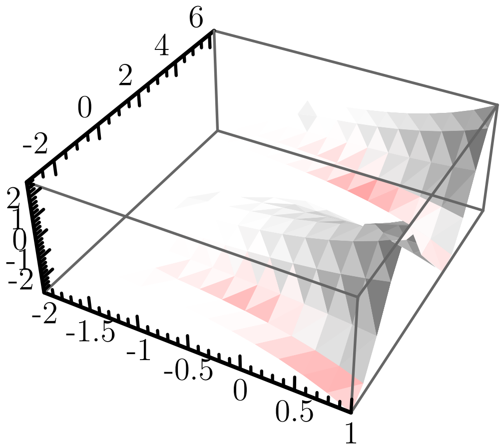
```
>> DensityPlot[x ^ 2 + 1 / y, {x, -1, 1}, {y, 1, 4}, ColorFunction -> (
   Blend[{Red, Green, Blue}, #]&)]
```



One problem with DensityPlot is that itś still very slow, basically due to function evaluation being pretty slow in general—and DensityPlot has to evaluate a lot of functions.

Three-dimensional plots are supported as well:

```
>> Plot3D[Exp[x] Cos[y], {x, -2, 1}, {y, -Pi, 2 Pi}]
```



3. Further Tutorial Examples

Here are some examples written by Jan Pöschko that come from the very first version.

Contents

3.1. Curve Sketching

Letś sketch the function

```
>> f[x_] := 4 x / (x ^ 2 + 3 x + 5)
```

The derivatives are:

```
>> {f'[x], f''[x], f'''[x]} // Together
```

$$\left\{\frac{-4\left(-5+x^2\right)}{\left(5+3x+x^2\right)^2},\frac{8\left(-15-15x+x^3\right)}{\left(5+3x+x^2\right)^3},\frac{-24\left(-20-60x-30x^2+x^4\right)}{\left(5+3x+x^2\right)^4}\right\}$$

To get the extreme values of `f`, compute the zeroes of the first derivatives:

```
>> extremes = Solve[f'[x] == 0, x]
```

$$\left\{\left\{x->-\sqrt{5}\right\},\left\{x->\sqrt{5}\right\}\right\}$$

And test the second derivative:

```
>> f''[x] /. extremes // N
```

$$\{1.65086,\ -0.064079\}$$

Thus, there is a local maximum at `x = Sqrt[5]` and a local minimum at `x = -Sqrt[5]`. Compute the inflection points numerically, chopping imaginary parts close to 0:

```
>> inflections = Solve[f''[x] == 0, x] // N // Chop
```

$$\{\{x->\ -1.0852\},\{x->\ -3.21463\},\{x->4.29983\}\}$$

Insert into the third derivative:

```
>> f'''[x] /. inflections
```
$\{-3.67683, 0.694905, 0.00671894\}$

Being different from 0, all three points are actual inflection points. `f` is not defined where its denominator is 0:

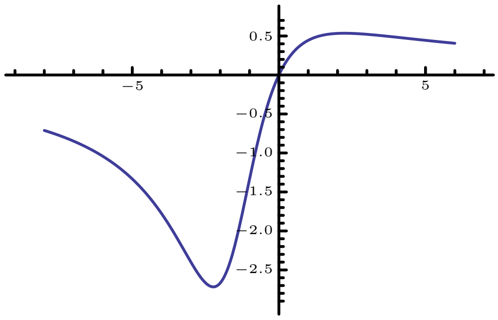
```
>> Solve[Denominator[f[x]] == 0, x]
```
$$\left\{\left\{x->-\frac{3}{2}-\frac{I}{2}\sqrt{11}\right\},\left\{x->-\frac{3}{2}+\frac{I}{2}\sqrt{11}\right\}\right\}$$

These are non-real numbers; consequently, `f` is defined on all real numbers. The behaviour of `f` at the boundaries of its definition:

```
>> Limit[f[x], x -> Infinity]
```
0

```
>> Limit[f[x], x -> -Infinity]
```
0

Finally, letś plot `f`:

```
>> Plot[f[x], {x, -8, 6}]
```


3.2. Linear algebra

Letś consider the matrix

```
>> A = {{1, 1, 0}, {1, 0, 1}, {0, 1, 1}};
```

```
>> MatrixForm[A]
```
$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

We can compute its eigenvalues and eigenvectors:

```
>> Eigenvalues[A]
```
$\{2, -1, 1\}$

```
>> Eigenvectors[A]
```
$\{\{1,1,1\},\{1,-2,1\},\{-1,0,1\}\}$

This yields the diagonalization of `A`:

```
>> T = Transpose[Eigenvectors[A]]; MatrixForm[T]
```
$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

```
>> Inverse[T] . A . T // MatrixForm
```
$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
>> % == DiagonalMatrix[Eigenvalues[A]]
```
True

We can solve linear systems:

```
>> LinearSolve[A, {1, 2, 3}]
```
$\{0,1,2\}$

```
>> A . %
```
$\{1,2,3\}$

In this case, the solution is unique:

```
>> NullSpace[A]
```
$\{\}$

Letś consider a singular matrix:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}};
```

```
>> MatrixRank[B]
```
2

```
>> s = LinearSolve[B, {1, 2, 3}]
```
$$\left\{-\frac{1}{3},\frac{2}{3},0\right\}$$

```
>> NullSpace[B]
```
$\{\{1,-2,1\}\}$

```
>> B . (RandomInteger[100] * %[[1]] + s)
```
$\{1,2,3\}$

3.3. Dice

Letś play with dice in this example. A `Dice` object shall represent the outcome of a series of rolling a dice with six faces, e.g.:

```
>> Dice[1, 6, 4, 4]
```

Dice [1, 6, 4, 4]

Like in most games, the ordering of the individual throws does not matter. We can express this by making `Dice` `Orderless`:

```
>> SetAttributes[Dice, Orderless]
```

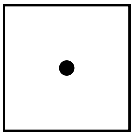
```
>> Dice[1, 6, 4, 4]
```

Dice [1, 4, 4, 6]

A dice object shall be displayed as a rectangle with the given number of points in it, positioned like on a traditional dice:

```
>> Format[Dice[n_Integer?(1 <= # <= 6 &)]] := Block[{p = 0.2, r = 0.05},
    Graphics[{EdgeForm[Black], White, Rectangle[], Black, EdgeForm[], If
   [OddQ[n], Disk[{0.5, 0.5}, r]], If[MemberQ[{2, 3, 4, 5, 6}, n], Disk
   [{p, p}, r]], If[MemberQ[{2, 3, 4, 5, 6}, n], Disk[{1 - p, 1 - p}, r
   ]], If[MemberQ[{4, 5, 6}, n], Disk[{p, 1 - p}, r]], If[MemberQ[{4, 5,
    6}, n], Disk[{1 - p, p}, r]], If[n === 6, {Disk[{p, 0.5}, r], Disk
   [{1 - p, 0.5}, r]}]}, ImageSize -> Tiny]]
```

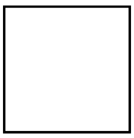
```
>> Dice[1]
```



The empty series of dice shall be displayed as an empty dice:

```
>> Format[Dice[]] := Graphics[{EdgeForm[Black], White, Rectangle[]},
   ImageSize -> Tiny]
```

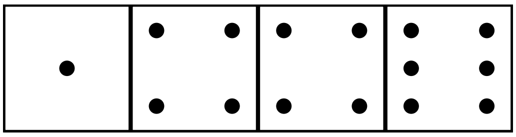
```
>> Dice[]
```



Any non-empty series of dice shall be displayed as a row of individual dice:

```
>> Format[Dice[d___Integer?(1 <= # <= 6 &)]] := Row[Dice /@ {d}]
```

```
>> Dice[1, 6, 4, 4]
```



Note that *Mathics3* will automatically sort the given format rules according to their "generality", so the rule for the empty dice does not get overridden by the rule for a series of dice. We can still see the original form by using `InputForm`:

```
>> Dice[1, 6, 4, 4] // InputForm
   Dice [1, 4, 4, 6]
```

We want to combine `Dice` objects using the + operator:

```
>> Dice[a___] + Dice[b___] ^:= Dice[Sequence @@ {a, b}]
```

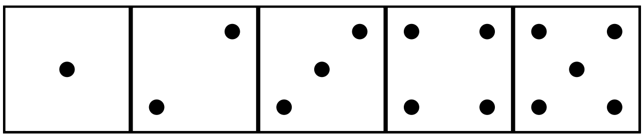
The `^:=` (`UpSetDelayed`) tells *Mathics3* to associate this rule with `Dice` instead of `Plus`.

`Plus` is protected—we would have to unprotect it first:

```
>> Dice[a___] + Dice[b___] := Dice[Sequence @@ {a, b}]
   Tag Plus in Dice[a___] + Dice[b___] is Protected.
   $Failed
```

We can now combine dice:

```
>> Dice[1, 5] + Dice[3, 2] + Dice[4]
```



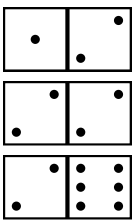
Letś write a function that returns the sum of the rolled dice:

```
>> DiceSum[Dice[d___]] := Plus @@ {d}
```

```
>> DiceSum @ Dice[1, 2, 5]
   8
```

And now letś put some dice into a table:

```
>> Table[{Dice[Sequence @@ d], DiceSum @ Dice[Sequence @@ d]}, {d, {{1,
   2}, {2, 2}, {2, 6}}}] // TableForm
```



3

4

8

It is not very sophisticated from a mathematical point of view, but itś beautiful.

4. Django-based Web Interface

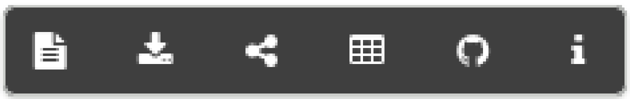
In the future, we plan on providing an interface to Jupyter as a separate package.

However, as part *Mathics3*, we currently distribute a browser-based interface using long-term-release (LTS) Django 4.

Since a Jupyter-based interface seems preferable to the home-grown interface described here, it is doubtful whether there will be future improvements to this interface.

When you enter Mathics in the top after the Mathics logo and the word "Mathics" you'll see a *menubar*.

It looks like this:



These save and load worksheets, share sessions, run a gallery of examples, go to the GitHub organization page, and provide information about the particular Mathics3 installation.

These are explained in the sections below.

Contents

4.1. URIs

For the most part, the application is a single-page application. Assuming you are running locally or on a host called `localhost` using the default port, 8000, here are some URLs and what they do:

`http://localhost:8000`
The single-page application; the main page.

`http://localhost:8000/about`
A page giving:

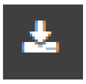
- the software versions of this package and the version information of important software this uses.
- directory path information for the current setup
- machine information
- system information
- customizable system settings

`http://localhost:8000/doc`
An online formatted version of the documentation, which includes this text. You can see this as a right-side frame of the main page, when clicking ”?” on the right-hand upper corner.

4.2. Saving, Loading, and Deleting Worksheets

<subsection title="Saving Worksheets">

Worksheets exist in the browser window only and are not stored on the server by default. To save all your queries and results, use the *Save* button, which is the middle graphic of the menu bar. It looks like this:



Depending on browser, desktop, and OS settings, the ”Ctrl+S” key combination may do the same thing.

<subsection title="Loading and Deleting Worksheets">

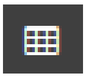
Saved worksheets can be loaded or deleted using the *FileOpen* button, which is the left-most button in the menu bar. It looks like this:

Depending on browser, desktop, and OS settings, the ”Ctrl+O” key combination may do the same thing.

A pop-up menu should appear with the list of saved worksheets, with an option to either load or delete the worksheet.

4.3. Gallery Examples

We have a number of examples showing off briefly some of the capabilities of the system. These are run when you hit the button that looks like this:



It is also shown in the pop-up text that appears when Mathics3 is first run.

4.4. Persistence of Mathics Definitions in a Session

When you use the Django-based Web interface of *Mathics3*, a browser session is created. Cookies have to be enabled to allow this. Your session holds a key that is used to access your definitions that are stored in a database on the server. As long as you do not clear the cookies in your browser, your definitions will remain even when you close and re-open the browser.

This implies that you should not store sensitive, private information in *Mathics3* variables when using the online Web interface. In addition to their values being stored in a database on the server, your queries might be saved for debugging purposes. However, the fact that they are transmitted over plain HTTP should make you aware that you should not transmit any sensitive information. When you want to do calculations with that kind of stuff, simply install *Mathics3* locally!

If you are using a public terminal, to erase all your definitions and close the browser window. When you use *Mathics3* in a browser, use the command `Quit[]` or its alias, `Exit[]`.

When you reload the current page in a browser using the default URL, e.g.,

```
http:localhost:8000
```

, all of the previous input and output disappears.

On the other hand, Definitions as described above do not, unless `Quit[]` or `Exit[]` is entered as described above.

If you want a URL that records the input entered, the *GenerateInputHash* button does this. The button looks like this:

For example, assuming you have a *Mathics3* server running at port 8000 on

```
localhost
```

, and you enter the URL

```
http://localhost:8000/\#cXVlcmllcz14
```

, you should see a single line of input containing `x` entered.

Of course, what the value of this is when evaluated depends on whether `x` has been previously defined.

4.5. Keyboard Commands

There are some keyboard commands you can use in the Django-based Web interface of *Mathics3*.

`Shift+Return`
: This evaluates the current cell (the most important one, for sure). On the right-hand side, you may also see an "=" button, which can be clicked to do the same thing.

`Ctrl+D`
: This moves the cursor over to the documentation pane on the right-hand side. From here you can perform a search for a pre-defined *Mathics3* function or symbol. Clicking on the "?" symbol on the right-hand side does the same thing.

`Ctrl+C`
: This moves the cursor back to the document code pane area where you type *Mathics3* expressions

`Ctrl+S`
: Save worksheet

`Ctrl+O`
: Open worksheet

`Right Click` on MathML output
: Opens MathJax Menu

Of special note is the last item on the list: right-click to open the MathJax menu. Under "Math Setting"/"Zoom Trigger", if the zoom trigger is set to a value other than "No Zoom", then when that trigger is applied to MathML-formatted output, the MathML formula pops up a window for the formula. The window can show the formula larger. Also, this is a way to see output that is too large to fit on the display since the window allows for scrolling.

Keyboard command behavior depends on the browser used, the operating system, desktop settings, and customization. We hook into the desktop "Open the current document" and "Save the current document" functions that many desktops provide. For example see: Finding keyboard shortcuts

Often, these shortcut keyboard commands are only recognized when a text field has focus; otherwise, the browser might do some browser-specific actions, like setting a bookmark, etc.

Part II.

Reference of Built-in Symbols

5. Arithmetic Functions

Arithmetic Functions are functions that work on individual numbers, lists, and arrays: in either symbolic or algebraic forms.

Contents

5.1. Basic Arithmetic

The functions here are the basic arithmetic operations that you might find on a calculator.

5.1.1. CubeRoot

Cube root (WMA)

`CubeRoot`[*n*]
finds the real-valued cube root of the given *n*.

```
>> CubeRoot[16]
```

$2 2^{\frac{1}{3}}$

5.1.2. Divide (÷)

Division (WMA link)

`Divide`[*a*, *b*]
a / *b*
represents the division of *a* by *b*.

>> `30 / 5`
6

>> `1 / 8`
$\frac{1}{8}$

>> `Pi / 4`
$\frac{\pi}{4}$

Use `N` or a decimal point to force numeric evaluation:

>> `Pi / 4.0`
0.785398

>> `1 / 8`
$\frac{1}{8}$

>> `N[%]`
0.125

Nested divisions:

>> `a / b / c`
$\frac{a}{bc}$

>> `a / (b / c)`
$\frac{ac}{b}$

>> `a / b / (c / (d / e))`
$\frac{ad}{bce}$

>> `a / (b ^ 2 * c ^ 3 / e)`
$\frac{ae}{b^2c^3}$

5.1.3. Minus (-)

Additive inverse (WMA)

`Minus[`*expr*`]`
: is the negation of *expr*.

>> `-a //FullForm`
Times [− 1, *a*]

`Minus` automatically distributes:

```
>> -(x - 2/3)
```

$\frac{2}{3} - x$

`Minus` threads over lists:

```
>> -Range[10]
```

$\{-1, -2, -3, -4, -5, -6, -7, -8, -9, -10\}$

5.1.4. Plus (+)

Addition (SymPy, WMA)

> `Plus`[a, b, ...]
> a + b + ...
> represents the sum of the terms a, b, ...

```
>> 1 + 2
```

3

`Plus` performs basic simplification of terms:

```
>> a + b + a
```

$2a + b$

```
>> a + a + 3 * a
```

$5a$

```
>> a + b + 4.5 + a + b + a + 2 + 1.5 b
```

$6.5 + 3a + 3.5b$

Apply `Plus` on a list to sum up its elements:

```
>> Plus @@ {2, 4, 6}
```

12

The sum of the first 1000 integers:

```
>> Plus @@ Range[1000]
```

500500

`Plus` has default value 0:

```
>> DefaultValues[Plus]
```

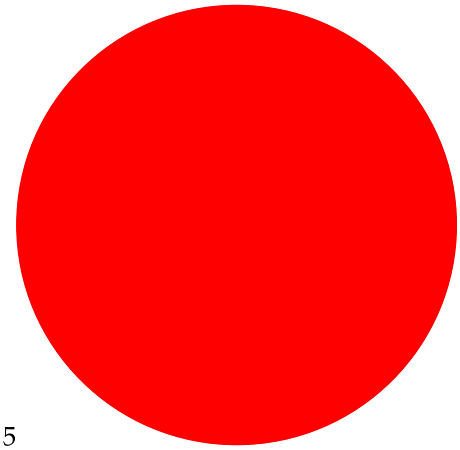
{HoldPattern [Default [Plus]] :>0}

```
>> a /. n_. + x_ :> {n, x}
```

$\{0, a\}$

The sum of 2 red circles and 3 red circles is...

```
>> 2 Graphics[{Red,Disk[]}] + 3 Graphics[{Red,Disk[]}]
```

5 

5.1.5. Power (^)

Exponentiation (SymPy, WMA)

`Power`[a, b]
a ^ b
    represents a raised to the power of b.

```
>> 4 ^ (1/2)
```

2

```
>> 4 ^ (1/3)
```

$2^{\frac{2}{3}}$

```
>> 3^123
```

48519278097689642681155855396759336072749841943521979872827

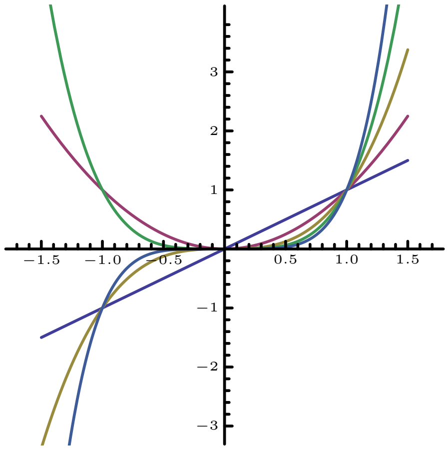
```
>> (y ^ 2)^ (1/2)
```

$\sqrt{y^2}$

```
>> (y ^ 2)^ 3
```

y^6

```
>> Plot[Evaluate[Table[x^y, {y, 1, 5}]], {x, -1.5, 1.5}, AspectRatio ->
   1]
```



Use a decimal point to force numeric evaluation:

```
>> 4.0 ^ (1/3)
```

1.5874

`Power` has default value 1 for its second argument:

```
>> DefaultValues[Power]
```

{HoldPattern [Default [Power, 2]] :>1}

```
>> a /. x_ ^ n_. :> {x, n}
```

$\{a, 1\}$

`Power` can be used with complex numbers:

```
>> (1.5 + 1.0 I)^ 3.5
```

$-3.68294 + 6.95139I$

```
>> (1.5 + 1.0 I)^ (3.5 + 1.5 I)
```

$-3.19182 + 0.645659I$

5.1.6. Sqrt

Square root (SymPy, WMA)

`Sqrt[`*expr*`]`
returns the square root of *expr*.

```
>> Sqrt[4]
```

2

>> `Sqrt[5]`

$\sqrt{5}$

>> `Sqrt[5] // N`

2.23607

>> `Sqrt[a]^2`

a

Complex numbers:

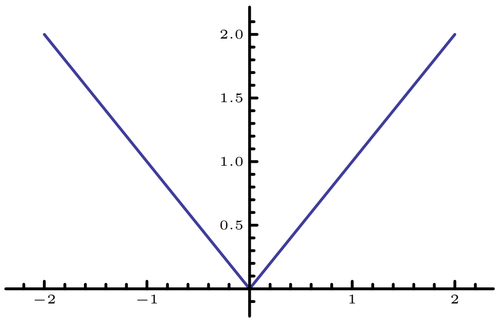
>> `Sqrt[-4]`

$2I$

>> `I == Sqrt[-1]`

True

>> `Plot[Sqrt[a^2], {a, -2, 2}]`



5.1.7. Subtract

Subtraction, (WMA)

> `Subtract`[a, b]
> a - b
> represents the subtraction of b from a.

>> `5 - 3`

2

>> `a - b // FullForm`

Plus [a, Times [$-$ 1, b]]

>> `a - b - c`

$a - b - c$

>> `a - (b - c)`

$a - b + c$

5.1.8. Times (×)

Multiplication (SymPy, WMA)

`Times`[a, b, ...]
$a * b * ...$
a b ...
  represents the product of the terms a, b, ...

```
>> 10 * 2
```
20

```
>> 10 2
```
20

```
>> a * a
```
a^2

```
>> x ^ 10 * x ^ -2
```
x^8

```
>> {1, 2, 3} * 4
```
$\{4,8,12\}$

```
>> Times @@ {1, 2, 3, 4}
```
24

```
>> IntegerLength[Times@@Range[5000]]
```
16326

`Times` has default value 1:

```
>> DefaultValues[Times]
```
{HoldPattern [Default [Times]] :>1}

```
>> a /. n_. * x_ :> {n, x}
```
$\{1,a\}$

5.2. Sums, Simple Statistics

These functions perform a simple arithmetic computation over a list.

5.2.1. Accumulate

WMA link

`Accumulate`[*list*]
  accumulates the values of *list*, returning a new list.

```
>> Accumulate[{1, 2, 3}]
```
$\{1,3,6\}$

5.2.2. Total

WMA link

`Total`[*list*]
: adds all values in *list*.

`Total`[*list*, *n*]
: adds all values up to level *n*.

`Total`[*list*, {*n*}]
: totals only the values at level {*n*}.

`Total`[*list*, {n_1, n_2}]
: totals at levels {n_1, n_2}.

```
>> Total[{1, 2, 3}]
```
6

```
>> Total[{{1, 2, 3}, {4, 5, 6}, {7, 8 ,9}}]
```
$\{12,15,18\}$

Total over rows and columns

```
>> Total[{{1, 2, 3}, {4, 5, 6}, {7, 8 ,9}}, 2]
```
45

Total over rows instead of columns

```
>> Total[{{1, 2, 3}, {4, 5, 6}, {7, 8 ,9}}, {2}]
```
$\{6,15,24\}$

6. Assignments

Assignments allow you to set or clear variables, indexed variables, structure elements, functions, and general transformations.

You can also get assignment and documentation information about symbols.

Contents

6.1. Clearing Assignments

6.1.1. Clear

WMA link

`Clear`[$symb_1$, $symb_2$, ...]
: clears all values of the given symbols. The arguments can also be given as strings containing symbol names.

```
>> x = 2;
```

```
>> Clear[x]
```

```
>> x
```

x

```
>> x = 2;
```

```
>> y = 3;
```

```
>> Clear["Global`*"]
```

```
>> x
```

x

```
>> y
```

y

`ClearAll` may not be called for `Protected` symbols.

```
>> Clear[Sin]
Symbol Sin is Protected.
```

The values and rules associated with built-in symbols will not get lost when applying `Clear` (after unprotecting them):

```
>> Unprotect[Sin]
```

```
>> Clear[Sin]
```

```
>> Sin[Pi]
```

0

`Clear` does not remove attributes, messages, options, and default values associated with the symbols. Use `ClearAll` to do so.

```
>> Attributes[r] = {Flat, Orderless};
```

```
>> Clear["r"]
```

```
>> Attributes[r]
```

{Flat, Orderless}

6.1.2. ClearAll

WMA link

`ClearAll`[$symb_1$, $symb_2$, ...]
: clears all values, attributes, messages and options associated with the given symbols. The arguments can also be given as strings containing symbol names.

```
>> x = 2;
```

```
>> ClearAll[x]
```

```
>> x
```

x

```
>> Attributes[r] = {Flat, Orderless};

>> ClearAll[r]

>> Attributes[r]
   {}
```

`ClearAll` may not be called for `Protected` or `Locked` symbols.

```
>> Attributes[lock] = {Locked};

>> ClearAll[lock]
   Symbol lock is locked.
```

6.1.3. Remove

WMA link

`Remove[x]`
    removes the definition associated to x.

```
>> a := 2

>> Names["Global`a"]
   {a}

>> Remove[a]

>> Names["Global`a"]
   {}
```

6.1.4. Unset (=.)

WMA link

`Unset[x]`
`$x$=.`
    removes any value belonging to x.

```
>> a = 2
   2

>> a =.

>> a
   a
```

Unsetting an already unset or never defined variable will not change anything:

```
>> a =.
```

```
>> b =.
```

`Unset` can unset particular function values. It will print a message if no corresponding rule is found.

```
>> f[x_] =.
   Assignment on f for f[x_] not found.
```
$Failed

```
>> f[x_] := x ^ 2
```

```
>> f[3]
```
9

```
>> f[x_] =.
```

```
>> f[3]
```
f[3]

You can also unset `OwnValues`, `DownValues`, `SubValues`, and `UpValues` directly. This is equivalent to setting them to `{}`.

```
>> f[x_] = x; f[0] = 1;
```

```
>> DownValues[f] =.
```

```
>> f[2]
```
f[2]

`Unset` threads over lists:

```
>> a = b = 3;
```

```
>> {a, {b}} =.
```
{Null, {Null}}

6.2. Forms of Assignment

6.2.1. LoadModule

`LoadModule[`*module*`]`
'Load Mathics definitions from the python module *module*

```
>> LoadModule["nomodule"]
   Python import errors with: No module named 'nomodule'.
```
$Failed

```
>> LoadModule["sys"]
   Python module "sys" is not a Mathics3 module.
```
$Failed

6.2.2. Set (=)

WMA link

`Set`[*expr*, *value*]
expr = *value*
    evaluates *value* and assigns it to *expr*.
$\{s_1, s_2, s_3\} = \{v_1, v_2, v_3\}$
    sets multiple symbols ($s_1, s_2, ...$) to the corresponding values ($v_1, v_2, ...$).

`Set` can be used to give a symbol a value:

```
>> a = 3
```
3

```
>> a
```
3

An assignment like this creates an ownvalue:

```
>> OwnValues[a]
```
{HoldPattern [*a*] :>3}

You can set multiple values at once using lists:

```
>> {a, b, c} = {10, 2, 3}
```
{10, 2, 3}

```
>> {a, b, {c, {d}}} = {1, 2, {{c1, c2}, {a}}}
```
{1, 2, {{c1, c2} , {10}}}

```
>> d
```
10

`Set` evaluates its right-hand side immediately and assigns it to the left-hand side:

```
>> a
```
1

```
>> x = a
```
1

```
>> a = 2
```
2

```
>> x
```
1

`Set` always returns the right-hand side, which you can again use in an assignment:

```
>> a = b = c = 2;
```

```
>> a == b == c == 2
```
True

`Set` supports assignments to parts:

```
>> A = {{1, 2}, {3, 4}};
```

```
>> A[[1, 2]] = 5
```
5

```
>> A
```
$\{\{1,5\},\{3,4\}\}$

```
>> A[[;;, 2]] = {6, 7}
```
$\{6,7\}$

```
>> A
```
$\{\{1,6\},\{3,7\}\}$

Set a submatrix:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}};
```

```
>> B[[1;;2, 2;;-1]] = {{t, u}, {y, z}};
```

```
>> B
```
$\{\{1,t,u\},\{4,y,z\},\{7,8,9\}\}$

6.2.3. SetDelayed (:=)

WMA link

> `SetDelayed`[*expr*, *value*]
> *expr* := *value*
> assigns *value* to *expr*, without evaluating *value*.

`SetDelayed` is like `Set`, except it has attribute `HoldAll`, thus it does not evaluate the right-hand side immediately, but evaluates it when needed.

```
>> Attributes[SetDelayed]
```
{HoldAll, Protected, SequenceHold}

```
>> a = 1
```
1

```
>> x := a
```

```
>> x
   1
```

Changing the value of a affects x:

```
>> a = 2
   2
```

```
>> x
   2
```

`Condition` (`/;`) can be used with `SetDelayed` to make an assignment that only holds if a condition is satisfied:

```
>> f[x_] := p[x] /; x>0
```

```
>> f[3]
```

$p[3]$

```
>> f[-3]
```

$f[-3]$

It also works if the condition is set in the LHS:

```
>> F[x_, y_] /; x < y /; x>0 := x / y;
```

```
>> F[x_, y_] := y / x;
```

```
>> F[2, 3]
```

$\frac{2}{3}$

```
>> F[3, 2]
```

$\frac{2}{3}$

```
>> F[-3, 2]
```

$-\frac{2}{3}$

We can use conditional delayed assignments to define symbols with values conditioned to the context. For example,

```
>> ClearAll[a,b]; a/; b>0:= 3
```

Set a to have a value of 3 if certain variable b is positive. So, if this variable is not set, a stays unevaluated:

```
>> a
```

a

If now we assign a positive value to b, then a is evaluated:

```
>> b=2; a
   3
```

6.2.4. TagSet

WMA link

`TagSet`[f, *expr*, *value*]
f /: *expr* = *value*
    assigns *value* to *expr*, associating the corresponding assignment with the symbol f.

Create an upvalue without using `UpSet`:

```
>> square /: area[square[s_]] := s^2
```

```
>> DownValues[square]
```
{}

```
>> UpValues[square]
```
{HoldPattern [area [square [s_]]] :> s^2}

The symbol f must appear as the ultimate head of *lhs* or as the head of an element in *lhs*:

```
>> x /: f[g[x]] = 3;
Tag x not found or too deep for an assigned rule.
```

```
>> g /: f[g[x]] = 3;
```

```
>> f[g[x]]
```
3

6.2.5. TagSetDelayed

WMA link

`TagSetDelayed`[f, *expr*, *value*]
`$f$ /: $expr$ := $value$`
    is the delayed version of `TagSet`.

6.2.6. UpSet (^=)

WMA link

f[x] ^= *expression*
    evaluates *expression* and assigns it to the value of $f[x]$, associating the value with x.

`UpSet` creates an upvalue:

```
>> a[b] ^= 3;
```

```
>> DownValues[a]
```
{}

```
>> UpValues[b]
```
{HoldPattern [a [b]] :>3}

You can use `UpSet` to specify special values like format values. However, these values will not be saved in `UpValues`:

```
>> Format[r] ^= "custom";
```

```
>> r
```
custom

```
>> UpValues[r]
```
{}

6.2.7. UpSetDelayed (`^:=`)

WMA link

> `UpSetDelayed[`*expression*, *value*`]`
> *expression* `^:=` *value*
>
> assigns *expression* to the value of $f[x]$ (without evaluating *expression*), associating the value with x.

```
>> a[b] ^:= x
```

```
>> x = 2;
```

```
>> a[b]
```
2

```
>> UpValues[b]
```
{HoldPattern [a [b]] :>x}

6.3. In-place binary assignment operator

There are a number operators and functions that combine assignment with some sort of binary operator.

Sometimes a value is returned *before* the assignment occurs. When there is an operator for this, the operator is a prefix operator and the function name starts with `Pre`.

Sometimes the binary operation occurs first, and *then* the assignment occurs. When there is an operator for this, the operator is a postfix operator.

Infix operators combined with assignment end in `By`, `From`, or `To`.

6.3.1. AddTo (+=)

WMA link

`AddTo`[x, dx]
x += dx
    is equivalent to x = x + dx.

```
>>  a = 10;
```

```
>>  a += 2
```

12

```
>>  a
```

12

6.3.2. Decrement (--)

WMA link

`Decrement`[x]
x--
    decrements x by 1, returning the original value of x.

```
>>  a = 5; a--
```

5

```
>>  a
```

4

Decrement a numerical value:

```
>>  a = 1.6; a--; a
```

0.6

Decrement all values in a list:

```
>>  a = {1, 3, 5}
```

$\{1,3,5\}$

```
>>  a--; a
```

$\{0,2,4\}$

Compare with PreDecrement 6.3.5 which returns the value before updating, and Increment 6.3.4 which goes the other way.

6.3.3. DivideBy (/=)

WMA link

`DivideBy`[x, dx]
x /= dx
    is equivalent to $x = x \ / \ dx$.

```
>> a = 10;
```

```
>> a /= 2
```
5

```
>> a
```
5

6.3.4. Increment

WMA link

`Increment`[x]
x++
    increments x by 1, returning the original value of x.

```
>> a = 1; a++
```
1

```
>> a
```
2

Increment a numeric value:

```
>> a = 1.5; a++
```
1.5

```
>> a
```
2.5

Increment a symbolic value:

```
>> y = 2 x; y++; y
```
$1 + 2x$

Increment all values in a list:

```
>> x = {1, 3, 5}
```
$\{1,3,5\}$

x++; x = {2, 4, 6}

Grouping of `Increment`, `PreIncrement` and `Plus`:

```
>> ++++a+++++2//Hold//FullForm
```
Hold [Plus [PreIncrement [PreIncrement [Increment [Increment [*a*]]]] , 2]]

Compare with PreIncrement 6.3.6 which returns the value before update.

6.3.5. PreDecrement

WMA link

`PreDecrement[`*x*`]`
`--`*x*
: decrements *x* by 1, returning the new value of *x*.

`--`*a* is equivalent to *a* = *a* `- 1`:

```
>> a = 2;
```

```
>> --a
```
1

```
>> a
```
1

Compare with Decrement 6.3.2 which returns the updated value, and Increment 6.3.4 which goes the other way.

6.3.6. PreIncrement (++)

WMA link

`PreIncrement[`*x*`]`
`++`*x*
: increments *x* by 1, returning the new value of *x*.

`++`*a* is equivalent to *a* = *a* `+ 1`:

```
>> a = 2
```
2

```
>> ++a
```
3

```
>> a
```
3

PreIncrement a numeric value:

```
>> a + 1.6
```
4.6

PreIncrement a symbolic value:

```
>> y = x; ++y
```
$1 + x$

```
>> y
```
$1 + x$

Compare with Increment 6.3.4 which returns the updated value, and PreDecrement 6.3.5 which goes the other way.

6.3.7. SubtractFrom (-=)

WMA link

`SubtractFrom`[x, dx]
x -= dx
    is equivalent to $x = x - dx$.

```
>> a = 10;
```

```
>> a -= 2
```
8

```
>> a
```
8

6.3.8. TimesBy (*=)

WMA link

`TimesBy`[x, dx]
x *= dx
    is equivalent to $x = x * dx$.

```
>> a = 10;
```

```
>> a *= 2
```
20

```
>> a
```
20

6.4. Types of Values

6.4.1. DefaultValues

WMA link

`DefaultValues`[*symbol*]
    gives the list of default values associated with *symbol*.
Note: this function is in Mathematica 5 but has been removed from current Mathematica.

```
>> Default[f, 1] = 4
```
4

```
>> DefaultValues[f]
```
$\{\text{HoldPattern}\left[\text{Default}\left[f,1\right]\right]:>4\}$

You can assign values to `DefaultValues`:

```
>> DefaultValues[g] = {Default[g] -> 3};
```

```
>> Default[g, 1]
```
3

```
>> g[x_.] := {x}
```

```
>> g[a]
```
$\{a\}$

```
>> g[]
```
$\{3\}$

6.4.2. Messages

WMA link

`Messages`[*symbol*]
    gives the list of messages associated with *symbol*.

```
>> a::b = "foo"
```
foo

```
>> Messages[a]
```
$\{\text{HoldPattern}[a\text{::b}]:>\text{foo}\}$

```
>> Messages[a] = {a::c :> "bar"};
```

```
>> a::c // InputForm
```
"bar"

```
>> Message[a::c]
bar
```

6.4.3. NValues

NValues[*symbol*]
gives the list of numerical values associated with *symbol*.
Note: this function is in Mathematica 5 but has been removed from current Mathematica.

```
>> NValues[a]
{}
```

```
>> N[a] = 3;
```

```
>> NValues[a]
{HoldPattern[N[a, MachinePrecision]] :>3}
```

You can assign values to `NValues`:

```
>> NValues[b] := {N[b, MachinePrecision] :> 2}
```

```
>> N[b]
2.
```

Be sure to use `SetDelayed`, otherwise the left-hand side of the transformation rule will be evaluated immediately, causing the head of `N` to get lost. Furthermore, you have to include the precision in the rules; `MachinePrecision` will not be inserted automatically:

```
>> NValues[c] := {N[c] :> 3}
```

```
>> N[c]
c
```

Mathics will assign any list of rules to `NValues`; however, inappropriate rules will never be used:

```
>> NValues[d] = {foo -> bar};
```

```
>> NValues[d]
{HoldPattern[foo] :>bar}
```

```
>> N[d]
d
```

6.4.4. SubValues

WMA link

`SubValues[`*symbol*`]`
: gives the list of subvalues associated with *symbol*.

Note: this function is not in current Mathematica.

```
>> f[1][x_] := x
```

```
>> f[2][x_] := x ^ 2
```

```
>> SubValues[f]
```

$\left\{\text{HoldPattern}\left[f[2][\text{x\_}]\right] :> x^2, \text{HoldPattern}\left[f[1][\text{x\_}]\right] :> x\right\}$

```
>> Definition[f]
```

$$f[2][\text{x\_}] = x^2$$
$$f[1][\text{x\_}] = x$$

6.5. UpValue-related assignments

An *UpValue* is a definition associated with a symbols that does not appear directly its head.

See Associating Definitions with Different Symbols.

6.5.1. UpValues

WMA link

`UpValues[`*symbol*`]`
: gives the list of transformation rules corresponding to upvalues define with *symbol*.

```
>> a + b ^= 2
```

2

```
>> UpValues[a]
```

$\{\text{HoldPattern}[a + b] :> 2\}$

```
>> UpValues[b]
```

$\{\text{HoldPattern}[a + b] :> 2\}$

You can assign values to `UpValues`:

```
>> UpValues[pi] := {Sin[pi] :> 0}
```

```
>> Sin[pi]
```

0

7. Atomic Elements of Expressions

Expressions are ultimately built from a small number of distinct types of atomic elements.

Contents

7.1. Atomic Primitives

7.1.1. AtomQ

WMA link

`AtomQ[`*expr*`]`
: returns `True` if *expr* is an expression which cannot be divided into subexpressions, or `False` otherwise.

An expression that cannot be divided into subparts is called called an "atom".

Strings and expressions that produce strings are atoms:

```
>> Map[AtomQ, {"x", "x" <> "y", StringReverse["live"]}]
```
{True, True, True}

Numeric literals are atoms:

```
>> Map[AtomQ, {2, 2.1, 1/2, 2 + I, 2^^101}]
```
{True, True, True, True, True}

So are Mathematical Constants:

```
>> Map[AtomQ, {Pi, E, I, Degree}]
```
{True, True, True, True}

A `Symbol` not bound to a value is an atom too:

```
>> AtomQ[x]
```
True

On the other hand, expressions with more than one `Part` after evaluation, even those resulting in numeric values, aren't atoms:

```
>> AtomQ[2 + Pi]
```
False

Similarly any compound `Expression`, even lists of literals, aren't atoms:

```
>> Map[AtomQ, {{}, {1}, {2, 3, 4}}]
```
{False, False, False}

Note that evaluation or the binding of "x" to an expression is taken into account:

```
>> x = 2 + Pi; AtomQ[x]
```
False

Again, note that the expression evaluation to a number occurs before `AtomQ` evaluated:

```
>> AtomQ[2 + 3.1415]
```
True

7.1.2. Head

WMA link

`Head[`*expr*`]`
  returns the head of the expression or atom *expr*.

```
>> Head[a * b]
```
Times

```
>> Head[6]
```
Integer

```
>> Head[x]
```
Symbol

7.2. Representation of Numbers

Integers and Real numbers with any number of digits, automatically tagging numerical precision when appropriate.

Precision is not "guarded" through the evaluation process. Only integer precision is supported.

However, things like `N[Pi, 100]` should work as expected.

7.2.1. Accuracy

Accuracy (WMA Accuracy)

`Accuracy[`*x*`]`
: examines the number of significant digits of *expr* after the decimal point in the number x.

Notice that the result could be slightly different from the result obtained in WMA, due to differences in the internal representation of the real numbers.

`Accuracy` of a real number is estimated from its value and its precision:

```
>> Accuracy[3.1416`2]
```
1.50298

Notice that the value is not exactly equal to the obtained in WMA: This is due to the different way in which `Precision` is handled in SymPy.

Accuracy for exact atoms is `Infinity`:

```
>> Accuracy[1]
```
∞

```
>> Accuracy[A]
```
∞

For Complex numbers, the accuracy is estimated as (minus) the base-10 log of the square root of the squares of the errors on the real and complex parts:

```
>> z=Complex[3.00``2, 4.00``2];
```

```
>> Accuracy[z] == -Log[10, Sqrt[10^(-2 Accuracy[Re[z]])+ 10^(-2 Accuracy
   [Im[z]])]]
```
True

Accuracy of expressions is given by the minimum accuracy of its elements:

```
>> Accuracy[F[1, Pi, A]]
```
∞

```
>> Accuracy[F[1.3, Pi, A]]
```
15.8406

`Accuracy` for the value 0 is a fixed-precision Real number:

```
>> 0``2
```
0.00

```
>> Accuracy[0.``2]
```
2.

For 0.‘, the accuracy satisfies:

```
>> Accuracy[0.`] == $MachinePrecision - Log[10, $MinMachineNumber]
```
True

In compound expressions, the `Accuracy` is fixed by the number with the lowest `Accuracy`:

```
>> Accuracy[{{1, 1.`},{1.``5, 1.``10}}]
```
5.

See also ’Precision’ 7.2.10.

7.2.2. IntegerExponent

WMA link

`IntegerExponent[`*n*, *b*`]`
    gives the highest exponent of *b* that divides *n*.

```
>> IntegerExponent[16, 2]
```
4

```
>> IntegerExponent[-510000]
```
4

```
>> IntegerExponent[10, b]
```
IntegerExponent[10, *b*]

7.2.3. IntegerLength

WMA link

`IntegerLength`[x]
: gives the number of digits in the base-10 representation of x.

`IntegerLength`[x, b]
: gives the number of base-b digits in x.

```
>> IntegerLength[123456]
```
6

```
>> IntegerLength[10^10000]
```
10001

```
>> IntegerLength[-10^1000]
```
1001

`IntegerLength` with base 2:

```
>> IntegerLength[8, 2]
```
4

Check that `IntegerLength` is correct for the first 100 powers of 10:

```
>> IntegerLength /@ (10 ^ Range[100])== Range[2, 101]
```
True

The base must be greater than 1:

```
>> IntegerLength[3, -2]
Base -2 is not an integer greater than 1.
```
IntegerLength [3, −2]

`0` is a special case:

```
>> IntegerLength[0]
```
0

7.2.4. $MachineEpsilon

WMA link

`$MachineEpsilon`
: is the distance between `1.0` and the next nearest representable machine-precision number.

```
>> $MachineEpsilon
```
2.22045*^ − 16

```
>> x = 1.0 + {0.4, 0.5, 0.6} $MachineEpsilon;
```

```
>> x - 1
```
{0., 0., 2.22045*^ − 16}

7.2.5. MachinePrecision

WMA link

`MachinePrecision`
: represents the precision of machine precision numbers.

```
>> N[MachinePrecision]
```
15.9546

```
>> N[MachinePrecision, 30]
```
15.9545897701910033463281614204

7.2.6. $MachinePrecision

WMA link

`$MachinePrecision`
: is the number of decimal digits of precision for machine-precision numbers.

```
>> $MachinePrecision
```
15.9546

7.2.7. $MaxPrecision

WMA link

`$MaxPrecision`
: represents the maximum number of digits of precision permitted in abitrary-precision numbers.

```
>> $MaxPrecision
```
∞

```
>> $MaxPrecision = 10;
```

```
>> N[Pi, 11]
Requested precision 11 is larger than $MaxPrecision. Using current
$MaxPrecision of 10. instead. $MaxPrecision = Infinity specifies that
any precision should be allowed.
```
3.141592654

7.2.8. $MinPrecision

WMA link

`$MinPrecision`
: represents the minimum number of digits of precision permitted in abitrary-precision numbers.

```
>> $MinPrecision
```
0

```
>> $MinPrecision = 10;
```

```
>> N[Pi, 9]
Requested precision 9 is smaller than $MinPrecision. Using current
$MinPrecision of 10. instead.
```
3.141592654

7.2.9. NumberDigit

WMA link

`NumberDigit`[x, n]
: returns the digit coefficient of 10^n for the real-valued number x.

`NumberDigit`[x, n, b]
: returns the coefficient of b^n in the base-b representation of x.

Get the 10^2 digit of a 210.345:

```
>> NumberDigit[210.345, 2]
```
2

Get the 10^-1 digit of a 210.345:

```
>> NumberDigit[210.345, -1]
```
3

```
>> BaseForm[N[Pi], 2]
```
SubscriptBox [11.00100100001111110, 2]

Get the 2^0 bit of the Pi: = 1

7.2.10. Precision

Precision WMA link

`Precision[`*expr*`]`
: examines the number of significant digits of *expr*.

Note that the result could be slightly different than the obtained in WMA, due to differences in the internal representation of the real numbers.

The precision of an exact number, e.g., an Integer, is `Infinity`:

```
>> Precision[1]
   ∞
```

A fraction is an exact number too, so its Precision is `Infinity`:

```
>> Precision[1/2]
   ∞
```

Numbers entered in the form *digits*'*p* are taken to have precision *p*:

```
>> Precision[1.23`10]
   10.
```

Precision of a machine-precision number is `MachinePrecision`:

```
>> Precision[0.5]
   MachinePrecision
```

In compound expressions, the `Precision` is fixed by the number with the lowest `Precision`:

```
>> Precision[{{1, 1.`},{1.`5, 1.`10}}]
   5.
```

In general, `Accuracy[`*z*`]` == `Precision[`*z*`]` + `Log[`*z*`]` for non-zero Real values:

```
>> (Accuracy[z] == Precision[z] + Log[z])/.z-> 37.`
   True
```

Following WMA, values in Machine Real representation starting with `0.` are values are special:

```
>> Precision[0.]
   MachinePrecision
```

On the other hand, for a Precision Real with fixed accuracy, the precision is evaluated to `0.`:

```
>> Precision[0.``3]
   0.
```

See also 'Accuracy' 7.2.1.

7.2.11. RealDigits

WMA link

`RealDigits[`n`]`
: returns the decimal representation for the real number n as list of digits, together with the number of digits that are to the left of the decimal point.

`RealDigits[`n, b`]`
: returns a list of the "digits" in base-b representation for the real number n.

`RealDigits[`n, b, len`]`
: returns a list of len digits.

`RealDigits[`n, b, len, p`]`
: return len digits starting with the coefficient of b^p.

Return the list of digits and exponent:

```
>> RealDigits[123.55555]
```
{{1,2,3,5,5,5,5,5,0,0,0,0,0,0,0,0},3}

Return an explicit recurring decimal form:

```
>> RealDigits[19 / 7]
```
{{2,{7,1,4,2,8,5}},1}

The 500th digit of Pi is 2:

```
>> RealDigits[Pi, 10, 1, -500]
```
{{2}, − 499}

11 digits starting with the coefficient of 10^-3:

```
>> RealDigits[Pi, 10, 11, -3]
```
{{1,5,9,2,6,5,3,5,8,9,7},−2}

RealDigits gives Indeterminate if more digits than the precision are requested:

```
>> RealDigits[123.45, 10, 18]
```
{{1,2,3,4,5,0,0,0,0,0,0,0,0,0,0,0,Indeterminate,Indeterminate},3}

Return 25 digits of in base 10:

```
>> RealDigits[Pi, 10, 25]
```
{{3,1,4,1,5,9,2,6,5,3,5,8,9,7,9,3,2,3,8,4,6,2,6,4,3},1}

```
>> RealDigits[10]
```
{{1,0},2}

7.3. String Manipulation

7.3.1. Alphabet

WMA link

`Alphabet[]`
    gives the list of lowercase letters a-z in the English alphabet .
`Alphabet[`*type*`]`
    gives the alphabet for the language or class *type*.

```
>> Alphabet[]
```
$\{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

```
>> Alphabet["German"]
```
{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, ß, ä, ö, ü}

7.3.2. $CharacterEncoding

WMA link

`$CharacterEncoding`
    specifies the default raw character encoding to use for input and output when no encoding is explicitly specified. Initially this is set to `$SystemCharacterEncoding`.

See the character encoding current is in effect and used in input and output functions functions like `OpenRead[]`:

```
>> $CharacterEncoding
```
ASCII

By setting its value to one of the values in `$CharacterEncodings`, operators are formatted differently. For example,

```
>> $CharacterEncoding = "ASCII"; a -> b
```
$a- > b$

```
>> $CharacterEncoding = "UTF-8"; a -> b
```
$a \to b$

Setting its value to `None` restore the value to `$SystemCharacterEncoding`:

```
>> $CharacterEncoding = None;
```

```
>> $SystemCharacterEncoding == $CharacterEncoding
```
True

See also $SystemCharacterEncoding 7.3.11.

7.3.3. $CharacterEncodings

WMA link

`$CharacterEncodings`
 stores the list of available character encodings.

```
>> $CharacterEncodings[[;;9]]
```
{ASCII, CP949, CP950, EUC-JP, IBM- 850, ISOLatin1, ISOLatin2, ISOLatin3, ISOLatin4}

7.3.4. HexadecimalCharacter

WMA link

`HexadecimalCharacter`
 represents the characters 0-9, a-f and A-F.

```
>> StringMatchQ[#, HexadecimalCharacter] & /@ {"a", "1", "A", "x", "H",
   " ", "."}
```
{True, True, True, False, False, False, False}

7.3.5. LetterNumber

WMA link

`LetterNumber[`c`]`
 returns the position of the character c in the English alphabet.

`LetterNumber[``string']'`
 returns a list of the positions of characters in string.

`LetterNumber[`“string”, $alpha$`]`
 returns a list of the positions of characters in string, regarding the alphabet $alpha$.

```
>> LetterNumber["b"]
```
2

LetterNumber also works with uppercase characters

```
>> LetterNumber["B"]
```
2

```
>> LetterNumber["ss2!"]
```
$\{19, 19, 0, 0\}$

Get positions of each of the letters in a string:

```
>> LetterNumber[Characters["Peccary"]]
```
$\{16, 5, 3, 3, 1, 18, 25\}$

```
>> LetterNumber[{"P", "Pe", "P1", "eck"}]
```
$\{16, \{16, 5\}, \{16, 0\}, \{5, 3, 11\}\}$

```
>> LetterNumber["\[Beta]", "Greek"]
```
2

7.3.6. NumberString

WMA link

`NumberString`
represents the characters in a number.

```
>> StringMatchQ["1234", NumberString]
```
True

```
>> StringMatchQ["1234.5", NumberString]
```
True

```
>> StringMatchQ["1.2`20", NumberString]
```
False

7.3.7. RemoveDiacritics

WMA link

`RemoveDiacritics[`*s*`]`
returns a version of *s* with all diacritics removed.

```
>> RemoveDiacritics["en prononccant p^echer et p'echer"]
```
en prononcant pecher et pecher

```
>> RemoveDiacritics["pi~nata"]
```
pinata

7.3.8. StringContainsQ

WMA link

`StringContainsQ["string", patt]`
: returns True if any part of *string* matches *patt*, and returns False otherwise.

`StringContainsQ[{``s1', ``s2'', ...}, patt]'`
: returns the list of results for each element of string list.

`StringContainsQ[patt]`
: represents an operator form of StringContainsQ that can be applied to an expression.

```
>> StringContainsQ["mathics", "m" ~~__ ~~"s"]
```
True

```
>> StringContainsQ["mathics", "a" ~~__ ~~"m"]
```
False

```
>> StringContainsQ[{"g", "a", "laxy", "universe", "sun"}, "u"]
```
{False, False, False, True, True}

```
>> StringContainsQ["e" ~~___ ~~"u"] /@ {"The Sun", "Mercury", "Venus", "
   Earth", "Mars", "Jupiter", "Saturn", "Uranus", "Neptune"}
```
{True, True, True, False, False, False, False, False, True}

7.3.9. StringRepeat

WMA link

`StringRepeat["string", n]`
: gives *string* repeated *n* times.

`StringRepeat["string", n, max]`
: gives *string* repeated *n* times, but not more than *max* characters.

```
>> StringRepeat["abc", 3]
```
abcabcabc

```
>> StringRepeat["abc", 10, 7]
```
abcabca

7.3.10. String

WMA link

`String`
: is the head of strings.

```
>> Head["abc"]
```
String

```
>> "abc"
```
abc

Use `InputForm` to display quotes around strings:

```
>> InputForm["abc"]
```
"abc"

`FullForm` also displays quotes:

```
>> FullForm["abc" + 2]
```
Plus [2, "abc"]

7.3.11. $SystemCharacterEncoding

WMA link

$SystemCharacterEncoding
    gives the default character encoding of the system.

On startup, the value of environment variable `MATHICS_CHARACTER_ENCODING` sets this value. However if that environment variable is not set, set the value is set in Python using `sys.getdefaultencoding()`.

```
>> $SystemCharacterEncoding
```
ASCII

7.3.12. ToExpression

WMA link

`ToExpression`[*input*]
    interprets a given string as Mathics3 input.

`ToExpression`[*input*, *form*]
    reads the given input in the specified *form*.

`ToExpression`[*input*, *form*, *h*]
    applies the head *h* to the expression before evaluating it.

```
>> ToExpression["1 + 2"]
```
3

```
>> ToExpression["{2, 3, 1}", InputForm, Max]
```
3

```
>> ToExpression["2 3", InputForm]
6
```

Note that newlines are like semicolons, not blanks. So so the return value is the second-line value.

```
>> ToExpression["2\[NewLine]3"]
3
```

7.3.13. ToString

WMA link

`ToString[`*expr*`]`
: returns a string representation of *expr*.

`ToString[`*expr*, *form*`]`
: returns a string representation of *expr* in the form *form*.

```
>> ToString[2]
2
>> ToString[2] // InputForm
"2"
>> ToString[a+b]
a + b
>> "U" <> 2
String expected.
U<>2
>> "U" <> ToString[2]
U2
>> ToString[Integrate[f[x],x], TeXForm]
\int f\left[x\right] \, dx
```

7.3.14. Transliterate

WMA link

`Transliterate[`*s*`]`
: transliterates a text in some script into an ASCII string.

ASCII translateration examples:

- Russian language

- Hiragana

7.3.15. Whitespace

WMA link

`Whitespace`
represents a sequence of whitespace characters.

```
>> StringMatchQ["\r \n", Whitespace]
```

True

```
>> StringSplit["a \n b \r\n c d", Whitespace]
```

$\{a, b, c, d\}$

```
>> StringReplace[" this has leading and trailing whitespace \n ", (
   StartOfString ~~Whitespace)| (Whitespace ~~EndOfString)-> ""] <> "
   removed" // FullForm
```

"this has leading and trailing whitespace removed"

7.4. Symbol Handling

Symbolic data. Every symbol has a unique name, exists in a certain context or namespace, and can have a variety of type of values and attributes.

7.4.1. Context

WMA link

`Context[`*symbol*`]`
yields the name of the context where *symbol* is defined in.
`Context[]`
returns the value of `$Context`.

```
>> Context[a]
```

Global'

```
>> Context[b`c]
```

b'

```
>> InputForm[Context[]]
```

"Global'"

7.4.2. Definition

WMA link

`Definition[`*symbol*`]`
    prints as the definitions given for *symbol*. This is in a form that can e stored in a package.

`Definition` does not print information for `ReadProtected` symbols. `Definition` uses `InputForm` to format values.

```
>> a = 2;
```

```
>> Definition[a]
```

$$a = 2$$

```
>> f[x_] := x ^ 2
```

```
>> g[f] ^:= 2
```

```
>> Definition[f]
```

$$f\,[\mathrm{x\_}] = x^2$$
$$g\,[f]\,{}^\wedge{=}2$$

Definition of a rather evolved (though meaningless) symbol:

```
>> Attributes[r] := {Orderless}
```

```
>> Format[r[args___]] := Infix[{args}, "~"]
```

```
>> N[r] := 3.5
```

```
>> Default[r, 1] := 2
```

```
>> r::msg := "My message"
```

```
>> Options[r] := {Opt -> 3}
```

```
>> r[arg_., OptionsPattern[r]] := {arg, OptionValue[Opt]}
```

Some usage:

```
>> r[z, x, y]
```

$x \sim y \sim z$

```
>> N[r]
```

3.5

```
>> r[]
```

$\{2,3\}$

```
>> r[5, Opt->7]
```

$\{5,7\}$

Its definition:

```
>> Definition[r]
```

$$
\begin{aligned}
\text{Attributes}[r] &= \{\text{Orderless}\} \\
\text{arg\_.} \sim \text{OptionsPattern}[r] &= \{\text{arg}, \text{OptionValue}[\text{Opt}]\} \\
N[r, \text{MachinePrecision}] &= 3.5 \\
\text{Format}[\text{args\_\_\_}, \text{MathMLForm}] &= \text{Infix}[\{\text{args}\}, \text{"}\sim\text{"}] \\
\text{Format}[\text{args\_\_\_}, \text{OutputForm}] &= \text{Infix}[\{\text{args}\}, \text{"}\sim\text{"}] \\
\text{Format}[\text{args\_\_\_}, \text{StandardForm}] &= \text{Infix}[\{\text{args}\}, \text{"}\sim\text{"}] \\
\text{Format}[\text{args\_\_\_}, \text{TeXForm}] &= \text{Infix}[\{\text{args}\}, \text{"}\sim\text{"}] \\
\text{Format}[\text{args\_\_\_}, \text{TraditionalForm}] &= \text{Infix}[\{\text{args}\}, \text{"}\sim\text{"}] \\
\text{Default}[r, 1] &= 2 \\
\text{Options}[r] &= \{\text{Opt}- > 3\}
\end{aligned}
$$

For `ReadProtected` symbols, `Definition` just prints attributes, default values and options:

```
>> SetAttributes[r, ReadProtected]
```

```
>> Definition[r]
```

$$
\begin{aligned}
\text{Attributes}[r] &= \{\text{Orderless}, \text{ReadProtected}\} \\
\text{Default}[r, 1] &= 2 \\
\text{Options}[r] &= \{\text{Opt}- > 3\}
\end{aligned}
$$

This is the same for built-in symbols:

```
>> Definition[Plus]
```

$$
\begin{aligned}
&\text{Attributes}[\text{Plus}] \\
&= \{\text{Flat}, \text{Listable}, \text{NumericFunction}, \text{OneIdentity}, \text{Orderless}, \text{Protected}\} \\
&\text{Default}[\text{Plus}] = 0
\end{aligned}
$$

```
>> Definition[Level]
```

$$
\begin{aligned}
\text{Attributes}[\text{Level}] &= \{\text{Protected}\} \\
\text{Options}[\text{Level}] &= \{\text{Heads}- > \text{False}\}
\end{aligned}
$$

`ReadProtected` can be removed, unless the symbol is locked:

```
>> ClearAttributes[r, ReadProtected]
```

`Clear` clears values:

```
>> Clear[r]
```

```
>> Definition[r]
```

$$
\begin{aligned}
\text{Attributes}[r] &= \{\text{Orderless}\} \\
\text{Default}[r, 1] &= 2 \\
\text{Options}[r] &= \{\text{Opt}- > 3\}
\end{aligned}
$$

`ClearAll` clears everything:

```
>> ClearAll[r]
```

```
>> Definition[r]
```
Null

If a symbol is not defined at all, `Null` is printed:

```
>> Definition[x]
```
Null

7.4.3. DownValues

WMA link

`DownValues`[*symbol*]
: gives the list of downvalues associated with *symbol*.

`DownValues` uses `HoldPattern` and `RuleDelayed` to protect the downvalues from being evaluated, and it has attribute `HoldAll` to get the specified symbol instead of its value.

```
>> f[x_] := x ^ 2
```

```
>> DownValues[f]
```
$\left\{\text{HoldPattern}\left[f\left[\text{x\_}\right]\right]:>x^2\right\}$

Mathics will sort the rules you assign to a symbol according to their specificity. If it cannot decide which rule is more special, the newer one will get higher precedence.

```
>> f[x_Integer] := 2
```

```
>> f[x_Real] := 3
```

```
>> DownValues[f]
```
$\left\{\text{HoldPattern}\left[f\left[\text{x\_Real}\right]\right]:>3,\text{HoldPattern}\left[f\left[\text{x\_Integer}\right]\right]:>2,\text{HoldPattern}\left[f\left[\text{x\_}\right]\right]:>x^2\right\}$

```
>> f[3]
```
2

```
>> f[3.]
```
3

```
>> f[a]
```
a^2

The default order of patterns can be computed using `Sort` with `PatternsOrderedQ`:

```
>> Sort[{x_, x_Integer}, PatternsOrderedQ]
```
{x_Integer, x_}

By assigning values to `DownValues`, you can override the default ordering:

```
>> DownValues[g] := {g[x_] :> x ^ 2, g[x_Integer] :> x}
```

```
>> g[2]
```
4

Fibonacci numbers:

```
>> DownValues[fib] := {fib[0] -> 0, fib[1] -> 1, fib[n_] :> fib[n - 1] +
    fib[n - 2]}
```

```
>> fib[5]
```
5

7.4.4. FormatValues

WMA link

`FormatValues`[*symbol*]
  gives the list of formatvalues associated with *symbol*.

```
>> Format[F[x_], OutputForm]:= Subscript[x, F]
```

```
>> FormatValues[F]
```
$\{\text{HoldPattern}\left[\text{Format}\left[F\left[\text{x\_}\right],\text{OutputForm}\right]\right] :> x_F\}$

7.4.5. Information (??)

WMA link

`Information`[*symbol*]
  Prints information about a *symbol*

`Information` does not print information for `ReadProtected` symbols.

`Information` uses `InputForm` to format values.

7.4.6. Names

WMA link

`Names`["*pattern*"]
  returns the list of names matching *pattern*.

```
>> Names["List"]
```
{List}

The wildcard `*` matches any character:

```
>> Names["List*"]
```
{List, ListLinePlot, ListLogPlot, ListPlot, ListQ, ListStepPlot, Listable}

The wildcard `@` matches only lowercase characters:

```
>> Names["List@"]
```
{Listable}

```
>> x = 5;
```

```
>> Names["Global`*"]
```
{x}

The number of built-in symbols:

```
>> Length[Names["System`*"]]
```
1506

7.4.7. OwnValues

WMA link

`OwnValues[`*symbol*`]`
: gives the list of ownvalue associated with *symbol*.

```
>> x = 3;
```

```
>> x = 2;
```

```
>> OwnValues[x]
```
{HoldPattern [x] :>2}

```
>> x := y
```

```
>> OwnValues[x]
```
{HoldPattern [x] :>y}

```
>> y = 5;
```

```
>> OwnValues[x]
```
{HoldPattern [x] :>y}

```
>> Hold[x] /. OwnValues[x]
```
Hold [y]

```
>> Hold[x] /. OwnValues[x] // ReleaseHold
```
5

7.4.8. SymbolName

WMA link

`SymbolName[`*s*`]`
: returns the name of the symbol *s* (without any leading context name).

```
>> SymbolName[x] // InputForm
```
"x"

7.4.9. SymbolQ

WMA link

`SymbolQ[`*x*`]`
: is `True` if *x* is a symbol, or `False` otherwise.

```
>> SymbolQ[a]
```
True

```
>> SymbolQ[1]
```
False

```
>> SymbolQ[a + b]
```
False

7.4.10. Symbol

WMA link

`Symbol`
: is the head of symbols.

```
>> Head[x]
```
Symbol

You can use `Symbol` to create symbols from strings:

```
>> Symbol["x"] + Symbol["x"]
```
$2x$

7.4.11. ValueQ

WMA link

`ValueQ`[*expr*]
: returns `True` if and only if *expr* is defined.

```
>> ValueQ[x]
```
False

```
>> x = 1;
```

```
>> ValueQ[x]
```
True

8. Binary Data

Binary data is a type of data that is represented in the binary, sequences of zeros or ones. Computer-generated information often comes in this form.

Contents

8.1. Binary Reading and Writing

8.1.1. BinaryRead

WMA link

`BinaryRead[`*stream*`]`
reads one byte from the stream as an integer from 0 to 255.
`BinaryRead[`*stream*, *type*`]`
reads one object of specified type from the stream.
`BinaryRead[`*stream*, {$type_1$, $type_2$, ...}`]`
reads a sequence of objects of specified types.

```
>> strm = OpenWrite[BinaryFormat -> True]
```
OutputStream [/tmp/tmpfli77xbw, 3]

```
>> BinaryWrite[strm, {97, 98, 99}]
```
OutputStream [/tmp/tmpfli77xbw, 3]

```
>> Close[strm];
```

```
>> strm = OpenRead[%, BinaryFormat -> True]
```
InputStream [/tmp/tmpfli77xbw, 3]

```
>> BinaryRead[strm]
```
97

```
>> BinaryRead[strm, {"Character8", "Character8"}]
```
{b, c}

If you read past the end of the file, you will get `EndOfFile` symbols:

```
>> BinaryRead[strm, {"Character8", "Character8"}]
```
{EndOfFile, EndOfFile}

```
>> DeleteFile[Close[strm]];
```

8.1.2. BinaryReadList

WMA link

`BinaryReadList[`*stream*`]`
: reads all remaining bytes from the stream or file as an integer from 0 to 255.

`BinaryReadList[`*stream*, *type*`]`
: reads objects of the specified type file a stream or file until the end of the file. The list of objects is returned.

`BinaryReadList[`*stream*, {$type_1$, $type_2$, ...}`]`
: reads a sequence of types, until the end of the file.

```
>> strm = OpenWrite[BinaryFormat -> True]
```
OutputStream [/tmp/tmp_hve1h7b, 3]

```
>> BinaryWrite[strm, {97, 98, 99}]
```
OutputStream [/tmp/tmp_hve1h7b, 3]

```
>> Close[strm];
```

```
>> strm = OpenRead[%, BinaryFormat -> True]
```
InputStream [/tmp/tmp_hve1h7b, 3]

```
>> BinaryReadList[strm]
```
{97, 98, 99}

```
>> DeleteFile[Close[strm]];
```

8.1.3. BinaryWrite

WMA link

`BinaryWrite[`*channel*, *b*`]`
: writes a single byte given as an integer from 0 to 255.

`BinaryWrite[`*channel*, {b1, b2, ...}`]`
: writes a sequence of byte.

`BinaryWrite[`*channel*, "string"`]`
: writes the raw characters in a string.

`BinaryWrite[`*channel*, *x*, *type*`]`
: writes x as the specified type.

`BinaryWrite[`*channel*, $\{x_1, x_2, ...\}$, *type*`]`
: writes a sequence of objects as the specified type.

`BinaryWrite[`*channel*, $\{x_1, x_2, ...\}$, $\{type_1, type_2, ...\}$`]`
: writes a sequence of objects using a sequence of specified types.

```
>> strm = OpenWrite[BinaryFormat -> True]
```
OutputStream [/tmp/tmpkyr5blqs, 3]

```
>> BinaryWrite[strm, {39, 4, 122}]
```
OutputStream [/tmp/tmpkyr5blqs, 3]

```
>> Close[strm];
```

```
>> strm = OpenRead[%, BinaryFormat -> True]
```
InputStream [/tmp/tmpkyr5blqs, 3]

```
>> BinaryRead[strm]
```
39

```
>> BinaryRead[strm, "Byte"]
```
4

```
>> BinaryRead[strm, "Character8"]
```
z

```
>> DeleteFile[Close[strm]];
```

Write a String

```
>> strm = OpenWrite[BinaryFormat -> True]
```
OutputStream [/tmp/tmp0tm8nwvc, 3]

```
>> BinaryWrite[strm, "abc123"]
```
OutputStream [/tmp/tmp0tm8nwvc, 3]

```
>> pathname = Close[%]
```
/tmp/tmp0tm8nwvc

Read as Bytes

```
>> strm = OpenRead[%, BinaryFormat -> True]
```
InputStream [/tmp/tmp0tm8nwvc, 3]

```
>> BinaryRead[strm, {"Character8", "Character8", "Character8", "
   Character8", "Character8", "Character8", "Character8"}]
```

$\{a, b, c, 1, 2, 3, \text{EndOfFile}\}$

```
>> pathname = Close[strm]
```

/tmp/tmp0tm8nwvc

Read as Characters

```
>> strm = OpenRead[%, BinaryFormat -> True]
```

InputStream [/tmp/tmp0tm8nwvc, 3]

```
>> BinaryRead[strm, {"Byte", "Byte", "Byte", "Byte", "Byte", "Byte", "
   Byte"}]
```

$\{97, 98, 99, 49, 50, 51, \text{EndOfFile}\}$

```
>> DeleteFile[Close[strm]];
```

Write Type

```
>> strm = OpenWrite[BinaryFormat -> True]
```

OutputStream [/tmp/tmpkend76ua, 3]

```
>> BinaryWrite[strm, 97, "Byte"]
```

OutputStream [/tmp/tmpkend76ua, 3]

```
>> BinaryWrite[strm, {97, 98, 99}, {"Byte", "Byte", "Byte"}]
```

OutputStream [/tmp/tmpkend76ua, 3]

```
>> DeleteFile[Close[%]];
```

8.2. Binary Types

8.2.1. Byte

WMA link

`Byte`
    is a data type for `Read`.

8.3. Byte Arrays

8.3.1. ByteArray

WMA link

`ByteArray[{`b_1, b_2, ...`}]`
    Represents a sequence of Bytes b_1, b_2, ...
`ByteArray["`*string*`"]`
    Constructs a byte array where bytes comes from decode a b64-encoded String

```
>> A=ByteArray[{1, 25, 3}]
```
ByteArray [<3>]

```
>> A[[2]]
```
25

```
>> Normal[A]
```
$\{1,25,3\}$

```
>> ToString[A]
```
ByteArray[<3>]

```
>> ByteArray["ARkD"]
```
ByteArray [<3>]

```
>> B=ByteArray["asy"]
The first argument in Bytearray[asy] should be a B64 encoded string
or a vector of integers.
```
$Failed

8.4. System-related binary handling

8.4.1. ByteOrdering

WMA link

`ByteOrdering`
    is an option for BinaryRead, BinaryWrite, and related functions that specifies what ordering of bytes should be assumed for your computer system..

8.4.2. $ByteOrdering

WMA link

`$ByteOrdering`
: returns the native ordering of bytes in binary data on your computer system.

9. Code Compilation

Code compilation allows Mathics functions to be run faster.

When LLVM and Python libraries are available, compilation produces LLVM code.

Contents

9.1. Compile

WMA link

`Compile`[{x_1, x_2, ...}, *expr*]
: Compiles *expr* assuming each *xi* is a *Real* number.

`Compile`[{{x_1, t_1} {x_2, t_1} ...}, *expr*]
: Compiles assuming each x_i matches type t_i.

Compilation is performed using llvmlite , or Python's builtin "compile" function.

```
>> cf = Compile[{x, y}, x + 2 y]

>> cf[2.5, 4.3]
   11.1

>> cf = Compile[{{x, _Real}}, Sin[x]]

>> cf[1.4]
   0.98545
```

Compile supports basic flow control:

```
>> cf = Compile[{{x, _Real}, {y, _Integer}}, If[x == 0.0 && y <= 0, 0.0,
    Sin[x ^ y] + 1 / Min[x, 0.5]] + 0.5]

>> cf[3.5, 2]
   2.18888
```

Loops and variable assignments are supported usinv Python builtin "compile" function:

```
>> Compile[{{a, _Integer}, {b, _Integer}}, While[b != 0, {a, b} = {b,
   Mod[a, b]}]; a] (* GCD of a, b *)
```

9.2. CompiledFunction

WMA link

`CompiledFunction[`*args*...`]`
: represents compiled code for evaluating a compiled function.

```
>> sqr = Compile[{x}, x x]
```

```
>> Head[sqr]
```

CompiledFunction

```
>> sqr[2]
```

4.

10. Colors

Programmatic support for symbolic colors.

Contents

10.1. Color Directives

There are many different way to specify color, and we support many of these.

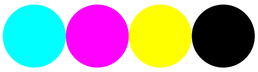
We can convert between the different color formats.

10.1.1. CMYKColor

CYMYK color model (WMA link)

CMYKColor[c, m, y, k]
    represents a color with the specified cyan, magenta, yellow and black components.

```
>> Graphics[MapIndexed[{CMYKColor @@ #1, Disk[2*#2 ~Join~{0}]} &,
   IdentityMatrix[4]], ImageSize->Small]
```



10.1.2. ColorDistance

Color difference (WMA link)

ColorDistance[c_1, c_2]
    returns a measure of color distance between the colors c_1 and c_2.
ColorDistance[*list*, c_2]
    returns a list of color distances between the colors in *list* and c_2.

The option DistanceFunction specifies the method used to measure the color distance. Available options are:

- CIE76: Euclidean distance in the LABColor space
- CIE94: Euclidean distance in the LCHColor space
- CIE2000 or CIEDE2000: CIE94 distance with corrections
- CMC: Color Measurement Committee metric (1984)
- DeltaL: difference in the L component of LCHColor
- DeltaC: difference in the C component of LCHColor
- DeltaH: difference in the H component of LCHColor

It is also possible to specify a custom distance.

```
>> ColorDistance[Magenta, Green]
```

2.2507

```
>> ColorDistance[{Red, Blue}, {Green, Yellow}, DistanceFunction -> {"CMC
   ", "Perceptibility"}]
```

$\{1.0495, 1.27455\}$

10.1.3. GrayLevel

WMA link

GrayLevel[g]
: represents a shade of gray specified by g, ranging from 0 (black) to 1 (white).

GrayLevel[g, a]
: represents a shade of gray specified by g with opacity a.

10.1.4. Hue

WMA link

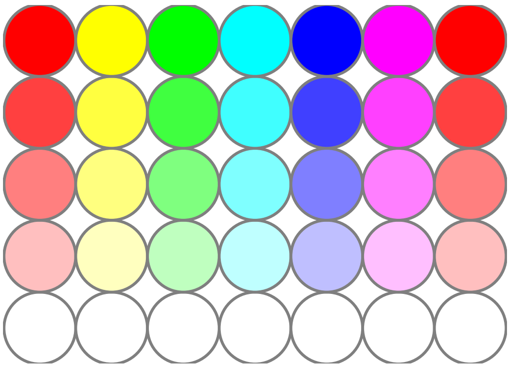
Hue[h, s, l, a]
: represents the color with hue h, saturation s, lightness l and opacity a.

Hue[h, s, l]
: is equivalent to `Hue[$h$, $s$, $l$, 1]`.

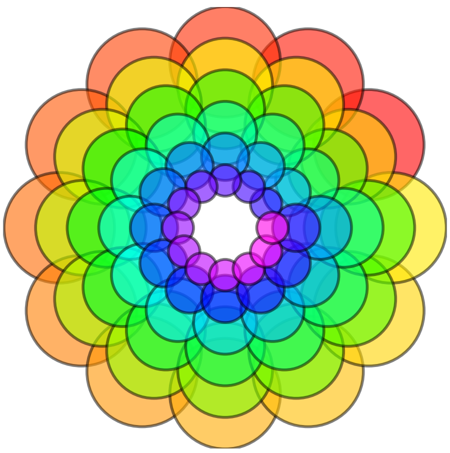
Hue[h, s]
: is equivalent to `Hue[$h$, $s$, 1, 1]`.

Hue[h]
: is equivalent to `Hue[$h$, 1, 1, 1]`.

```
>> Graphics[Table[{EdgeForm[Gray], Hue[h, s], Disk[{12h, 8s}]}, {h, 0,
   1, 1/6}, {s, 0, 1, 1/4}]]
```



```
>> Graphics[Table[{EdgeForm[{GrayLevel[0, 0.5]}], Hue[(-11+q+10r)/72, 1,
    1, 0.6], Disk[(8-r){Cos[2Pi q/12], Sin[2Pi q/12]}, (8-r)/3]}, {r,
   6}, {q, 12}]]
```



10.1.5. LABColor

WMA link

`LABColor`[l, a, b]
: represents a color with the specified lightness, red/green and yellow/blue components in the CIE 1976 L*a*b* (CIELAB) color space.

10.1.6. LCHColor

WMA link

`LCHColor`[l, c, h]
: represents a color with the specified lightness, chroma and hue components in the CIELCh CIELab cube color space.

10.1.7. LUVColor

WMA link

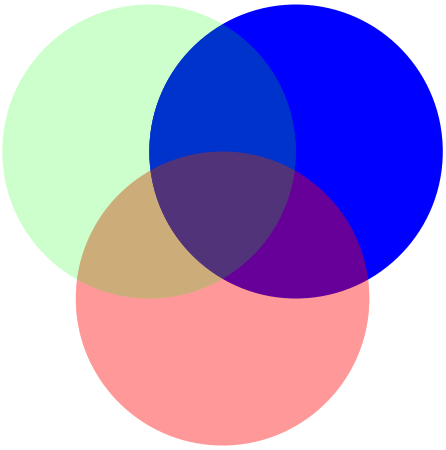
`LCHColor`[l, u, v]
: represents a color with the specified components in the CIE 1976 L*u*v* (CIELUV) color space.

10.1.8. Opacity

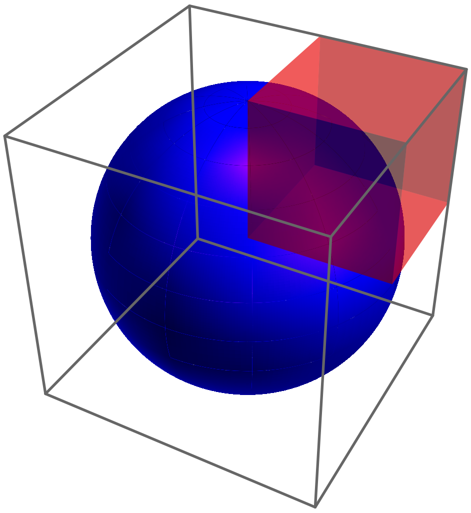
Alpha compositing (WMA link)

`Opacity`[*level*]
: is a graphics directive that sets the opacity to *level*; *level* is a value between 0 and 1.

```
>> Graphics[{Blue, Disk[{.5, 1}, 1], Opacity[.4], Red, Disk[], Opacity
   [.2], Green, Disk[{-.5, 1}, 1]}]
```

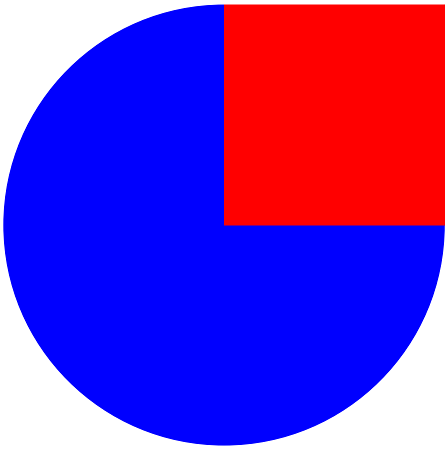


```
>> Graphics3D[{Blue, Sphere[], Opacity[.4], Red, Cuboid[]}]
```



Notice that `Opacity` does not overwrite the value of the alpha channel if it is set in a color directive:

```
>> Graphics[{Blue, Disk[], RGBColor[1,0,0,1],Opacity[.2], Rectangle
   [{0,0},{1,1}]}]
```



10.1.9. RGBColor

RGB color model (WMA link)

`RGBColor`[r, g, b]
: represents a color with the specified red, green and blue components. These values should be a number between 0 and 1. Unless specified using the form below or using Opacity 10.1.8, default opacity is 1, a solid opaque color.

`RGBColor`[r, g, b, a]
: Same as above but an opacity value is specified. a must have value between 0 and 1. `RGBColor[$r$,$g$,$b$,$a$]` is equivalent to `{RGBColor[$r$,$g$,$b$],Opacity[$a$]}`.

A swatch of color green:

```
>> RGBColor[0, 1, 0]
```

Let's show what goes on in the process of boxing the above to make this display:

```
>> RGBColor[0, 1, 0] // ToBoxes
```

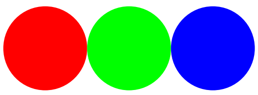
StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [0, 1, 0] , RectangleBox [{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle − > {} , Background− > Automatic, ImageSize− > 16, LabelStyle − > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle − > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

A swatch of color green which is 1/8 opaque:

```
>> RGBColor[0, 1, 0, 0.125]
```

A series of small disks of the primary colors:

```
>> Graphics[MapIndexed[{RGBColor @@ #1, Disk[2*#2 ~Join~{0}]} &,
   IdentityMatrix[3]], ImageSize->Small]
```



10.1.10. XYZColor

WMA link

`XYZColor[`x, y, z`]`
: represents a color with the specified components in the CIE 1931 XYZ color space.

10.2. Color Operations

Functions for manipulating colors and color images.

10.2.1. Blend

WMA link

`Blend[{`c_1, c_2`}]`
: represents the color between c_1 and c_2.

`Blend[{`c_1, c_2`}, `x`]`
: represents the color formed by blending c_1 and c_2 with factors 1 - x and x respectively.

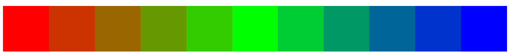
`Blend[{`$c_1, c_2, ..., c_n$`}, `x`]`
: blends between the colors c_1 to c_n according to the factor x.

```
>> Blend[{Red, Blue}]
```

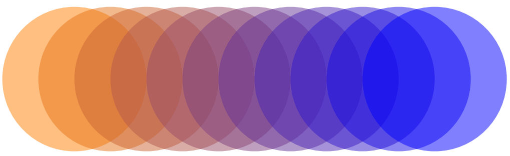
```
>> Blend[{Red, Blue}, 0.3]
```

```
>> Blend[{Red, Blue, Green}, 0.75]
```

```
>> Graphics[Table[{Blend[{Red, Green, Blue}, x], Rectangle[{10 x, 0}]},
   {x, 0, 1, 1/10}]]
```



```
>> Graphics[Table[{Blend[{RGBColor[1, 0.5, 0, 0.5], RGBColor[0, 0, 1,
   0.5]}, x], Disk[{5x, 0}]}, {x, 0, 1, 1/10}]]
```



10.2.2. ColorConvert

WMA link

`ColorConvert[`*c*, *colspace*`]`
: returns the representation of *c* in the color space *colspace*. *c* may be a color or an image.

Valid values for *colspace* are:

CMYK: convert to CMYKColor Grayscale: convert to GrayLevel HSB: convert to Hue LAB: concert to LABColor LCH: convert to LCHColor LUV: convert to LUVColor RGB: convert to RGBColor XYZ: convert to XYZColor

10.2.3. ColorNegate

Color Inversion (WMA link)

`ColorNegate[`*color*`]`
: returns the negative of a color, that is, the RGB color subtracted from white.

`ColorNegate[`*image*`]`
: returns an image where each pixel has its color negated.

Yellow is `RGBColor[1.0, 1.0, 0.0]` So when inverted or subtracted from `White,` we get blue:

```
>> ColorNegate[Yellow] == Blue
```
True

```
>> ColorNegate[Import["ExampleData/sunflowers.jpg"]]
```

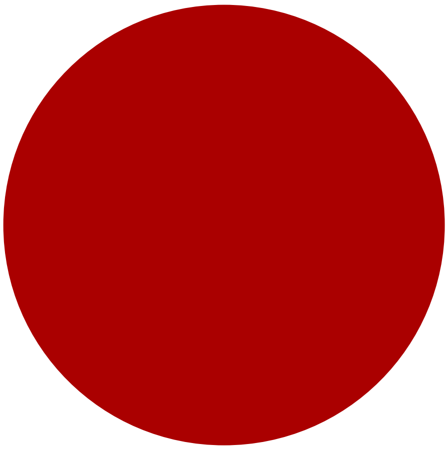

10.2.4. Darker

WMA link

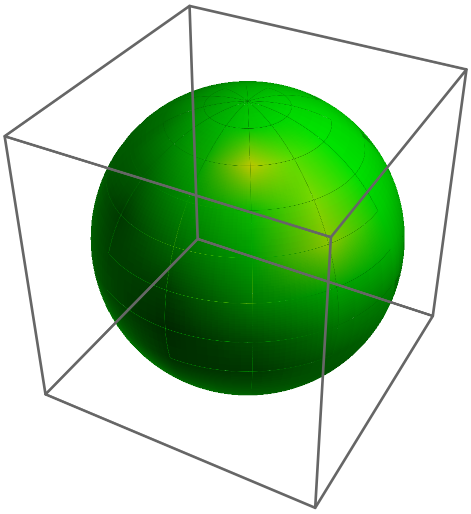
`Darker[`*c*, *f*`]`
: is equivalent to `Blend[{`*c*, `Black}`, *f*`]`.

`Darker[`*c*`]`
: is equivalent to `Darker[`*c*, `1/3]`.

```
>> Graphics[{Darker[Red], Disk[]}]
```



```
>> Graphics3D[{Darker[Green], Sphere[]}]
```



```
>> Graphics[Table[{Darker[Yellow, x], Disk[{12x, 0}]}, {x, 0, 1, 1/6}]]
```

10.2.5. DominantColors

WMA link

`DominantColors[`*image*`]`
: gives a list of colors which are dominant in the given image.

`DominantColors[`*image*, *n*`]`
: returns at most *n* colors.

`DominantColors[`*image*, *n*, *prop*`]`
: returns the given property *prop*, which may be:

- "Color": return RGB colors,
- "LABColor": return LAB colors,
- "Count": return the number of pixels a dominant color covers,
- "Coverage": return the fraction of the image a dominant color covers, or
- "CoverageImage": return a black and white image indicating with white the parts that are covered by a dominant color.

The option "ColorCoverage" specifies the minimum amount of coverage needed to include a dominant color in the result.

The option "MinColorDistance" specifies the distance (in LAB color space) up to which colors are merged and thus regarded as belonging to the same dominant color.

```
>> img = Import["ExampleData/hedy.tif"]
```

```
>> DominantColors[img]
```

{■,□,■}

```
>> DominantColors[img, 3]
```

{■,□,■}

```
>> DominantColors[img, 3, "Coverage"]
```

$$\left\{\frac{68817}{103360}, \frac{62249}{516800}, \frac{37953}{516800}\right\}$$

```
>> DominantColors[img, 3, "CoverageImage"]
```

```
>> DominantColors[img, 3, "Count"]
```

{344085, 62249, 37953}

```
>> DominantColors[img, 2, "LABColor"]
```

{LABColor [0.00581591, 0.00207458, −0.00760911] , }

```
>> DominantColors[img, MinColorDistance -> 0.5]
```

{ , }

```
>> DominantColors[img, ColorCoverage -> 0.15]
```

{ }

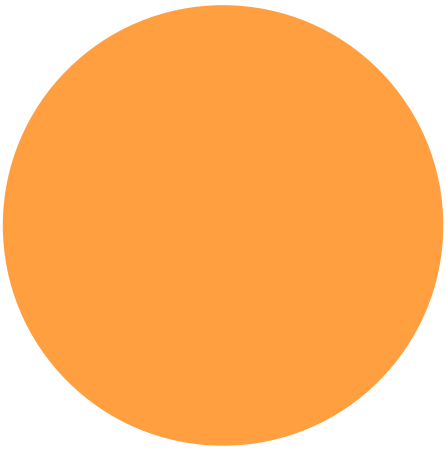
10.2.6. Lighter

WMA link

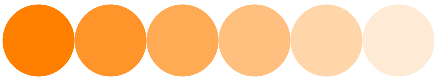
`Lighter`[c, f]
    is equivalent to `Blend`[{c, `White`}, f].
`Lighter`[c]
    is equivalent to `Lighter`[c, `1/3`].

```
>> Lighter[Orange, 1/4]
```

```
>> Graphics[{Lighter[Orange, 1/4], Disk[]}]
```



```
>> Graphics[Table[{Lighter[Orange, x], Disk[{12x, 0}]}, {x, 0, 1, 1/6}]]
```



10.3. Named Colors

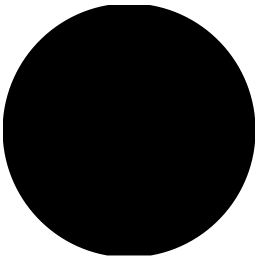
Mathics has definitions for the most common color names which can be used in a graphics or style specification.

10.3.1. Black

WMA link

`Black`
: represents the color black in graphics.

>> `Graphics[{EdgeForm[Black], Black, Disk[]}, ImageSize->Small]`



>> `Black // ToBoxes`

StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0, 0, 0] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

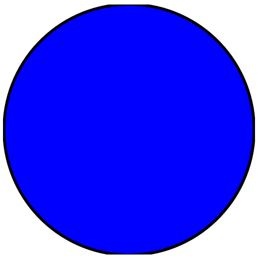
>> `Black`

10.3.2. Blue

WMA link

`Blue`
: represents the color blue in graphics.

```
>> Graphics[{EdgeForm[Black], Blue, Disk[]}, ImageSize->Small]
```



```
>> Blue // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0, 0, 1] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

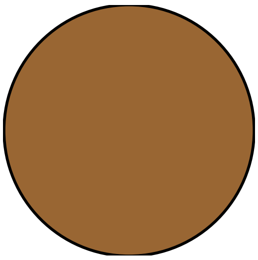
```
>> Blue
```

10.3.3. Brown

WMA link

`Brown`
represents the color brown in graphics.

```
>> Graphics[{EdgeForm[Black], Brown, Disk[]}, ImageSize->Small]
```



```
>> Brown // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0.6, 0.4, 0.2] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

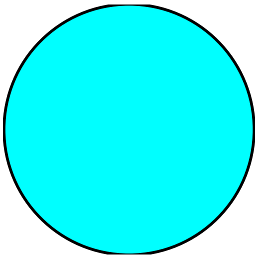
```
>> Brown
```

10.3.4. Cyan

WMA link

`Cyan`
: represents the color cyan in graphics.

```
>> Graphics[{EdgeForm[Black], Cyan, Disk[]}, ImageSize->Small]
```



```
>> Cyan // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [0, 1, 1] , RectangleBox [{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle − > {} , Background− > Automatic, ImageSize− > 16, LabelStyle − > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle − > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

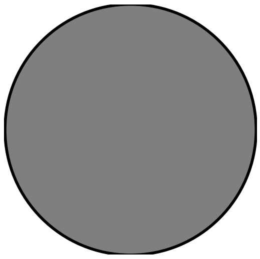
```
>> Cyan
```

10.3.5. Gray

WMA link

`Gray`
: represents the color gray in graphics.

>> `Graphics[{EdgeForm[Black], Gray, Disk[]}, ImageSize->Small]`



>> `Gray // ToBoxes`

StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , GrayLevel [0.5] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

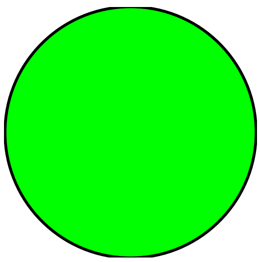
>> `Gray`

10.3.6. Green

WMA link

`Green`
: represents the color green in graphics.

>> `Graphics[{EdgeForm[Black], Green, Disk[]}, ImageSize->Small]`



>> `Green // ToBoxes`

StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0, 1, 0] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

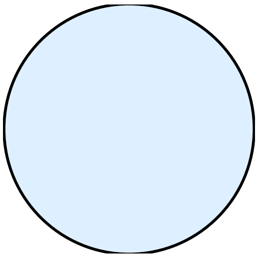
>> `Green`

10.3.7. LightBlue

WMA link

`LightBlue`
: represents the color light blue in graphics.

>> `Graphics[{EdgeForm[Black], LightBlue, Disk[]}, ImageSize->Small]`

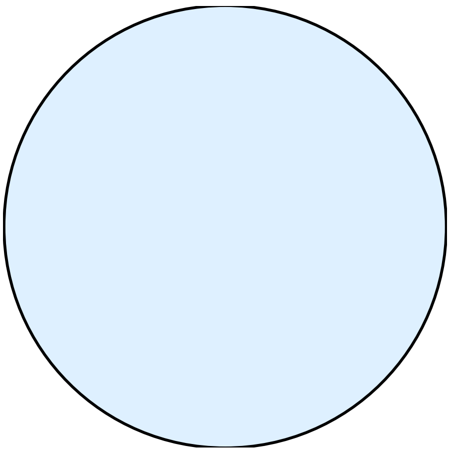


>> `LightBlue // ToBoxes`

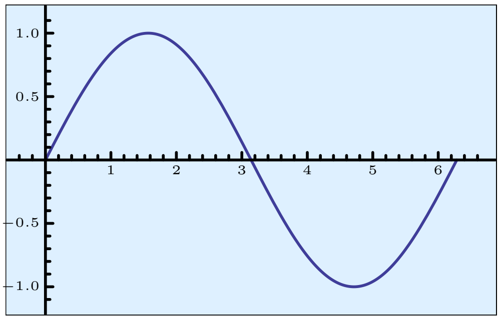
StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0.87, 0.94, 1] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

>> `Graphics[{LightBlue, EdgeForm[Black], Disk[]}]`



```
>> Plot[Sin[x], {x, 0, 2 Pi}, Background -> LightBlue]
```

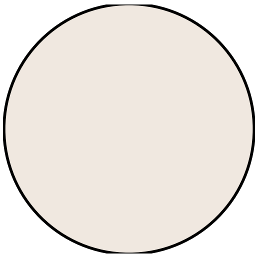


10.3.8. LightBrown

WMA link

`LightBrown`
represents the color light brown in graphics.

```
>> Graphics[{EdgeForm[Black], LightBrown, Disk[]}, ImageSize->Small]
```



```
>> LightBrown // ToBoxes
```

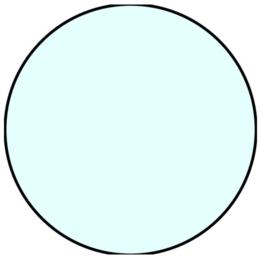
StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0.94, 0.91, 0.88] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.9. LightCyan

WMA link

`LightCyan`
represents the color light cyan in graphics.

```
>> Graphics[{EdgeForm[Black], LightCyan, Disk[]}, ImageSize->Small]
```



```
>> LightCyan // ToBoxes
```

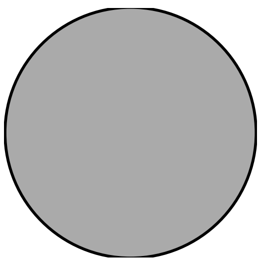
StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0.9, 1., 1.] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.10. LightGray

WMA link

`LightGray`
represents the color light gray in graphics.

```
>> Graphics[{EdgeForm[Black], LightGray, Disk[]}, ImageSize->Small]
```



```
>> LightGray // ToBoxes
```

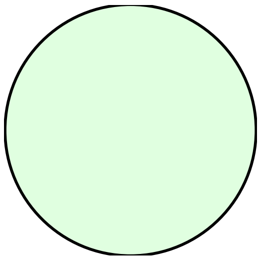
StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , GrayLevel [0.666667, 1.] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.11. LightGreen

WMA link

`LightGreen`
: represents the color light green in graphics.

```
>> Graphics[{EdgeForm[Black], LightGreen, Disk[]}, ImageSize->Small]
```



```
>> LightGreen // ToBoxes
```

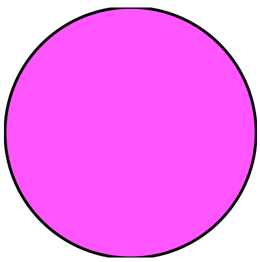
StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0.88, 1., 0.88] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.12. LightMagenta

WMA link

`LightMagenta`
: represents the color light magenta in graphics.

```
>> Graphics[{EdgeForm[Black], LightMagenta, Disk[]}, ImageSize->Small]
```



```
>> LightMagenta // ToBoxes
```

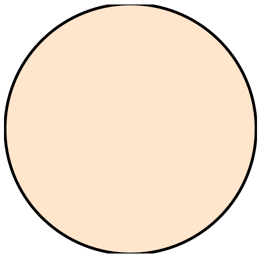
StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [1., 0.333333, 1.] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.13. LightOrange

WMA link

`LightOrange`
    represents the color light orange in graphics.

>> `Graphics[{EdgeForm[Black], LightOrange, Disk[]}, ImageSize->Small]`



>> `LightOrange // ToBoxes`

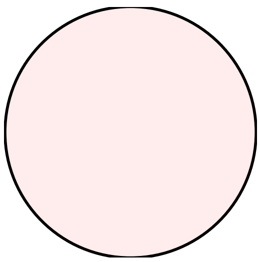
StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [1, 0.9, 0.8] , RectangleBox [{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle − > {} , Background− > Automatic, ImageSize− > 16, LabelStyle − > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle − > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.14. LightPink

WMA link

`LightPink`
    represents the color light pink in graphics.

>> `Graphics[{EdgeForm[Black], LightPink, Disk[]}, ImageSize->Small]`



`>>` **`LightPink // ToBoxes`**

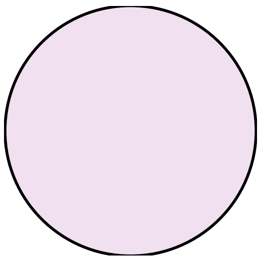
StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [1., 0.925, 0.925] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.15. LightPurple

WMA link

`LightPurple`
represents the color light purple in graphics.

`>>` **`Graphics[{EdgeForm[Black], LightPurple, Disk[]}, ImageSize->Small]`**



`>>` **`LightPurple // ToBoxes`**

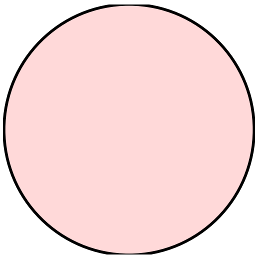
StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [0.94, 0.88, 0.94] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.16. LightRed

WMA link

`LightRed`
represents the color light red in graphics.

```
>> Graphics[{EdgeForm[Black], LightRed, Disk[]}, ImageSize->Small]
```



```
>> LightRed // ToBoxes
```

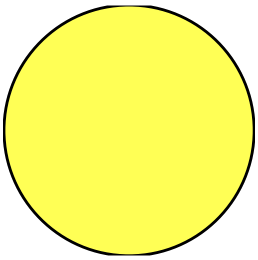
StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [1., 0.85, 0.85] , RectangleBox [{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle − > {} , Background− > Automatic, ImageSize− > 16, LabelStyle − > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle − > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.17. LightYellow

WMA link

`LightYellow`
represents the color light yellow in graphics.

```
>> Graphics[{EdgeForm[Black], LightYellow, Disk[]}, ImageSize->Small]
```



```
>> LightYellow // ToBoxes
```

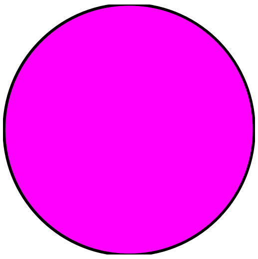
StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [1., 1., 0.333333] , RectangleBox [{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle − > {} , Background− > Automatic, ImageSize− > 16, LabelStyle − > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle − > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.18. Magenta

WMA link

```
Magenta
```
represents the color magenta in graphics.

```
>> Graphics[{EdgeForm[Black], Magenta, Disk[]}, ImageSize->Small]
```



```
>> Magenta // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [1, 0, 1] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

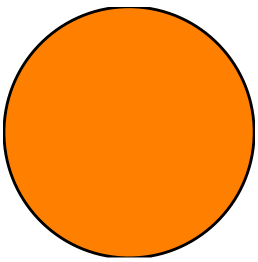
```
>> Magenta
```

10.3.19. Orange

WMA link

```
Orange
```
represents the color orange in graphics.

```
>> Graphics[{EdgeForm[Black], Orange, Disk[]}, ImageSize->Small]
```



>> `Orange // ToBoxes`

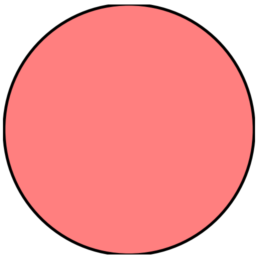
StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [1, 0.5, 0] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.20. Pink

WMA link

`Pink`
represents the color pink in graphics.

>> `Graphics[{EdgeForm[Black], Pink, Disk[]}, ImageSize->Small]`



>> `Pink // ToBoxes`

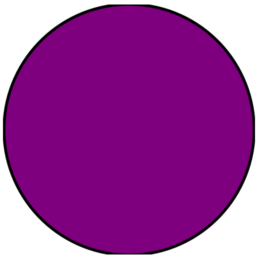
StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [1., 0.5, 0.5] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.21. Purple

WMA link

`Purple`
represents the color purple in graphics.

```
>> Graphics[{EdgeForm[Black], Purple, Disk[]}, ImageSize->Small]
```



```
>> Purple // ToBoxes
```

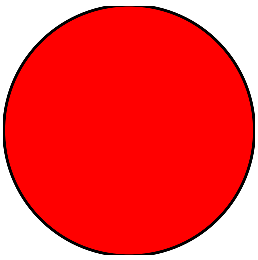
StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [0.5, 0, 0.5] , RectangleBox [{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle − > {} , Background− > Automatic, ImageSize− > 16, LabelStyle − > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle − > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

10.3.22. Red

WMA link

`Red`
: represents the color red in graphics.

```
>> Graphics[{EdgeForm[Black], Red, Disk[]}, ImageSize->Small]
```



```
>> Red // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , RGBColor [1, 0, 0] , RectangleBox [{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle − > {} , Background− > Automatic, ImageSize− > 16, LabelStyle − > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle − > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

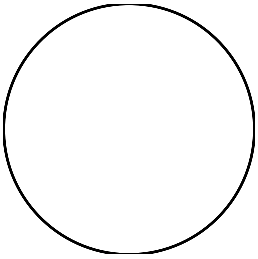
```
>> Red
```

10.3.23. White

WMA link

`White`
: represents the color white in graphics.

```
>> Graphics[{EdgeForm[Black], White, Disk[]}, ImageSize->Small]
```



```
>> White // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [RGBColor [0, 0, 0]] , GrayLevel [1] , RectangleBox [{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle − > {} , Background− > Automatic, ImageSize− > 16, LabelStyle − > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle − > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

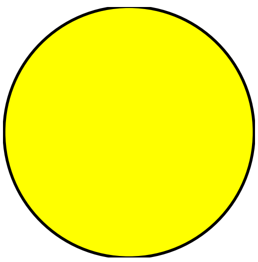
```
>> White
```

10.3.24. Yellow

WMA link

`Yellow`
: represents the color yellow in graphics.

```
>> Graphics[{EdgeForm[Black], Yellow, Disk[]}, ImageSize->Small]
```



```
>> Yellow // ToBoxes
```

StyleBox [GraphicsBox [{EdgeForm [RGBColor [
0, 0, 0]] , RGBColor [1, 1, 0] , RectangleBox [
{0, 0}] } , AspectRatio− > Automatic, Axes− > False, AxesStyle
− > {} , Background− > Automatic, ImageSize− > 16, LabelStyle
− > {} , PlotRange− > Automatic, PlotRangePadding− > Automatic, TicksStyle
− > {}] , ImageSizeMultipliers− > {1, 1} , ShowStringCharacters− > True]

WMA link

```
>> Yellow
```

11. Compress Functions

Contents

11.1. Compress

WMA link

`Compress[`*expr*`]`
: gives a compressed string representation of *expr*.

```
>> Compress[N[Pi, 10]]
```

eJwz1jM0MTS1NDIzNQEADRsCNw==

11.2. Uncompress

WMA link

`Uncompress["`*string*`"]`
: recovers an expression from a string generated by `Compress`.

```
>> Compress["Mathics is cool"]
```

eJxT8k0sychMLlbILFZIzs/PUQIANFwF1w==

```
>> Uncompress[%]
```

Mathics is cool

```
>> a = x ^ 2 + y Sin[x] + 10 Log[15];
```

```
>> b = Compress[a];
```

```
>> Uncompress[b]
```

$x^2 + y\text{Sin}[x] + 10\text{Log}[15]$

12. Date and Time

Dates and times are represented symbolically; computations can be performed on them.

Date object can also input and output dates and times in a wide range of formats, as well as handle calendars.

Contents

12.1. $DateStringFormat

WMA link

`$DateStringFormat`
gives the format used for dates generated by `DateString`.

```
>> $DateStringFormat
```
{DateTimeShort}

12.2. $SystemTimeZone

WMA link

`$SystemTimeZone`
gives the current time zone for the computer system on which Mathics is being run.

```
>> $SystemTimeZone
```
$-5.$

12.3. $TimeZone

Time Zone (WMA)

`$TimeZone`
  gives the current time zone to assume for dates and times.

```
>> $TimeZone
```
−5.

12.4. AbsoluteTime

WMA link

`AbsoluteTime[]`
  gives the local time in seconds since epoch January 1, 1900, in your time zone.

`AbsoluteTime[{`*y*, *m*, *d*, *h*, *m*, *s*`}]`
  gives the absolute time specification corresponding to a date list.

`AbsoluteTime[`"*string*"`]`
  gives the absolute time specification for a given date string.

`AbsoluteTime[{`"*string*",{e_1, e_2, ...}`}]`
  takgs the date string to contain the elements "*ei*".

```
>> AbsoluteTime[]
```
3.96592*^9

```
>> AbsoluteTime[{2000}]
```
3155673600

```
>> AbsoluteTime[{"01/02/03", {"Day", "Month", "YearShort"}}]
```
3253046400

```
>> AbsoluteTime["6 June 1991"]
```
2885155200

```
>> AbsoluteTime[{"6-6-91", {"Day", "Month", "YearShort"}}]
```
2885155200

12.5. AbsoluteTiming

WMA link

`AbsoluteTiming[`*expr*`]`
  evaluates *expr*, returning a list of the absolute number of seconds in real time that have elapsed, together with the result obtained.

`>>` `AbsoluteTiming[50!]`

$\{0.000074625, 30414093201713378043612608166064768844377641568960512000000000000\}$

`>>` `Attributes[AbsoluteTiming]`

$\{\text{HoldAll}, \text{Protected}\}$

12.6. DateDifference

WMA link

`DateDifference[`$date_1$, $date_2$`]`
  returns the difference between $date_1$ and $date_2$ in days.
`DateDifference[`$date_1$, $date_2$, *unit*`]`
  returns the difference in the specified *unit*.
`DateDifference[`$date_1$, $date_2$, {$unit_1$, $unit_2$, ...}`]`
  represents the difference as a list of integer multiples of each *unit*, with any remainder expressed in the smallest unit.

`>>` `DateDifference[{2042, 1, 4}, {2057, 1, 1}]`

5476

`>>` `DateDifference[{1936, 8, 14}, {2000, 12, 1}, "Year"]`

$\{64.3425, \text{Year}\}$

`>>` `DateDifference[{2010, 6, 1}, {2015, 1, 1}, "Hour"]`

$\{40200, \text{Hour}\}$

`>>` `DateDifference[{2003, 8, 11}, {2003, 10, 19}, {"Week", "Day"}]`

$\{\{9, \text{Week}\}, \{6, \text{Day}\}\}$

12.7. DateList

WMA link

`DateList[]`
  returns the current local time in the form {*year*, *month*, *day*, *hour*, *minute*, *second*}.
`DateList[`*time*`]`
  returns a formatted date for the number of seconds *time* since epoch Jan 1 1900.
`DateList[`{*y*, *m*, *d*, *h*, *m*, *s*}`]`
  converts an incomplete date list to the standard representation.

`>>` `DateList[0]`

$\{1900, 1, 1, 0, 0, 0.\}$

```
>> DateList[3155673600]
```
$\{2000, 1, 1, 0, 0, 0.\}$

```
>> DateList[{2003, 5, 0.5, 0.1, 0.767}]
```
$\{2003, 4, 30, 12, 6, 46.02\}$

```
>> DateList[{2012, 1, 300., 10}]
```
$\{2012, 10, 26, 10, 0, 0.\}$

```
>> DateList["31/10/1991"]
```
$\{1991, 10, 31, 0, 0, 0.\}$

```
>> DateList["1/10/1991"]
The interpretation of 1/10/1991 is ambiguous.
```
$\{1991, 1, 10, 0, 0, 0.\}$

```
>> DateList[{"31/10/91", {"Day", "Month", "YearShort"}}]
```
$\{1991, 10, 31, 0, 0, 0.\}$

```
>> DateList[{"31 10/91", {"Day", " ", "Month", "/", "YearShort"}}]
```
$\{1991, 10, 31, 0, 0, 0.\}$

If not specified, the current year assumed

```
>> DateList[{"5/18", {"Month", "Day"}}]
```
$\{2025, 5, 18, 0, 0, 0.\}$

12.8. DateObject

WMA link

`DateObject[...]`
: Returns an object codifying DateList....

```
>> DateObject[{2020, 4, 15}]
```
$[\text{Wed 15 Apr 2020 00:00:00 GTM} - 5]$

12.9. DatePlus

WMA link

`DatePlus[`*date*, *n*`]`
: finds the date n days after *date*.

`DatePlus[`*date*, {*n*, "*unit*"}`]`
: finds the date n units after *date*.

`DatePlus[`*date*, {{n_1, "$unit_1$"}, {n_2, "$unit_2$"}, ...}`]`
: finds the date which is n_i specified units after *date*.

`DatePlus[`*n*`]`
: finds the date n days after the current date.

`DatePlus[`*offset*`]`
: finds the date which is offset from the current date.

Add 73 days to Feb 5, 2010:

```
>> DatePlus[{2010, 2, 5}, 73]
```

$\{2010, 4, 19\}$

Add 8 weeks and 1 day to March 16, 1999:

```
>> DatePlus[{2010, 2, 5}, {{8, "Week"}, {1, "Day"}}]
```

$\{2010, 4, 3\}$

12.10. DateString

WMA link

`DateString[]`
: returns the current local time and date as a string.

`DateString[`*elem*`]`
: returns the time formatted according to *elems*.

`DateString[`{e_1, e_2, ...}`]`
: concatenates the time formatted according to elements *ei*.

`DateString[`*time*`]`
: returns the date string of an AbsoluteTime.

`DateString[`{y, m, d, h, m, s}`]`
: returns the date string of a date list specification.

`DateString[`*string*`]`
: returns the formatted date string of a date string specification.

`DateString[`*spec*, *elems*`]`
: formats the time in turns of *elems*. Both *spec* and *elems* can take any of the above formats.

The current date and time:

```
>> DateString[];
```

```
>> DateString[{1991, 10, 31, 0, 0}, {"Day", " ", "MonthName", " ", "Year
   "}]
```

31 October 1991

```
>> DateString[{2007, 4, 15, 0}]
```
Sun 15 Apr 2007 00:00:00

```
>> DateString[{1979, 3, 14}, {"DayName", " ", "Month", "-", "YearShort
"}]
```
Wednesday 03-79

Non-integer values are accepted too:

```
>> DateString[{1991, 6, 6.5}]
```
Thu 6 Jun 1991 12:00:00

12.11. EasterSunday

Date of Easter (WMA link)

`EasterSunday[`*year*`]`
    returns the date of the Gregorian Easter Sunday as {year, month, day}.

```
>> EasterSunday[2000]
```
$\{2000, 4, 23\}$

```
>> EasterSunday[2030]
```
$\{2030, 4, 21\}$

12.12. Now

WMA link

`Now`
    gives the current time on the system.

```
>> Now
```
$[\text{Wed 3 Sep 2025 19:23:05 GTM} - 5]$

12.13. SessionTime

WMA link

`SessionTime[]`
: returns the total time in seconds since this session started.

```
>> SessionTime[]
75.9413
```

12.14. TimeConstrained

WMA link

`TimeConstrained[`*expr*, *t*`]`
: evaluates $expr$, stopping after t seconds.

`TimeConstrained[`*expr*, *t*, *failexpr*`]`
: returns $failexpr$ if the time constraint is not met.

Possible issues: for certain time-consuming functions (like simplify) which are based on sympy or other libraries, it is possible that the evaluation continues after the timeout. However, at the end of the evaluation, the function will return `$Aborted` and the results will not affect the state of the Mathics3 kernel.

12.15. TimeRemaining

WMA link

`TimeRemaining[]`
: Gives the number of seconds remaining until the earliest enclosing `TimeConstrained` will request the current computation to stop.

`TimeConstrained[`*expr*, *t*, *failexpr*`]`
: returns *failexpr* if the time constraint is not met.

If TimeConstrained is called out of a TimeConstrained expression, returns `Infinity`:

```
>> TimeRemaining[]
∞
```

```
>> TimeConstrained[1+2; Print[TimeRemaining[]], 0.9]
0.899472
```

12.16. TimeUsed

WMA link

TimeUsed[]
: returns the total CPU time used for this session, in seconds.

>> **TimeUsed[]**

80.3014

12.17. Timing

WMA link

`Timing[`*expr*`]`
: measures the processor time taken to evaluate *expr*. It returns a list containing the measured time in seconds and the result of the evaluation.

>> **Timing[50!]**

$\{0.000078113, 30414093201713378043612608166064768844377641568960512000000000000\}$

>> **Attributes[Timing]**

$\{\text{HoldAll}, \text{Protected}\}$

13. Definition Attributes

While a definition like `cube[x_] = x^3` gives a way to specify *values* of a function, *attributes* allow a way to specify general properties of functions and symbols. This is independent of the parameters they take and the values they produce.

The builtin-attributes having a predefined meaning in *Mathics3* which are described below.

However in contrast to *Mathematica®*, you can set any symbol as an attribute.

Contents

13.1. Attributes

WMA link

`Attributes`[*symbol*]
    returns the attributes of *symbol*.
`Attributes`[“*string*”]
    returns the attributes of `Symbol`[“*string*”].
`Attributes`[*symbol*] = {$attr_1$, $attr_2$}
    sets the attributes of *symbol*, replacing any existing attributes.

```
>> Attributes[Plus]
```
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}

```
>> Attributes["Plus"]
```
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}

`Attributes` always considers the head of an expression:

```
>> Attributes[a + b + c]
```
$\{\text{Flat}, \text{Listable}, \text{NumericFunction}, \text{OneIdentity}, \text{Orderless}, \text{Protected}\}$

You can assign values to `Attributes` to set attributes:

```
>> Attributes[f] = {Flat, Orderless}
```
$\{\text{Flat}, \text{Orderless}\}$

```
>> f[b, f[a, c]]
```
$f[a, b, c]$

Attributes must be symbols:

```
>> Attributes[f] := {a + b}
Argument a + b at position 1 is expected to be a symbol.
```
$Failed

Use `Symbol` to convert strings to symbols:

```
>> Attributes[f] = Symbol["Listable"]
```
Listable

```
>> Attributes[f]
```
$\{\text{Listable}\}$

13.2. ClearAttributes

WMA link

`ClearAttributes`[*symbol*, *attrib*]
removes *attrib* from *symbol*'s attributes.

```
>> SetAttributes[f, Flat]
```

```
>> Attributes[f]
```
$\{\text{Flat}\}$

```
>> ClearAttributes[f, Flat]
```

```
>> Attributes[f]
```
$\{\}$

Attributes that are not even set are simply ignored:

```
>> ClearAttributes[{f}, {Flat}]
```

```
>> Attributes[f]
```
$\{\}$

13.3. Constant

WMA link

`Constant`
: is an attribute that indicates that a symbol is a constant.

Mathematical constants like `E` have attribute `Constant`:

```
>> Attributes[E]
```
{Constant, Protected, ReadProtected}

Constant symbols cannot be used as variables in `Solve` and related functions:

```
>> Solve[x + E == 0, E]
E is not a valid variable.
```
Solve $[x + E{=}{=}0, E]$

13.4. Flat

WMA link

`Flat`
: is an attribute that specifies that nested occurrences of a function should be automatically flattened.

A symbol with the `Flat` attribute represents an associative mathematical operation:

```
>> SetAttributes[f, Flat]
```

```
>> f[a, f[b, c]]
```
$f\,[a, b, c]$

`Flat` is taken into account in pattern matching:

```
>> f[a, b, c] /. f[a, b] -> d
```
$f\,[d, c]$

13.5. HoldAll

WMA link

`HoldAll`
: is an attribute specifying that all arguments of a function should be left unevaluated.

```
>> Attributes[Function]
```

{HoldAll, Protected}

13.6. HoldAllComplete

WMA link

`HoldAllComplete`
: is an attribute that includes the effects of `HoldAll` and `SequenceHold`, and also protects the function from being affected by the upvalues of any arguments.

`HoldAllComplete` even prevents upvalues from being used, and includes `SequenceHold`.

```
>> SetAttributes[f, HoldAllComplete]
```

```
>> f[a] ^= 3;
```

```
>> f[a]
```

$f[a]$

```
>> f[Sequence[a, b]]
```

$f\left[\text{Sequence}[a, b]\right]$

13.7. HoldFirst

WMA link

`HoldFirst`
: is an attribute specifying that the first argument of a function should be left unevaluated.

```
>> Attributes[Set]
```

{HoldFirst, Protected, SequenceHold}

13.8. HoldRest

WMA link

`HoldRest`
: is an attribute specifying that all but the first argument of a function should be left unevaluated.

```
>> Attributes[If]
```
{HoldRest, Protected}

13.9. Listable

WMA link

`Listable`
: is an attribute specifying that a function should be automatically applied to each element of a list.

```
>> SetAttributes[f, Listable]
```

```
>> f[{1, 2, 3}, {4, 5, 6}]
```
$\{f[1,4], f[2,5], f[3,6]\}$

```
>> f[{1, 2, 3}, 4]
```
$\{f[1,4], f[2,4], f[3,4]\}$

```
>> {{1, 2}, {3, 4}} + {5, 6}
```
$\{\{6,7\}, \{9,10\}\}$

13.10. Locked

WMA link

`Locked`
: is an attribute that prevents attributes on a symbol from being modified.

The attributes of `Locked` symbols cannot be modified:

```
>> Attributes[lock] = {Flat, Locked};
```

```
>> SetAttributes[lock, {}]
Symbol lock is locked.
```

```
>> ClearAttributes[lock, Flat]
   Symbol lock is locked.

>> Attributes[lock] = {}
   Symbol lock is locked.
   {}

>> Attributes[lock]
   {Flat, Locked}
```

However, their values might be modified (as long as they are not `Protected` too):

```
>> lock = 3
   3
```

13.11. NHoldAll

WMA link

`NHoldAll`
: is an attribute that protects all arguments of a function from numeric evaluation.

```
>> N[f[2, 3]]
   f[2., 3.]

>> SetAttributes[f, NHoldAll]

>> N[f[2, 3]]
   f[2, 3]
```

13.12. NHoldFirst

WMA link

`NHoldFirst`
: is an attribute that protects the first argument of a function from numeric evaluation.

13.13. NHoldRest

WMA link

`NHoldRest`
: is an attribute that protects all but the first argument of a function from numeric evaluation.

13.14. NumericFunction

WMA link

`NumericFunction`
: is an attribute that indicates that a symbol is the head of a numeric function.

Mathematical functions like `Sqrt` have attribute `NumericFunction`:

```
>> Attributes[Sqrt]
```
{Listable, NumericFunction, Protected}

Expressions with a head having this attribute, and with all the elements being numeric expressions, are considered numeric expressions:

```
>> NumericQ[Sqrt[1]]
```
True

```
>> NumericQ[a]=True; NumericQ[Sqrt[a]]
```
True

```
>> NumericQ[a]=False; NumericQ[Sqrt[a]]
```
False

13.15. OneIdentity

WMA link

`OneIdentity`
: is an attribute assigned to a symbol, say f, indicating that $f[x]$, $f[f[x]]$, etc. are all equivalent to x in pattern matching.

```
>> a /. f[x_:0, u_] -> {u}
```
a

Here is how `OneIdentity` changes the pattern matched above :

```
>> SetAttributes[f, OneIdentity]
```

```
>> a /. f[x_:0, u_] -> {u}
```
$\{a\}$

However, without a default argument, the pattern does not match:

```
>> a /. f[u_] -> {u}
```
a

`OneIdentity` does not change evaluation:

```
>> f[a]
```
$f[a]$

13.16. Orderless

WMA link

`Orderless`
: is an attribute that can be assigned to a symbol f to indicate that the elements ei in expressions of the form $f[e_1, e_2, ...]$ should automatically be sorted into canonical order. This property is accounted for in pattern matching.

The elements of an `Orderless` function are automatically sorted:

```
>> SetAttributes[f, Orderless]
```

```
>> f[c, a, b, a + b, 3, 1.0]
```
$f[1., 3, a, b, c, a + b]$

A symbol with the `Orderless` attribute represents a commutative mathematical operation.

```
>> f[a, b] == f[b, a]
```
True

`Orderless` affects pattern matching:

```
>> SetAttributes[f, Flat]
```

```
>> f[a, b, c] /. f[a, c] -> d
```
$f[b, d]$

13.17. Protect

WMA link

`Protect`[s_1, s_2, ...]
: sets the attribute `Protected` for the symbols *si*.

`Protect`[str_1, str_2, ...]
: protects all symbols whose names textually match *stri*.

```
>> A = {1, 2, 3};

>> Protect[A]

>> A[[2]] = 4;
   Symbol A is Protected.

>> A
```

$\{1,2,3\}$

13.18. Protected

WMA link

`Protected`
: is an attribute that prevents values on a symbol from being modified.

Values of `Protected` symbols cannot be modified:

```
>> Attributes[p] = {Protected};

>> p = 2;
   Symbol p is Protected.

>> f[p] ^= 3;
   Tag p in f[p] is Protected.

>> Format[p] = "text";
   Symbol p is Protected.
```

However, attributes might still be set:

```
>> SetAttributes[p, Flat]

>> Attributes[p]
```

{Flat, Protected}

Thus, you can easily remove the attribute `Protected`:

```
>> Attributes[p] = {};

>> p = 2
```

2

You can also use `Protect` or `Unprotect`, resp.

```
>> Protect[p]
```

```
>> Attributes[p]
```

{Protected}

```
>> Unprotect[p]
```

If a symbol is `Protected` and `Locked`, it can never be changed again:

```
>> SetAttributes[p, {Protected, Locked}]
```

```
>> p = 2
Symbol p is Protected.
```

2

```
>> Unprotect[p]
Symbol p is locked.
```

13.19. ReadProtected

WMA link

`ReadProtected`
is an attribute that prevents values on a symbol from being read.

Values associated with `ReadProtected` symbols cannot be seen in `Definition`:

```
>> ClearAll[p]
```

```
>> p = 3;
```

```
>> Definition[p]
```

$$p = 3$$

```
>> SetAttributes[p, ReadProtected]
```

```
>> Definition[p]
```

$$\text{Attributes}\left[p\right] = \{\text{ReadProtected}\}$$

13.20. SequenceHold

WMA link

`SequenceHold`
: is an attribute that prevents `Sequence` objects from being spliced into a function's arguments.

Normally, `Sequence` will be spliced into a function:

```
>> f[Sequence[a, b]]
```
$f[a,b]$

It does not for `SequenceHold` functions:

```
>> SetAttributes[f, SequenceHold]
```

```
>> f[Sequence[a, b]]
```
$f\left[\text{Sequence}[a,b]\right]$

E.g., `Set` has attribute `SequenceHold` to allow assignment of sequences to variables:

```
>> s = Sequence[a, b];
```

```
>> s
```
$\text{Sequence}[a,b]$

```
>> Plus[s]
```
$a+b$

13.21. SetAttributes

WMA link

`SetAttributes[`*symbol*, *attrib*`]`
: adds *attrib* to the list of *symbol*'s attributes.

```
>> SetAttributes[f, Flat]
```

```
>> Attributes[f]
```
$\{\text{Flat}\}$

Multiple attributes can be set at the same time using lists:

```
>> SetAttributes[{f, g}, {Flat, Orderless}]
```

```
>> Attributes[g]
```
$\{\text{Flat},\text{Orderless}\}$

13.22. Unprotect

WMA link

`Unprotect`[s_1, s_2, ...]
: removes the attribute `Protected` for the symbols *si*.

`Unprotect`[*str*]
: unprotects symbols whose names textually match *str*.

14. Descriptive Statistics

Function which operate on explicit data and symbolic representations of statistical distributions.

Contents

14.1. Dependency and Dispursion Statistics

14.1.1. Correlation

Pearson correlation coefficient (WMA)

`Correlation[`*a*, *b*`]`
computes Pearson's correlation of two equal-sized vectors *a* and *b*.

An example from Wikipedia:

```
>> Correlation[{10, 8, 13, 9, 11, 14, 6, 4, 12, 7, 5}, {8.04, 6.95,
   7.58, 8.81, 8.33, 9.96, 7.24, 4.26, 10.84, 4.82, 5.68}]
```

0.816421

14.1.2. Covariance

Covariance (WMA)

`Covariance[`*a*, *b*`]`
computes the covariance between the equal-sized vectors *a* and *b*.

```
>> Covariance[{0.2, 0.3, 0.1}, {0.3, 0.3, -0.2}]
```
0.025

14.2. General Statistics

14.2.1. CentralMoment

Central moment (WMA)

`CentralMoment`[*list*, *r*]
gives the the *r*th central moment (i.e. the *r*th moment about the mean) of *list*.

```
>> CentralMoment[{1.1, 1.2, 1.4, 2.1, 2.4}, 4]
```
0.100845

14.3. Location Statistics

14.3.1. Mean

WMA link

`Mean`[*list*]
returns the statistical mean of *list*.

```
>> Mean[{26, 64, 36}]
```
42

```
>> Mean[{1, 1, 2, 3, 5, 8}]
```
$\frac{10}{3}$

```
>> Mean[{a, b}]
```
$\frac{a+b}{2}$

14.4. Order Statistics

In statistics, an order statistic gives the k-th smallest value.

Together with rank statistics these are fundamental tools in non-parametric statistics and inference.

Important special cases of order statistics are finding minimum and maximum value of a sample and sample quantiles.

14.4.1. Quantile

Quantile (WMA link)

In statistics and probability, quantiles are cut points dividing the range of a probability distribution into continuous intervals with equal probabilities, or dividing the observations in a sample in the same way.

Quantile is also known as value at risk (VaR) or fractile.

`Quantile[`*list*, *q*`]`
: returns the qth quantile of *list*.

`Quantile[`*list*, *q*, $\{\{a, b\}, \{c, d\}\}$`]`
: uses the quantile definition specified by parameters a, b, c, d.

For a list of length n, `Quantile[`*list*, *q*, $\{\{a, b\}, \{c, d\}\}$`]` depends on $x = a + (n + b)q$.
If x is an integer, the result is $s[[x]]$, where s=`Sort[list,Less]` .
Otherwise, the result is: `s[[Floor[x]]] + (s[[Ceiling[x]]] - s[[Floor[x]]])(c + d FractionalPart[x])`, with the indices taken to be 1 or n if they are out of range.
The default choice of parameters is `{{0,0},{1,0}}`.

Common choices of parameters include:

- `{{0, 0}, {1, 0}}` inverse empirical CDF (default)
- `{{0, 0}, {0, 1}}` linear interpolation (California method)

`Quantile[list,q]` always gives a result equal to an element of list.

```
>> Quantile[Range[11], 1/3]
```
4

```
>> Quantile[Range[16], 1/4]
```
4

```
>> Quantile[{1, 2, 3, 4, 5, 6, 7}, {1/4, 3/4}]
```
$\{2, 6\}$

14.4.2. Quartiles

Quartile (WMA link)

`Quartiles[`*list*`]`
: returns the 1/4, 1/2, and 3/4 quantiles of *list*.

```
>> Quartiles[Range[25]]
```
$\left\{\frac{27}{4}, 13, \frac{77}{4}\right\}$

14.4.3. RankedMax

WMA link

`RankedMax`[*list*, *n*]
: returns the *n*th largest element of *list* (with $n = 1$ yielding the largest element, $n = 2$ yielding the second largest element, and so on).

```
>> RankedMax[{482, 17, 181, -12}, 2]
```
181

14.4.4. RankedMin

WMA link

`RankedMin`[*list*, *n*]
: returns the *n*th smallest element of *list* (with $n = 1$ yielding the smallest element, $n = 2$ yielding the second smallest element, and so on).

```
>> RankedMin[{482, 17, 181, -12}, 2]
```
17

14.4.5. ReverseSort

WMA link

`ReverseSort`[*list*]
: sorts *list* (or the elements of any other expression) according to reverse canonical ordering.

`ReverseSort`[*list*, *p*]
: sorts using *p* to determine the order of two elements.

```
>> ReverseSort[{c, b, d, a}]
```
$\{d, c, b, a\}$

You can specify a binary comparison function:

```
>> ReverseSort[{1, 2, 0, 3}, Less]
```
$\{3, 2, 1, 0\}$

Using `Greater` for the above, reverses the reverse sort:

```
>> ReverseSort[{1, 2, 0, 3}, Greater]
```
$\{0, 1, 2, 3\}$

See also Sort 14.4.6.

14.4.6. Sort

WMA link

`Sort[`*list*`]`
: sorts *list* (or the elements of any other expression) according to canonical ordering.

`Sort[`*list*, *p*`]`
: sorts using *p* to determine the order of two elements.

```
>> Sort[{4, 1.0, a, 3+I}]
```
$\{1., 3 + I, 4, a\}$

Sort uses `OrderedQ` to determine ordering by default. You can sort patterns according to their precedence using `PatternsOrderedQ`:

```
>> Sort[{items___, item_, OptionsPattern[], item_symbol, item_?test},
   PatternsOrderedQ]
```
{item_symbol, item_? test, item_, items___, OptionsPattern []}

When sorting patterns, values of atoms do not matter:

```
>> Sort[{a, b/;t}, PatternsOrderedQ]
```
$\{b/;t, a\}$

```
>> Sort[{2+c_, 1+b__}, PatternsOrderedQ]
```
{2 + c_, 1 + b__}

```
>> Sort[{x_ + n_*y_, x_ + y_}, PatternsOrderedQ]
```
{x_ + n_y_, x_ + y_}

See also ReverseSort 14.4.5.

14.4.7. TakeLargest

WMA link

`TakeLargest[`*list*, *f*, *n*`]`
: returns the a sorted list of the *n* largest items in *list*.

List the largest two numbers of a list:

```
>> TakeLargest[{100, -1, 50, 10}, 2]
```
$\{100, 50\}$

None, Null, Indeterminate and expressions with head Missing are ignored by default:

```
>> TakeLargest[{-8, 150, Missing[abc]}, 2]
```
$\{150, -8\}$

You may specify which items are ignored using the option `ExcludedForms`:

```
>> TakeLargest[{-8, 150, Missing[abc]}, 2, ExcludedForms -> {}]
```
{Missing [abc] , 150}

14.4.8. TakeSmallest

WMA link

`TakeSmallest`[*list*, *n*]
: returns the a sorted list of the *n* smallest items in *list*.

List the smallest two numbers of a list:

```
>> TakeSmallest[{100, -1, 50, 10}, 2]
```
$\{-1, 10\}$

For details on how to use the `ExcludedForms` option, see TakeLargest 14.4.7.

14.5. Shape Statistics

14.5.1. Kurtosis

Kurtosis (WMA)

`Kurtosis`[*list*]
: gives the Pearson measure of kurtosis for *list* (a measure of existing outliers).

```
>> Kurtosis[{1.1, 1.2, 1.4, 2.1, 2.4}]
```
1.42098

14.5.2. Skewness

Skewness (WMA)

`Skewness[`*list*`]`
: gives Pearson's moment coefficient of skewness for *list* (a measure for estimating the symmetry of a distribution).

```
>> Skewness[{1.1, 1.2, 1.4, 2.1, 2.4}]
```
0.407041

15. Directories and Directory Operations

Contents

15.1. Directory Names

15.1.1. DirectoryName

WMA link

`DirectoryName["name"]`
extracts the directory name from a filename.

```
>> DirectoryName["a/b/c"]
a/b
>> DirectoryName["a/b/c", 2]
a
```

15.1.2. DirectoryQ

WMA link

`DirectoryQ["name"]`
returns `True` if the directory called *name* exists and `False` otherwise.

```
>> DirectoryQ["ExampleData/"]
```
True

```
>> DirectoryQ["ExampleData/MythicalSubdir/"]
```
False

15.1.3. FileNameDepth

WMA link

`FileNameDepth[`"*name*"`]`
: gives the number of path parts in the given filename.

```
>> FileNameDepth["a/b/c"]
```
3

```
>> FileNameDepth["a/b/c/"]
```
3

15.1.4. FileNameJoin

WMA link

`FileNameJoin[`{"dir_1", "dir_2", ...}`]`
: joins the dir_i together into one path.

`FileNameJoin[..., OperatingSystem->``os']'`
: yields a file name in the format for the specified operating system. Possible choices are "Windows", "MacOSX", and "Unix".

```
>> FileNameJoin[{"dir1", "dir2", "dir3"}]
```
dir1/dir2/dir3

```
>> FileNameJoin[{"dir1", "dir2", "dir3"}, OperatingSystem -> "Unix"]
```
dir1/dir2/dir3

```
>> FileNameJoin[{"dir1", "dir2", "dir3"}, OperatingSystem -> "Windows"]
```
dir1\dir2\dir3

15.1.5. FileNameSplit

WMA link

`FileNameSplit[`"*filenames*"`]`
: splits a *filename* into a list of parts.

```
>> FileNameSplit["example/path/file.txt"]
```
{example, path, file.txt}

15.1.6. ParentDirectory

WMA link

`ParentDirectory[]`
returns the parent of the current working directory.
`ParentDirectory["dir"]`
returns the parent *dir*.

```
>> ParentDirectory[]
```
/src/external-vcs/github/Mathics3/mathics-core

15.2. Directory Operations

15.2.1. CreateDirectory

WMA link

`CreateDirectory["dir"]`
creates a directory called *dir*.
`CreateDirectory[]`
creates a temporary directory.

```
>> dir = CreateDirectory[]
```
/tmp/mtzq8v694

15.2.2. DeleteDirectory

WMA link

`DeleteDirectory["dir"]`
deletes a directory called *dir*.

```
>> dir = CreateDirectory[]
```
/tmp/maruh8t4t

```
>> DeleteDirectory[dir]
```

```
>> DirectoryQ[dir]
```
False

15.2.3. RenameDirectory

WMA link

`RenameDirectory["dir1", "dir2"]`
: renames directory dir_1 to dir_2.

15.3. System File Directories

15.3.1. $BaseDirectory

WMA link

`$BaseDirectory`
: returns the folder where user configurations are stored.

```
>> $BaseDirectory
```
/src/external-vcs/github/Mathics3/mathics-core/mathics

15.3.2. $InitialDirectory

WMA link

`$InitialDirectory`
: returns the directory from which emphMathics3 was started.

```
>> $InitialDirectory
```
/src/external-vcs/github/Mathics3/mathics-core/mathics

15.3.3. $InstallationDirectory

WMA link

`$InstallationDirectory`
returns the top-level directory in which emphMathics3 was installed.

>> `$InstallationDirectory`

/src/external-vcs/github/Mathics3/mathics-core/mathics

15.3.4. $RootDirectory

WMA link

`$RootDirectory`
returns the system root directory.

>> `$RootDirectory`

/

15.3.5. $TemporaryDirectory

WMA link

`$TemporaryDirectory`
returns the directory used for temporary files.

>> `$TemporaryDirectory`

/tmp

15.4. User File Directories

15.4.1. $HomeDirectory

WMA link

`$HomeDirectory`
returns the users HOME directory.

>> `$HomeDirectory`

/home/rocky

15.4.2. $Path

WMA link

`$Path`
: returns the list of directories to search when looking for a file.

```
>> $Path
```
{., /home/rocky, /home/rocky/.local/var/Mathics3/Packages, /src/external-vcs/github/Mathics3/mathics-cor

15.4.3. $UserBaseDirectory

WMA link

`$UserBaseDirectory`
: returns the folder where user configurations are stored.

```
>> $UserBaseDirectory
```
/home/rocky/.mathics

16. Distance and Similarity Measures

Different measures of distance or similarity for different types of analysis.

Contents

16.1. Cluster Analysis

16.1.1. ClusteringComponents

WMA link

`ClusteringComponents[`*list*`]`
forms clusters from *list* and returns a list of cluster indices, in which each element shows the index of the cluster in which the corresponding element in *list* ended up.
`ClusteringComponents[`*list*, *k*`]`
forms *k* clusters from *list* and returns a list of cluster indices, in which each element shows the index of the cluster in which the corresponding element in *list* ended up.

For more detailed documentation regarding options and behavior, see FindClusters[].

```
>> ClusteringComponents[{1, 2, 3, 1, 2, 10, 100}]
```
$\{1,1,1,1,1,1,2\}$

```
>> ClusteringComponents[{10, 100, 20}, Method -> "KMeans"]
```
$\{1,0,1\}$

16.1.2. FindClusters

WMA link

`FindClusters[`*list*`]`
: returns a list of clusters formed from the elements of *list*. The number of cluster is determined automatically.

`FindClusters[`*list*, *k*`]`
: returns a list of *k* clusters formed from the elements of *list*.

```
>> FindClusters[{1, 2, 20, 10, 11, 40, 19, 42}]
```
$\{\{1,2,20,10,11,19\},\{40,42\}\}$

```
>> FindClusters[{25, 100, 17, 20}]
```
$\{\{25,17,20\},\{100\}\}$

```
>> FindClusters[{3, 6, 1, 100, 20, 5, 25, 17, -10, 2}]
```
$\{\{3,6,1,5,-10,2\},\{100\},\{20,25,17\}\}$

```
>> FindClusters[{1, 2, 10, 11, 20, 21}]
```
$\{\{1,2\},\{10,11\},\{20,21\}\}$

```
>> FindClusters[{1, 2, 10, 11, 20, 21}, 2]
```
$\{\{1,2,10,11\},\{20,21\}\}$

```
>> FindClusters[{1 -> a, 2 -> b, 10 -> c}]
```
$\{\{a,b\},\{c\}\}$

```
>> FindClusters[{1, 2, 5} -> {a, b, c}]
```
$\{\{a,b\},\{c\}\}$

```
>> FindClusters[{1, 2, 3, 1, 2, 10, 100}, Method -> "Agglomerate"]
```
$\{\{1,2,3,1,2,10\},\{100\}\}$

```
>> FindClusters[{1, 2, 3, 10, 17, 18}, Method -> "Agglomerate"]
```
$\{\{1,2,3\},\{10\},\{17,18\}\}$

```
>> FindClusters[{{1}, {5, 6}, {7}, {2, 4}}, DistanceFunction -> (Abs[
   Length[#1] - Length[#2]]&)]
```
$\{\{\{1\},\{7\}\},\{\{5,6\},\{2,4\}\}\}$

```
>> FindClusters[{"meep", "heap", "deep", "weep", "sheep", "leap", "keep
   "}, 3]
```
{{meep, deep, weep, keep}, {heap, leap}, {sheep}}

FindClusters' automatic distance function detection supports scalars, numeric tensors, boolean vectors and strings.

The Method option must be either "Agglomerate" or "Optimize". If not specified, it defaults to "Optimize". Note that the Agglomerate and Optimize methods usually produce different clusterings.

The runtime of the Agglomerate method is quadratic in the number of clustered points n, builds the clustering from the bottom up, and is exact (no element of randomness). The Optimize method's runtime is linear in n, Optimize builds the clustering from top down, and uses random sampling.

16.1.3. Nearest

WMA link

`Nearest`[*list*, *x*]
: returns the one item in *list* that is nearest to *x*.

`Nearest`[*list*, *x*, *n*]
: returns the *n* nearest items.

`Nearest`[*list*, *x*, {*n*, *r*}]
: returns up to *n* nearest items that are not farther from *x* than *r*.

`Nearest`[{p_1 -> q_1, p_2 -> q_2, ...}, *x*]
: returns q_1, q_2, ... but measures the distances using p_1, p_2, ...

`Nearest`[{p_1, p_2, ...} -> {q_1, q_2, ...}, *x*]
: returns q_1, q_2, ... but measures the distances using p_1, p_2, ...

```
>> Nearest[{5, 2.5, 10, 11, 15, 8.5, 14}, 12]
```

{11}

Return all items within a distance of 5:

```
>> Nearest[{5, 2.5, 10, 11, 15, 8.5, 14}, 12, {All, 5}]
```

{11, 10, 14}

```
>> Nearest[{Blue -> "blue", White -> "white", Red -> "red", Green -> "
   green"}, {Orange, Gray}]
```

{{red}, {white}}

```
>> Nearest[{{0, 1}, {1, 2}, {2, 3}} -> {a, b, c}, {1.1, 2}]
```

{*b*}

16.2. Numerical Data

16.2.1. BrayCurtisDistance

Bray-Curtis Dissimilarity (WMA)

`BrayCurtisDistance`[*u*, *v*]
: returns the Bray-Curtis distance between *u* and *v*.

The Bray-Curtis distance is equivalent to `Total[Abs[u-v]]/Total[Abs[u+v]]`.

```
>> BrayCurtisDistance[-7, 5]
```
6

```
>> BrayCurtisDistance[{-1, -1}, {10, 10}]
```
$\frac{11}{9}$

16.2.2. CanberraDistance

Canberra distance (WMA)

`CanberraDistance`[u, v]
: returns the canberra distance between u and v, which is a weighted version of the Manhattan distance.

```
>> CanberraDistance[-7, 5]
```
1

```
>> CanberraDistance[{-1, -1}, {1, 1}]
```
2

16.2.3. ChessboardDistance

Chebyshev distance (WMA)

`ChessboardDistance`[u, v]
: returns the chessboard distance (also known as Chebyshev distance) between u and v, which is the number of moves a king on a chessboard needs to get from square u to square v.

```
>> ChessboardDistance[-7, 5]
```
12

```
>> ChessboardDistance[{-1, -1}, {1, 1}]
```
2

16.2.4. CosineDistance

Cosine similarity (WMA)

`CosineDistance`[u, v]
: returns the angular cosine distance between vectors u and v.

The cosine distance is equivalent to $1 - (u.Conjugate[v])/(Norm[u]Norm[v])$.

```
>> N[CosineDistance[{7, 9}, {71, 89}]]
```
0.0000759646

When the length of either vector is 0, the result is 0:

```
>> CosineDistance[{0.0, 0.0}, {x, y}]
```
0

```
>> CosineDistance[{1, 0}, {x, y}]
```
$$1 - \frac{\text{Conjugate}[x]}{\sqrt{\text{Abs}[x]^2 + \text{Abs}[y]^2}}$$

The order of the vectors influences the result:

```
>> CosineDistance[{x, y}, {1, 0}]
```
$$1 - \frac{x}{\sqrt{\text{Abs}[x]^2 + \text{Abs}[y]^2}}$$

Cosine distance includes a dot product scaled by norms:

```
>> CosineDistance[{a, b, c}, {x, y, z}]
```
$$1 + \frac{-a\text{Conjugate}[x] - b\text{Conjugate}[y] - c\text{Conjugate}[z]}{\sqrt{\text{Abs}[a]^2 + \text{Abs}[b]^2 + \text{Abs}[c]^2}\sqrt{\text{Abs}[x]^2 + \text{Abs}[y]^2 + \text{Abs}[z]^2}}$$

A Cosine distance applied to complex numbers, uses `Abs[]` for `Norm[]` and complex multiplication for dot product, 1 - u * Conjugate[v] / `(Abs[$u$] Abs[$v$])`:

```
>> CosineDistance[1+2I, 5]
```
$$1 - \left(\frac{1}{5} + \frac{2I}{5}\right)\sqrt{5}$$

16.2.5. EuclideanDistance

Euclidean similarity (WMA)

`EuclideanDistance[`u, v`]`
returns the euclidean distance between u and v.

```
>> EuclideanDistance[-7, 5]
```
12

```
>> EuclideanDistance[{-1, -1}, {1, 1}]
```
$2\sqrt{2}$

```
>> EuclideanDistance[{a, b}, {c, d}]
```

$\sqrt{\text{Abs}[a-c]^2 + \text{Abs}[b-d]^2}$

16.2.6. ManhattanDistance

Manhattan distance (WMA)

`ManhattanDistance`[u, v]
: returns the Manhattan distance between u and v, which is the number of horizontal or vertical moves in the gridlike Manhattan city layout to get from u to v.

```
>> ManhattanDistance[-7, 5]
```

12

```
>> ManhattanDistance[{-1, -1}, {1, 1}]
```

4

16.2.7. SquaredEuclideanDistance

WMA link

`SquaredEuclideanDistance`[u, v]
: returns squared the euclidean distance between u and v.

```
>> SquaredEuclideanDistance[-7, 5]
```

144

```
>> SquaredEuclideanDistance[{-1, -1}, {1, 1}]
```

8

16.3. String Distances and Similarity Measures

16.3.1. DamerauLevenshteinDistance

WMA link

`DamerauLevenshteinDistance`[a, b]
: returns the Damerau-Levenshtein distance of a and b, which is defined as the minimum number of transpositions, insertions, deletions and substitutions needed to transform one into the other. In contrast to EditDistance, DamerauLevenshteinDistance counts transposition of adjacent items (e.g. “ab” into “ba”) as one operation of change.

```
>> DamerauLevenshteinDistance["kitten", "kitchen"]
   2
>> DamerauLevenshteinDistance["abc", "ac"]
   1
>> DamerauLevenshteinDistance["abc", "acb"]
   1
>> DamerauLevenshteinDistance["azbc", "abxyc"]
   3
```

The IgnoreCase option makes DamerauLevenshteinDistance ignore the case of letters:

```
>> DamerauLevenshteinDistance["time", "Thyme"]
   3
>> DamerauLevenshteinDistance["time", "Thyme", IgnoreCase -> True]
   2
```

DamerauLevenshteinDistance also works on lists:

```
>> DamerauLevenshteinDistance[{1, E, 2, Pi}, {1, E, Pi, 2}]
   1
```

16.3.2. EditDistance

WMA link

`EditDistance[`*a*, *b*`]`
: returns the Levenshtein distance of *a* and *b*, which is defined as the minimum number of insertions, deletions and substitutions on the constituents of *a* and *b* needed to transform one into the other.

```
>> EditDistance["kitten", "kitchen"]
   2
>> EditDistance["abc", "ac"]
   1
>> EditDistance["abc", "acb"]
   2
>> EditDistance["azbc", "abxyc"]
   3
```

The IgnoreCase option makes EditDistance ignore the case of letters:

```
>> EditDistance["time", "Thyme"]
   3
```

```
>> EditDistance["time", "Thyme", IgnoreCase -> True]
   2
```

EditDistance also works on lists:

```
>> EditDistance[{1, E, 2, Pi}, {1, E, Pi, 2}]
   2
```

16.3.3. HammingDistance

WMA link

`HammingDistance[`*u*, *v*`]`
: returns the Hamming distance between *u* and *v*, i.e. the number of different elements. *u* and *v* may be lists or strings.

```
>> HammingDistance[{1, 0, 1, 0}, {1, 0, 0, 1}]
   2
>> HammingDistance["time", "dime"]
   1
>> HammingDistance["TIME", "dime", IgnoreCase -> True]
   1
```

17. Drawing Graphics

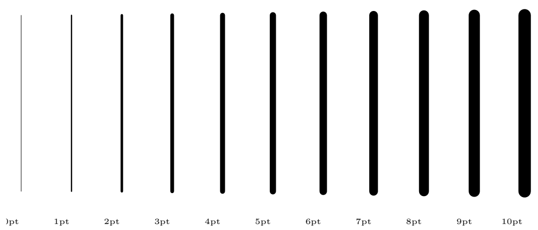
Contents

17.1. AbsoluteThickness

WMA link

`AbsoluteThickness[`p`]`
: sets the line thickness for subsequent graphics primitives to p points.

```
>> Graphics[Table[{AbsoluteThickness[t], Line[{{20 t, 10}, {20 t, 80}}],
    Text[ToString[t]<>"pt", {20 t, 0}]}, {t, 0, 10}]]
```



17.2. Arrow

WMA link

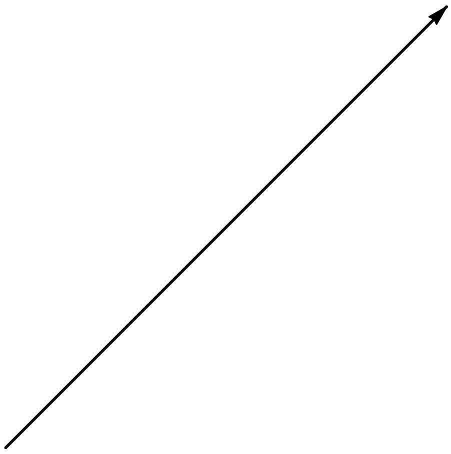
`Arrow[{`p_1, p_2`}]`
: represents a line from p_1 to p_2 that ends with an arrow at p_2.

`Arrow[{`p_1, p_2`}, `s`]`
: represents a line with arrow that keeps a distance of s from p_1 and p_2.

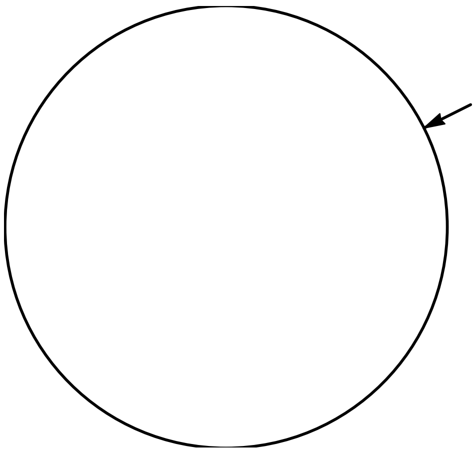
`Arrow[{`*point*$_1$, *point*$_2$`}, {`s_1, s_2`}]`
: represents a line with arrow that keeps a distance of s_1 from p_1 and a distance of s_2 from p_2.

`Arrow[{`*point*$_1$, *point*$_2$`}, {`s_1, s_2`}]`
: represents a line with arrow that keeps a distance of s_1 from p_1 and a distance of s_2 from p_2.

```
>> Graphics[Arrow[{{0,0}, {1,1}}]]
```

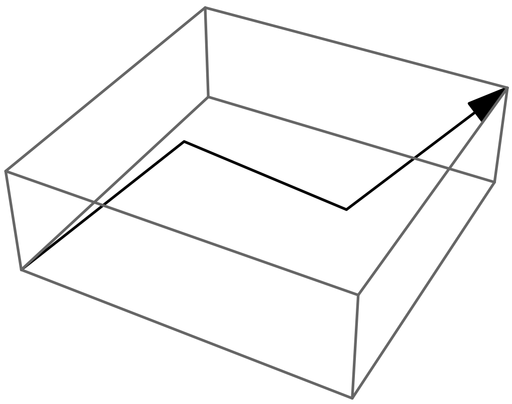


```
>> Graphics[{Circle[], Arrow[{{2, 1}, {0, 0}}, 1]}]
```



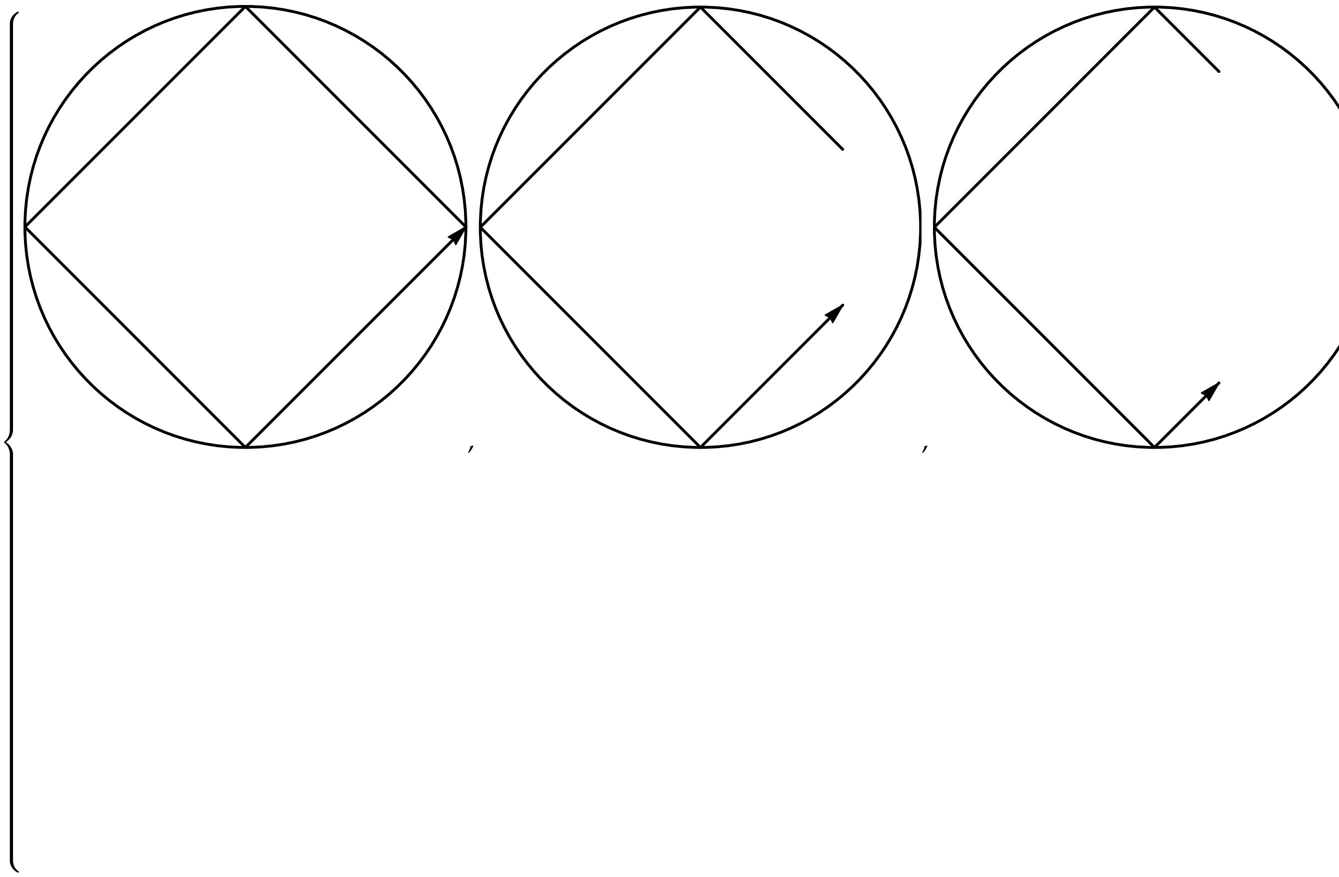
Arrows can also be drawn in 3D by giving point in three dimensions:

```
>> Graphics3D[Arrow[{{1, 1, -1}, {2, 2, 0}, {3, 3, -1}, {4, 4, 0}}]]
```



Keeping distances may happen across multiple segments:

```
>> Table[Graphics[{Circle[], Arrow[Table[{Cos[phi],Sin[phi]},{phi,0,2*Pi
   ,Pi/2}],{d, d}]}],{d,0,2,0.5}]
```



17.3. Arrowheads

WMA link

`Arrowheads[`*s*`]`
: specifies that Arrow[] draws one arrow of size *s* (relative to width of image, defaults to 0.04).

`Arrowheads[{`*spec*$_1$, *spec*$_2$, ..., *spec*$_n$`}]`
: specifies that Arrow[] draws n arrows as defined by *spec*$_1$, *spec*$_2$, ... *spec*$_n$.

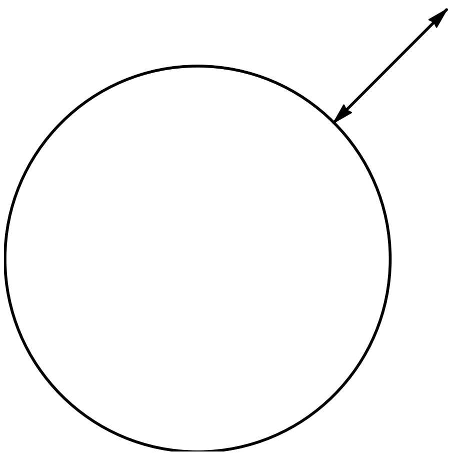
`Arrowheads[{{`*s*`}}]`
: specifies that one arrow of size *s* should be drawn.

`Arrowheads[{{`*s*, *pos*`}}]`
: specifies that one arrow of size *s* should be drawn at position *pos* (for the arrow to be on the line, *pos* has to be between 0, i.e. the start for the line, and 1, i.e. the end of the line).

`Arrowheads[{{`*s*, *pos*, *g*`}}]`
: specifies that one arrow of size *s* should be drawn at position *pos* using Graphics *g*.

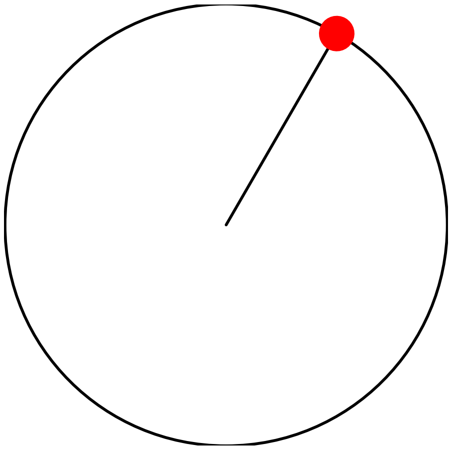
Arrows on both ends can be achieved using negative sizes:

```
>> Graphics[{Circle[],Arrowheads[{-0.04, 0.04}], Arrow[{{0, 0}, {2, 2}},
    {1,1}]}]
```

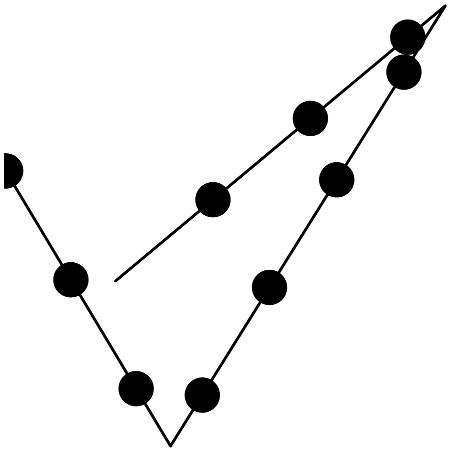


You may also specify our own arrow shapes:

```
>> Graphics[{Circle[], Arrowheads[{{0.04, 1, Graphics[{Red, Disk[]}]}}],
    Arrow[{{0, 0}, {Cos[Pi/3],Sin[Pi/3]}}]}]
```



```
>> Graphics[{Arrowheads[Table[{0.04, i/10, Graphics[Disk[]]},{i,1,10}]],
    Arrow[{{0, 0}, {6, 5}, {1, -3}, {-2, 2}}]}]
```



17.4. Circle

WMA link

`Circle[{`c_x, c_y`}, `r`]`
: draws a circle with center `($c_x$, $c_y$)` and radius r.

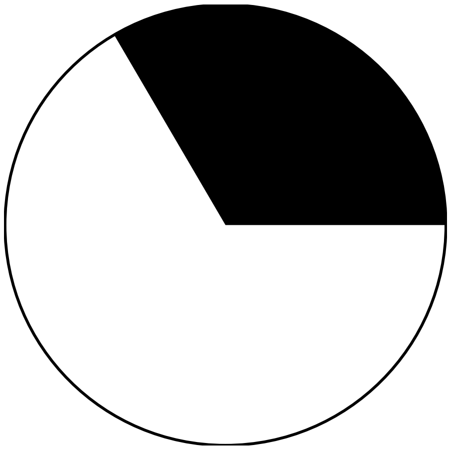
`Circle[{`c_x, c_y`}, {`r_x, r_y`}]`
: draws an ellipse.

`Circle[{`c_x, c_y`}]`
: chooses radius 1.

`Circle[]`
: chooses center `(0, 0)` and radius 1.

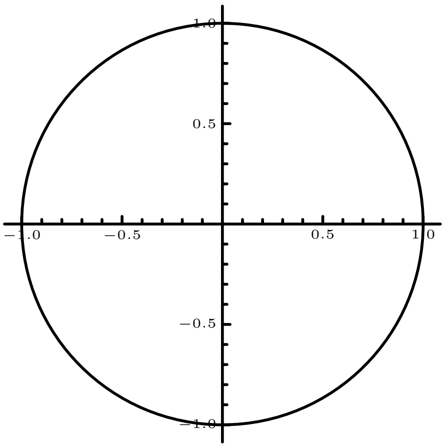
```
>> Graphics[{Red, Circle[{0, 0}, {2, 1}]}]
```

```
>> Graphics[{Circle[], Disk[{0, 0}, {1, 1}, {0, 2.1}]}]
```



Target practice:

```
>> Graphics[Circle[], Axes-> True]
```



17.5. Directive

WMA link

`Directive`[g_1, g_2, ...]
  represents a single graphics directive composed of the directives g_1, g_2, ...

17.6. Disk

WMA link

`Disk[{`c_x, c_y`}, `r`]`
: fills a circle with center (c_x, c_y) and radius r.

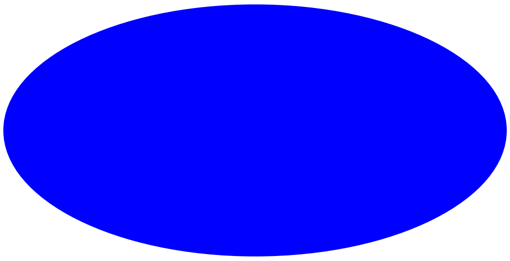
`Disk[{`c_x, c_y`}, {`r_x, r_y`}]`
: fills an ellipse.

`Disk[{`c_x, c_y`}]`
: chooses radius 1.

`Disk[]`
: chooses center $(0, 0)$' and radius 1.

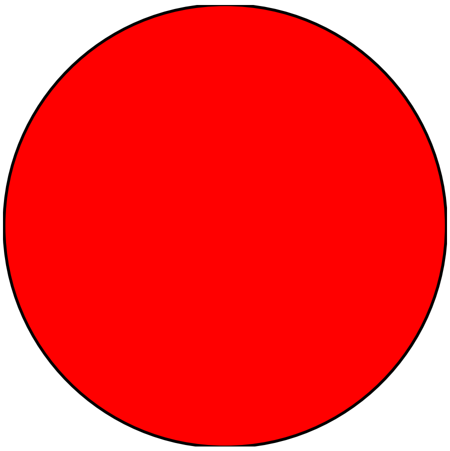
`Disk[{`x, y`}, ..., {`t_1, t_2`}]`
: is a sector from angle t_1 to t_2.

```
>> Graphics[{Blue, Disk[{0, 0}, {2, 1}]}]
```



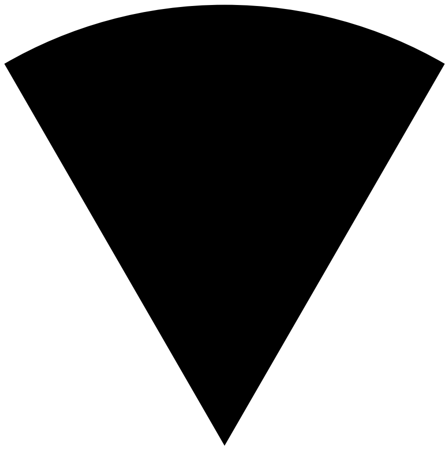
The outer border can be drawn using `EdgeForm`:

```
>> Graphics[{EdgeForm[Black], Red, Disk[]}]
```

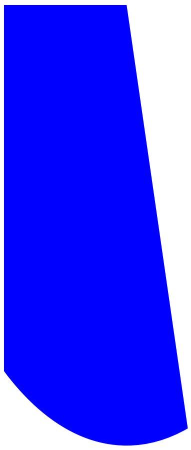


Disk can also draw sectors of circles and ellipses

```
>> Graphics[Disk[{0, 0}, 1, {Pi / 3, 2 Pi / 3}]]
```



```
>> Graphics[{Blue, Disk[{0, 0}, {1, 2}, {Pi / 3, 5 Pi / 3}]}]
```

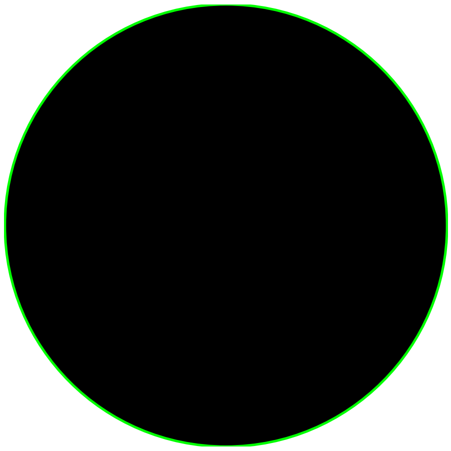


17.7. EdgeForm

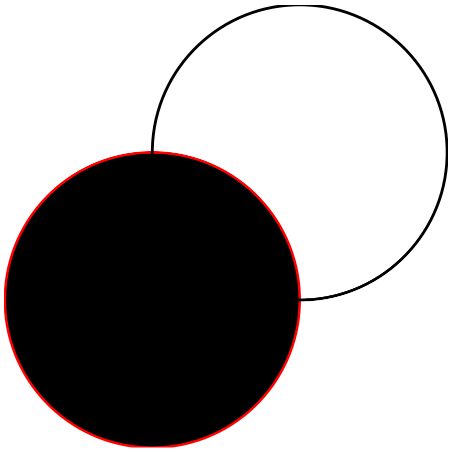
WMA link

`EdgeForm[`*g*`]`
: is a graphics directive that specifies that edges of filled graphics objects are to be drawn using the graphics directive or list of directives *g*.

```
>> Graphics[{EdgeForm[{Thick, Green}], Disk[]}]
```



```
>> Graphics[{Style[Disk[],EdgeForm[{Thick,Red}]], Circle[{1,1}]}]
```



17.8. FaceForm

WMA link

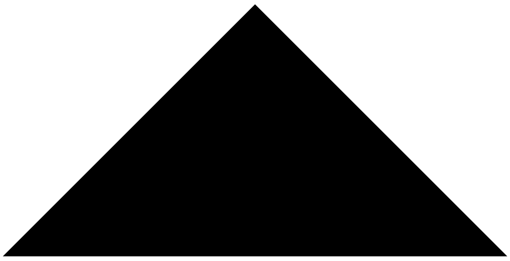
`FaceForm`[*g*]
: is a graphics directive that specifies that faces of filled graphics objects are to be drawn using the graphics directive or list of directives *g*.

17.9. FilledCurve

WMA link

`FilledCurve`[{$segment_1$, $segment_2$ ...}]
: represents a filled curve.

```
>> Graphics[FilledCurve[{Line[{{0, 0}, {1, 1}, {2, 0}}]}]]
```



```
>> Graphics[FilledCurve[{BezierCurve[{{0, 0}, {1, 1}, {2, 0}}], Line
   [{{3, 0}, {0, 2}}]}]]
```

17.10. FontColor

WMA link

`FontColor`
: is an option for Style to set the font color.

17.11. Graphics

WMA link

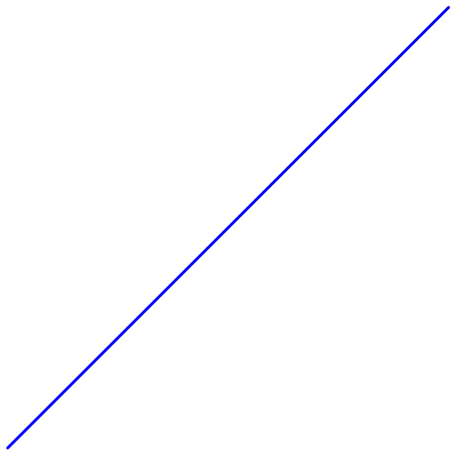
`Graphics[`*primitives, options*`]`
: represents a graphic.

Options include:

- Axes
- TicksStyle
- AxesStyle
- LabelStyle

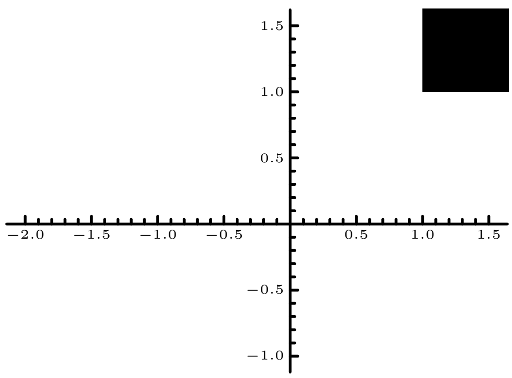
- AspectRatio
- PlotRange
- PlotRangePadding
- ImageSize
- Background

```
>> Graphics[{Blue, Line[{{0,0}, {1,1}}]}]
```



`Graphics` supports `PlotRange`:

```
>> Graphics[{Rectangle[{1, 1}]}, Axes -> True, PlotRange -> {{-2, 1.5},
   {-1, 1.5}}]
```



```
>> Graphics[{Rectangle[],Red,Disk[{1,0}]},PlotRange->{{0,1},{0,1}}]
```

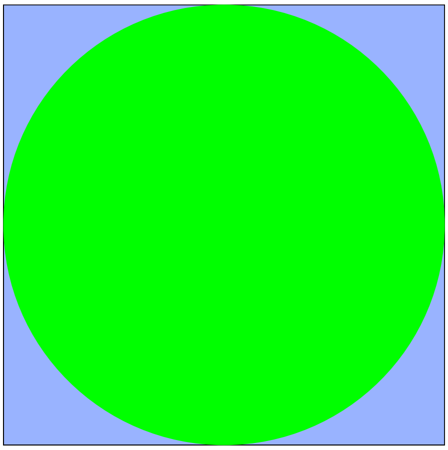
`Graphics` produces `GraphicsBox` boxes:

```
>> Graphics[Rectangle[]] // ToBoxes // Head
```

GraphicsBox

The `Background` option allows to set the color of the background:

```
>> Graphics[{Green, Disk[]}, Background->RGBColor[.6, .7, 1.]]
```



In `TeXForm`, `Graphics` produces Asymptote figures:

```
>> Graphics[Circle[]] // TeXForm
```

\begin{asy}
usepackage(”amsmath”);
size(5.869cm, 5.8333cm);
draw(ellipse((175,175),175,175), rgb(0, 0, 0)+linewidth(1.0667));
clip(box((-0.53333,0.53333), (350.53,349.47)));
\end{asy}

17.12. Inset

WMA link

`Text[`*obj*`]`
: represents an object *obj* inset in a graphic.

`Text[`*obj*, *pos*`]`
: represents an object *obj* inset in a graphic at position *pos*.

`Text[`*obj*, *pos*, *opos*`]`
: represents an object *obj* inset in a graphic at position *pos*, in away that the position *opos* of *obj* coincides with *pos* in the enclosing graphic.

17.13. Large

WMA link

`ImageSize -> Large`
: produces a large image.

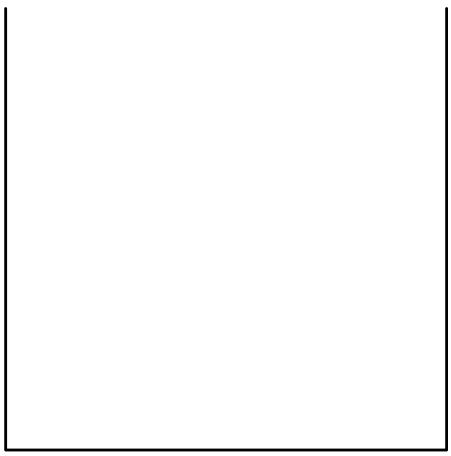
17.14. Line

WMA link

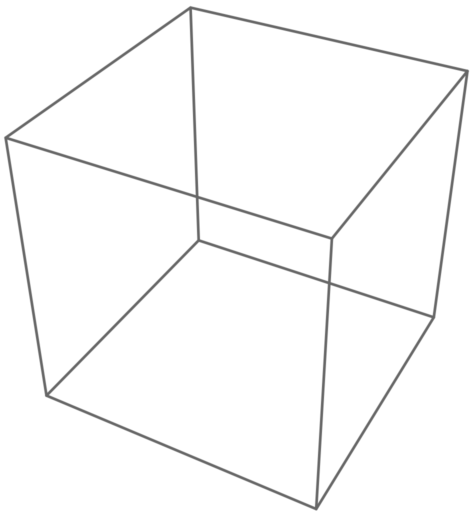
`Line[{`$point_1$, $point_2$ ...`}]`
: represents the line primitive.

`Line[{{`$point_{11}$, $point_{12}$, ...}, {$point_{21}$, $point_{22}$, ...}, ...`}]`
: represents a number of line primitives.

```
>> Graphics[Line[{{0,1},{0,0},{1,0},{1,1}}]]
```



```
>> Graphics3D[Line[{{0,0,0},{0,1,1},{1,0,0}}]]
```



17.15. Medium

WMA link

`ImageSize -> Medium`
: produces a medium-sized image.

17.16. Offset

WMA link

`Offset`[{d_x, d_y}, *position*]
: gives the position of a graphical object obtained by starting at the specified *position* and then moving by absolute offset {d_x,d_y}.

17.17. Point

WMA link

`Point`[{$point_1$, $point_2$ ...}]
: represents the point primitive.

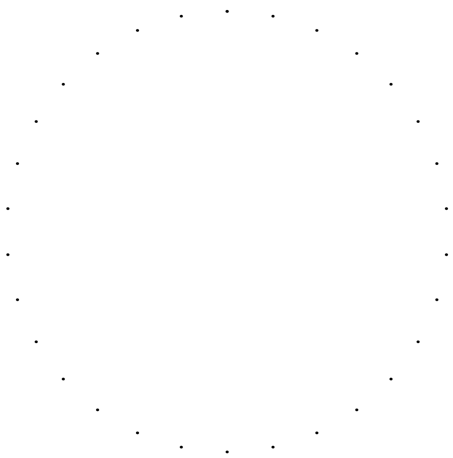
`Point`[{{$p_1$1, $p_1$2, ...}, {$p_2$1, $p_2$2, ...}, ...}]
: represents a number of point primitives.

Points are rendered if possible as circular regions. Their diameters can be specified using `PointSize`.

Points can be specified as $\{x, y\}$:

```
>> Graphics[Point[{0, 0}]]
```

```
>> Graphics[Point[Table[{Sin[t], Cos[t]}, {t, 0, 2. Pi, Pi / 15.}]]]
```



or as $\{x, y, z\}$:

```
>> Graphics3D[{Orange, PointSize[0.05], Point[Table[{Sin[t], Cos[t], 0},
    {t, 0, 2 Pi, Pi / 15.}]]}]
```

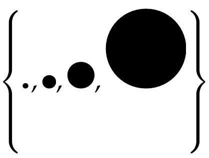
17.18. PointSize

WMA link

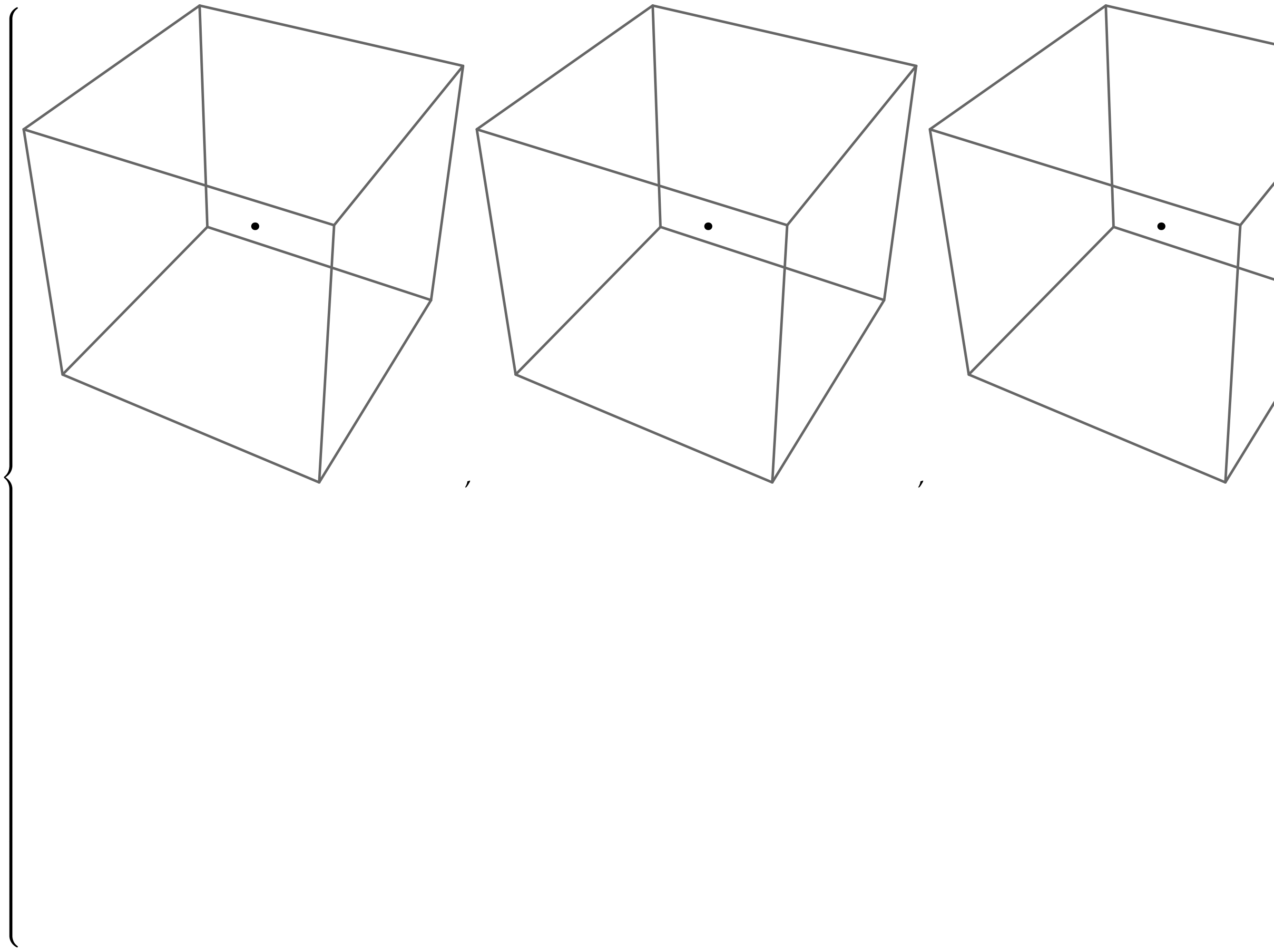
`PointSize[`*t*`]`
: sets the diameter of points to *t*, which is relative to the overall width.

`PointSize` can be used for both two- and three-dimensional graphics. The initial default pointsize is 0.008 for two-dimensional graphics and 0.01 for three-dimensional graphics.

```
>> Table[Graphics[{PointSize[r], Point[{0, 0}]}], {r, {0.02, 0.05, 0.1,
   0.3}}]
```



```
>> Table[Graphics3D[{PointSize[r], Point[{0, 0, 0}]}], {r, {0.05, 0.1,
   0.8}}]
```



17.19. Polygon

WMA link

`Polygon`[{$point_1$, $point_2$ ...}]
: represents the filled polygon primitive.

`Polygon`[{{$p_1 1$, $p_1 2$, ...}, {$p_2 1$, $p_2 2$, ...}, ...}]
: represents a number of filled polygon primitives.

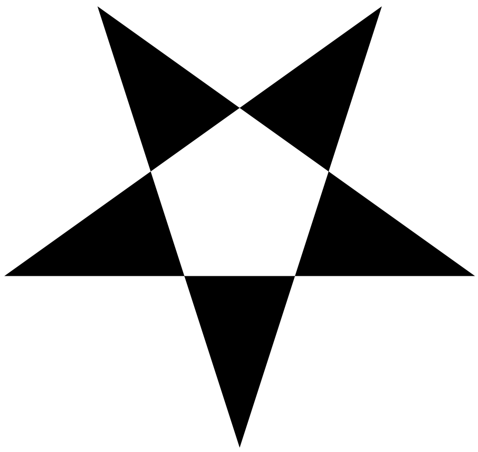
A Right Triangle:

```
>> Graphics[Polygon[{{1,0},{0,0},{0,1}}]]
```

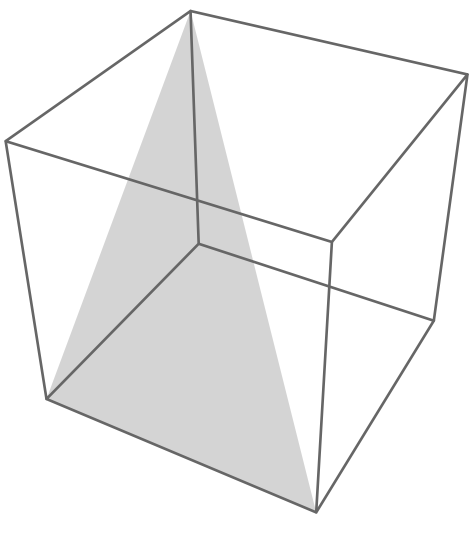
Notice that there is a line connecting from the last point to the first one.

A point is an element of the polygon if a ray from the point in any direction in the plane crosses the boundary line segments an odd number of times.

```
>> Graphics[Polygon[{{150,0},{121,90},{198,35},{102,35},{179,90}}]]
```



```
>> Graphics3D[Polygon[{{0,0,0},{0,1,1},{1,0,0}}]]
```



17.20. Rectangle

WMA link

`Rectangle[{`x_{min}, y_{min}`}]`
: represents a unit square with bottom-left corner at {x_{min}, y_{min}}.

'Rectangle[{x_{min}, y_{min}}, {x_{max}, y_{max}}]
: is a rectangle extending from {x_{min}, y_{min}} to {x_{max}, y_{max}}.

```
>> Graphics[Rectangle[]]
```

```
>> Graphics[{Blue, Rectangle[{0.5, 0}], Orange, Rectangle[{0, 0.5}]}]
```

17.21. RegularPolygon

WMA link

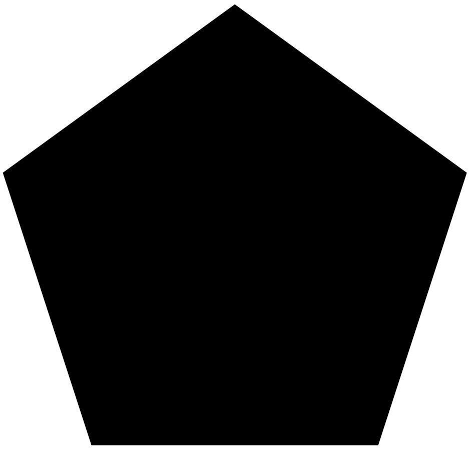
`RegularPolygon[`n`]`
: gives the regular polygon with n edges.

`RegularPolygon[`r, n`]`
: gives the regular polygon with n edges and radius r.

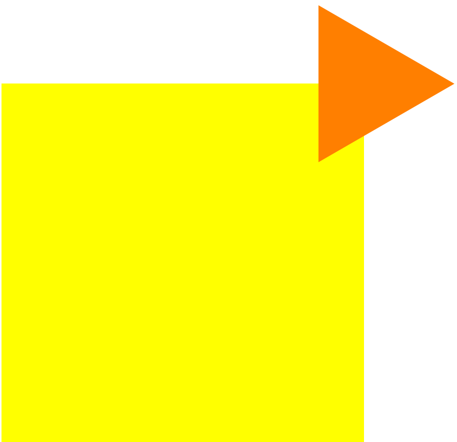
`RegularPolygon[{`r, ϕ`}, `n`]`
: gives the regular polygon with radius r with one vertex drawn at angle ϕ.

`RegularPolygon[{`x, y`}, `r, n`]`
: gives the regular polygon centered at the position $\{x, y\}$.

```
>> Graphics[RegularPolygon[5]]
```



```
>> Graphics[{Yellow, Rectangle[], Orange, RegularPolygon[{1, 1}, {0.25,
   0}, 3]}]
```

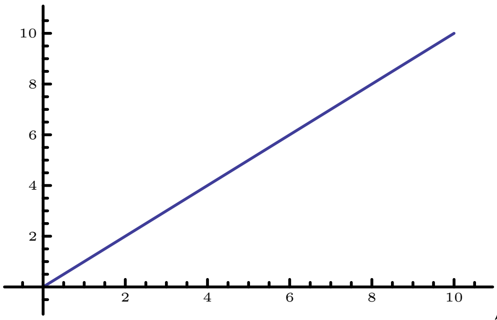
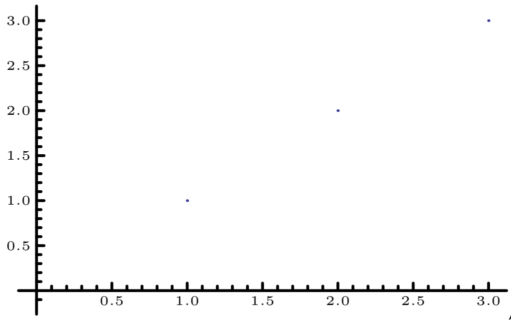


17.22. Show

WMA link

`Show[`*graphics*, *options*`]`
: shows a list of graphics with the specified options added.

```
>> Show[{Plot[x, {x, 0, 10}], ListPlot[{1,2,3}]}]
```

{ ,  , AspectRatio

− > Automatic, Axes− > False, AxesStyle− > {} , Background
− > Automatic, ImageSize− > Automatic, LabelStyle− > {} , PlotRange
− > Automatic, PlotRangePadding− > Automatic, TicksStyle− > {} }

17.23. Small

WMA link

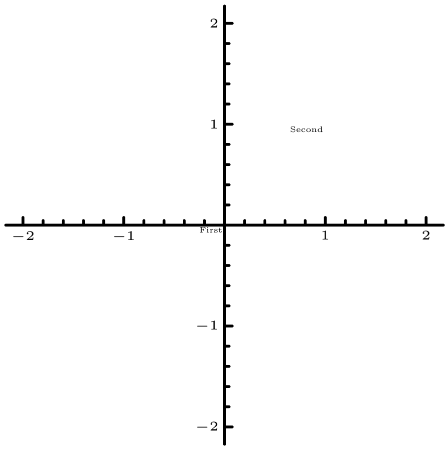
```
ImageSize -> Small
```
produces a small image.

17.24. Text

WMA link

`Text`["*text*", {x, y}]
: draws *text* centered on position {x, y}.

```
>> Graphics[{Text["First", {0, 0}], Text["Second", {1, 1}]}, Axes->True,
   PlotRange->{{-2, 2}, {-2, 2}}]
```



17.25. Thick

WMA link

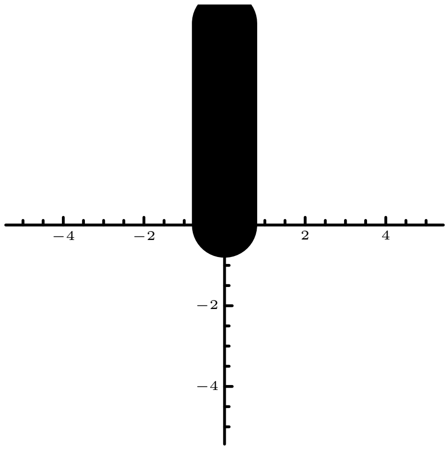
`Thick`
: sets the line width for subsequent graphics primitives to 2pt.

17.26. Thickness

WMA link

`Thickness`[t]
: sets the line thickness for subsequent graphics primitives to t times the size of the plot area.

```
>> Graphics[{Thickness[0.2], Line[{{0, 0}, {0, 5}}]}, Axes->True,
   PlotRange->{{-5, 5}, {-5, 5}}]
```



17.27. Thin

WMA link

`Thin`
: sets the line width for subsequent graphics primitives to 0.5pt.

17.28. Tiny

WMA link

`ImageSize -> Tiny`
: produces a tiny image.

18. Evaluation Control

Mathics3 takes an expression that it is given, and evaluates it. Built into the evaluation are primitives that allow finer control over the process of evaluation in cases where it is needed.

Contents

18.1. $IterationLimit

WMA link

`$IterationLimit`
: specifies the maximum number of times a reevaluation of an expression may happen.

Calculations terminated by `$IterationLimit` return `$Aborted`:

```
>> $IterationLimit
   4096
```

The iteration limit protects against runaway definitions:

```
>> Block[{$IterationLimit = 20}, yin := yang; yang := yin; yin]
   Iteration limit of 20 exceeded.
   $Aborted
```

18.2. $RecursionLimit

WMA link

`$RecursionLimit`
: specifies the maximum allowable recursion depth after which a calculation is terminated.

Calculations terminated by `$RecursionLimit` return `$Aborted`:

```
>> a = a + a
Recursion depth of 200 exceeded.
```
$Aborted

```
>> $RecursionLimit
```
200

```
>> $RecursionLimit = x;
Cannot set $RecursionLimit to x; value must be an integer between 20
and 512; use the MATHICS_MAX_RECURSION_DEPTH environment variable to
allow higher limits.
```

```
>> $RecursionLimit = 512
```
512

```
>> a = a + a
Recursion depth of 512 exceeded.
```
$Aborted

18.3. Evaluate

WMA link

`Evaluate[`*expr*`]`
  forces evaluation of *expr*, even if it occurs inside a held argument or a `Hold` form.

Create a function f with a held argument:

```
>> SetAttributes[f, HoldAll]
```

```
>> f[1 + 2]
```
$f[1+2]$

`Evaluate` forces evaluation of the argument, even though f has the `HoldAll` attribute:

```
>> f[Evaluate[1 + 2]]
```
$f[3]$

```
>> Hold[Evaluate[1 + 2]]
```
Hold [3]

```
>> HoldComplete[Evaluate[1 + 2]]
```
HoldComplete [Evaluate [1 + 2]]

```
>> Evaluate[Sequence[1, 2]]
```
Sequence [1, 2]

18.4. Hold

WMA link

`Hold`[*expr*]
    prevents *expr* from being evaluated.

>> **`Attributes[Hold]`**
    {HoldAll, Protected}

18.5. HoldComplete

WMA link

`HoldComplete`[*expr*]
    prevents *expr* from being evaluated, and also prevents `Sequence` objects from being spliced into argument lists.

>> **`Attributes[HoldComplete]`**
    {HoldAllComplete, Protected}

18.6. HoldForm

WMA link

`HoldForm`[*expr*]
    is equivalent to `Hold[$expr$]`, but prints as *expr*.

>> **`HoldForm[1 + 2 + 3]`**
    $1 + 2 + 3$

`HoldForm` has attribute `HoldAll`:

>> **`Attributes[HoldForm]`**
    {HoldAll, Protected}

18.7. ReleaseHold

WMA link

`ReleaseHold[`*expr*`]`
: removes any `Hold`, `HoldForm`, `HoldPattern` or `HoldComplete` head from *expr*.

```
>> x = 3;
```

```
>> Hold[x]
```
Hold [x]

```
>> ReleaseHold[Hold[x]]
```
3

```
>> ReleaseHold[y]
```
y

18.8. Sequence

WMA link

`Sequence[`x_1, x_2, ...`]`
: represents a sequence of arguments to a function.

`Sequence` is automatically spliced in, except when a function has attribute `SequenceHold` (like assignment functions).

```
>> f[x, Sequence[a, b], y]
```
$f\left[x, a, b, y\right]$

```
>> Attributes[Set]
```
{HoldFirst, Protected, SequenceHold}

```
>> a = Sequence[b, c];
```

```
>> a
```
Sequence [b, c]

Apply `Sequence` to a list to splice in arguments:

```
>> list = {1, 2, 3};
```

```
>> f[Sequence @@ list]
```
f [1, 2, 3]

Inside `Hold` or a function with a held argument, `Sequence` is spliced in at the first level of the argument:

```
>> Hold[a, Sequence[b, c], d]
```
Hold [a, b, c, d]

If `Sequence` appears at a deeper level, it is left unevaluated:

```
>> Hold[{a, Sequence[b, c], d}]
```
Hold [{a, Sequence [b, c], d}]

18.9. Unevaluated

WMA link

`Unevaluated[`*expr*`]`
: temporarily leaves *expr* in an unevaluated form when it appears as a function argument.

`Unevaluated` is automatically removed when function arguments are evaluated:

```
>> Sqrt[Unevaluated[x]]
```
$\sqrt{x}$

```
>> Length[Unevaluated[1+2+3+4]]
```
4

`Unevaluated` has attribute `HoldAllComplete`:

```
>> Attributes[Unevaluated]
```
{HoldAllComplete, Protected}

`Unevaluated` is maintained for arguments to non-executed functions:

```
>> f[Unevaluated[x]]
```
f [Unevaluated [x]]

Likewise, its kept in flattened arguments and sequences:

```
>> Attributes[f] = {Flat};
```

```
>> f[a, Unevaluated[f[b, c]]]
```
f [a, Unevaluated [b], Unevaluated [c]]

```
>> g[a, Sequence[Unevaluated[b], Unevaluated[c]]]
```
g [a, Unevaluated [b], Unevaluated [c]]

However, unevaluated sequences are kept:

```
>> g[Unevaluated[Sequence[a, b, c]]]
```
g [Unevaluated [Sequence [a, b, c]]]

19. Expression Structure

Contents

19.1. Expression Sizes and Signatures

19.1.1. ByteCount

WMA link

`ByteCount`[*expr*]
    gives the internal memory space used by *expr*, in bytes.

The results may heavily depend on the Python implementation in use.

19.1.2. Hash

Hash function (WMA link)

`Hash`[*expr*]
    returns an integer hash for the given *expr*.
`Hash`[*expr*, *type*]
    returns an integer hash of the specified *type* for the given *expr*.
    The types supported are “MD5”, “Adler32”, “CRC32”, “SHA”, “SHA224”, “SHA256”, “SHA384”, and “SHA512”.
`Hash`[*expr*, *type*, *format*]
    Returns the hash in the specified format.

```
>> Hash["The Adventures of Huckleberry Finn"]
```
213425047836523694663619736686226550816

```
>> Hash["The Adventures of Huckleberry Finn", "SHA256"]
```
95092649594590384288057183408609254918934351811669818342876362244564858646638

```
>> Hash[1/3]
```
56073172797010645108327809727054836008

```
>> Hash[{a, b, {c, {d, e, f}}}]
```
135682164776235407777080772547528225284

```
>> Hash[SomeHead[3.1415]]
```
47205238268993602951487675588386522878

```
>> Hash[{a, b, c}, "xyzstr"]
```
$\text{Hash}\left[\{a, b, c\}, \text{xyzstr}, \text{Integer}\right]$

19.1.3. LeafCount

WMA link

`LeafCount[`*expr*`]`
: returns the total number of indivisible subexpressions in *expr*.

```
>> LeafCount[1 + x + y^a]
```
6

```
>> LeafCount[f[x, y]]
```
3

```
>> LeafCount[{1 / 3, 1 + I}]
```
7

```
>> LeafCount[Sqrt[2]]
```
5

```
>> LeafCount[100!]
```
1

19.2. Head-Related Operations

19.2.1. Operate

WMA link

`Operate[`*p*, *expr*`]`
: applies *p* to the head of *expr*.

`Operate[`*p*, *expr*, *n*`]`
: applies *p* to the *n*th head of *expr*.

>> `Operate[p, f[a, b]]`

$p[f][a,b]$

The default value of n is 1:

>> `Operate[p, f[a, b], 1]`

$p[f][a,b]$

With n=0, `Operate` acts like `Apply`:

>> `Operate[p, f[a][b][c], 0]`

$p[f[a][b][c]]$

19.2.2. Through

WMA link

`Through`[p[f][x]]
gives p[f[x]].

>> `Through[f[g][x]]`

$f[g[x]]$

>> `Through[p[f, g][x]]`

$p[f[x], g[x]]$

19.3. Structural Expression Functions

19.3.1. Depth

WMA link

`Depth`[$expr$]
gives the depth of $expr$.

The depth of an expression is defined as one plus the maximum number of `Part` indices required to reach any part of $expr$, except for heads.

>> `Depth[x]`

1

>> `Depth[x + y]`

2

```
>> Depth[{{{{x}}}}]
   5
```

Complex numbers are atomic, and hence have depth 1:

```
>> Depth[1 + 2 I]
   1
```

`Depth` ignores heads:

```
>> Depth[f[a, b][c]]
   2
```

19.3.2. FreeQ

WMA link

`FreeQ`[*expr*, *x*]
returns `True` if *expr* does not contain the expression *x*.

```
>> FreeQ[y, x]
   True
>> FreeQ[a+b+c, a+b]
   False
>> FreeQ[{1, 2, a^(a+b)}, Plus]
   False
>> FreeQ[a+b, x_+y_+z_]
   True
>> FreeQ[a+b+c, x_+y_+z_]
   False
>> FreeQ[x_+y_+z_][a+b]
   True
```

19.3.3. Level

WMA link

`Level`[*expr*, *levelspec*]
gives a list of all subexpressions of *expr* at the level(s) specified by *levelspec*.

Level uses standard level specifications:

n
    levels 1 through *n*
`Infinity`
    all levels from level 1
`{$n$}`
    level *n* only
`{$m$, $n$}`
    levels *m* through *n*

Level 0 corresponds to the whole expression.

A negative level `-$n$` consists of parts with depth *n*.

Level -1 is the set of atoms in an expression:

```
>> Level[a + b ^ 3 * f[2 x ^ 2], {-1}]
```
$\{a, b, 3, 2, x, 2\}$

```
>> Level[{{{{a}}}}, 3]
```
$\{\{a\}, \{\{a\}\}, \{\{\{a\}\}\}\}$

```
>> Level[{{{{a}}}}, -4]
```
$\{\{\{\{a\}\}\}\}$

```
>> Level[{{{{a}}}}, -5]
```
$\{\}$

```
>> Level[h0[h1[h2[h3[a]]]], {0, -1}]
```
$\{a, \text{h3}[a], \text{h2}[\text{h3}[a]], \text{h1}[\text{h2}[\text{h3}[a]]], \text{h0}[\text{h1}[\text{h2}[\text{h3}[a]]]]\}$

Use the option `Heads -> True` to include heads:

```
>> Level[{{{{a}}}}, 3, Heads -> True]
```
$\{\text{List}, \text{List}, \text{List}, \{a\}, \{\{a\}\}, \{\{\{a\}\}\}\}$

```
>> Level[x^2 + y^3, 3, Heads -> True]
```
$\left\{\text{Plus}, \text{Power}, x, 2, x^2, \text{Power}, y, 3, y^3\right\}$

```
>> Level[a ^ 2 + 2 * b, {-1}, Heads -> True]
```
$\{\text{Plus}, \text{Power}, a, 2, \text{Times}, 2, b\}$

```
>> Level[f[g[h]][x], {-1}, Heads -> True]
```
$\{f, g, h, x\}$

```
>> Level[f[g[h]][x], {-2, -1}, Heads -> True]
```
$\{f, g, h, g[h], x, f[g[h]][x]\}$

19.3.4. MapApply (`@@@`)

WMA link

`MapApply`[f, $expr$]
`$f$ @@@ $expr$`
: is equivalent to `Apply[$f$, $expr$, {1}]`.

>> `f @@@ {{a, b}, {c, d}}`
$\{f[a,b], f[c,d]\}$

19.3.5. Null

WMA link

`Null`
: is the implicit result of expressions that do not yield a result.

>> `FullForm[a:=b]`
Null

It is not displayed in StandardForm,

>> `a:=b`

in contrast to the empty string:

>> `""`

19.3.6. SortBy

WMA link

`SortBy`[*list*, f]
: sorts *list* (or the elements of any other expression) according to canonical ordering of the keys that are extracted from the *list*'s elements using f. Chunks of elements that appear the same under f are sorted according to their natural order (without applying f).

`SortBy`[f]
: creates an operator function that, when applied, sorts by f.

>> `SortBy[{{5, 1}, {10, -1}}, Last]`
$\{\{10, -1\}, \{5, 1\}\}$

>> `SortBy[Total][{{5, 1}, {10, -9}}]`
$\{\{10, -9\}, \{5, 1\}\}$

20. File Formats

Built-in Importers.

Contents

20.1. HTML

Basic implementation for a HTML importer.

20.1.1. HTML‘DataImport

```
HTML`DataImport[``filename']'
```
imports data from a HTML file.

```
>> Import["ExampleData/PrimeMeridian.html", "Data"][[1, 1, 2, 3]]
```
{Washington, D.C., 77°03 56.07 W (1897) or 77°04 02.24 W (NAD 27) or 77°04 01.16 W (NAD 83), New Naval Observatory meridian}

20.1.2. HTML‘FullDataImport

```
HTML`FullDataImport[``filename']'
```
imports data from a HTML file.

20.1.3. HTML‘Parser‘HTMLGet

HTMLGet[`str`]
Parses `str` as HTML code.

20.1.4. HTML‘Parser‘HTMLGetString

`` HTML`Parser`HTMLGetString[``string']' ``
parses HTML code contained in “string”.

20.1.5. HTML‘HyperlinksImport

`` HTML`HyperlinksImport[``filename']' ``
imports hyperlinks from a HTML file.

```
>> Import["ExampleData/PrimeMeridian.html", "Hyperlinks"][[1]]
```
/wiki/Prime_meridian_(Greenwich)

20.1.6. HTML‘ImageLinksImport

`` HTML`ImageLinksImport[``filename']' ``
imports links to the images included in a HTML file.

```
>> Import["ExampleData/PrimeMeridian.html", "ImageLinks"][[6]]
```
//upload.wikimedia.org/wikipedia/commons/thumb/d/d5/Prime_meridian.jpg/180px-Prime_meridian.jpg

20.1.7. HTML‘PlaintextImport

`` HTML`PlaintextImport[``filename']' ``
imports plane text from a HTML file.

```
>> DeleteDuplicates[StringCases[Import["ExampleData/PrimeMeridian.html
   "], RegularExpression["Wiki[a-z]+"]]]
```
{Wikipedia, Wikidata, Wikibase, Wikimedia}

20.1.8. HTML‘SourceImport

```
HTML`SourceImport[``filename']'
```
imports source code from a HTML file.

```
>> DeleteDuplicates[StringCases[Import["ExampleData/PrimeMeridian.html",
    "Source"], RegularExpression["<t[a-z]+>"]]]
```
{<title>, <tr>, <th>, <td>}

20.1.9. HTML‘TitleImport

```
HTML`TitleImport[``filename']'
```
imports the title string from a HTML file.

```
>> Import["ExampleData/PrimeMeridian.html", "Title"]
```
Prime meridian - Wikipedia

20.1.10. HTML‘XMLObjectImport

```
HTML`XMLObjectImport[``filename']'
```
imports XML objects from a HTML file.

```
>> Part[Import["ExampleData/PrimeMeridian.html", "XMLObject"], 2, 3, 1,
   3, 2]
```
XMLElement [title, {} , {Prime meridian - Wikipedia}]

20.2. XML

Basic implementation for an XML importer.

20.2.1. XML‘PlaintextImport

WMA link

```
XML`PlaintextImport[``string']'
```
parses “string” as XML code, and returns it as plain text.

```
>> StringReplace[StringTake[Import["ExampleData/InventionNo1.xml", "
   Plaintext"],31],FromCharacterCode[10]->"/"]
```

MuseScore 1.2/2012-09-12/5.7/40

20.2.2. XML‘TagsImport

`XML`TagsImport[``string']'`
parses “string” as XML code, and returns a list with the tags found.

```
>> Take[Import["ExampleData/InventionNo1.xml", "Tags"], 10]
```

{accidental, alter, arpeggiate, articulations, attributes, backup, bar-style, barline, beam, beat-type}

20.2.3. XMLElement

WMA link

`XMLElement`[*tag*, {$attr_1$, val_1, ...}, {*data*, ...}]
represents an element in symbolic XML.

20.2.4. XML‘Parser‘XMLGet

`XMLGet[...]`
Internal. Document me.

20.2.5. XML‘Parser‘XMLGetString

`XML`Parser`XMLGetString[``string']'`
parses “string” as XML code, and returns an XMLObject.

```
>> Head[XML`Parser`XMLGetString["<a></a>"]]
```

XMLObject [Document]

20.2.6. XMLObject

WMA link

```
XMLObject[``type']'
```
represents the head of an XML object in symbolic XML.

20.2.7. XML‘XMLObjectImport

```
XML`XMLObjectImport[``string']'
```
parses “string” as XML code, and returns a list of XMLObjects found.

```
>> Part[Import["ExampleData/InventionNo1.xml", "XMLObject"], 2, 3, 1]
```

XMLElement [identification, {} , {XMLElement [
encoding, {} , {XMLElement [software, {} , {MuseScore
1.2}] , XMLElement [encoding-date, {} , {2012-09-12}] }] }]

```
>> Part[Import["ExampleData/Namespaces.xml"], 2]
```

XMLElement [book, {{http://www.w3.org/2000/xmlns/, xmlns}
− > urn:loc.gov:books} , {XMLElement [
title, {} , {Cheaper by the Dozen}] , XMLElement [
{urn:ISBN:0- 395-36 341-6, number} , {} , {1568491379}] , XMLElement [
notes, {} , {XMLElement [
p, {{http://www.w3.org/2000/xmlns/, xmlns} − > http://www.w3.org/1999/xhtml} , {This
is a, XMLElement [i, {} , {funny, book!}] }] }] }]

21. File Operations

Contents

21.1. File Path Manipulation

21.1.1. FileNameDrop

WMA link

`FileNameDrop`[“*path*”, *n*]
drops the first *n* path elements in the file name *path*.
`FileNameDrop`[“*path*”, -*n*]
drops the last *n* path elements in the file name *path*.
`FileNameDrop`[“*path*”, {*m*, *n*}]
drops elements *m* through *n* path elements in the file name *path*.
`FileNameDrop`[“*path*”]
drops the last path elements in the file name *path*.

```
>> path = FileNameJoin[{"a","b","c"}]
```
a/b/c

```
>> FileNameDrop[path, -1]
```
a/b

A shorthand for the above:

```
>> FileNameDrop[path]
```
a/b

21.2. File Properties

21.2.1. FileDate

WMA link

`FileDate`[*file*, *types*]
: returns the time and date at which the file was last modified.

```
>> FileDate["ExampleData/sunflowers.jpg"]
```
$\{2023, 1, 14, 22, 50, 39.\}$

```
>> FileDate["ExampleData/sunflowers.jpg", "Access"]
```
$\{2025, 9, 3, 3, 42, 53.\}$

```
>> FileDate["ExampleData/sunflowers.jpg", "Creation"]
```
Missing [NotApplicable]

```
>> FileDate["ExampleData/sunflowers.jpg", "Change"]
```
$\{2024, 10, 5, 12, 50, 53.\}$

```
>> FileDate["ExampleData/sunflowers.jpg", "Modification"]
```
$\{2023, 1, 14, 22, 50, 39.\}$

```
>> FileDate["ExampleData/sunflowers.jpg", "Rules"]
```
{Access− > {2025, 9, 3, 3, 42, 53.} , Creation− > Missing [NotApplicable] , Change− > {2024, 10, 5, 12, 50, 53.} , Modification − > {2023, 1, 14, 22, 50, 39.}}

21.2.2. FileHash

WMA link

`FileHash`[*file*]
: returns an integer hash for the given *file*.

`FileHash`[*file*, *type*]
: returns an integer hash of the specified *type* for the given *file*.
The types supported are “MD5”, “Adler32”, “CRC32”, “SHA”, “SHA224”, “SHA256”, “SHA384”, and “SHA512”.

`FileHash`[*file*, *type*, *format*]
: gives a hash code in the specified format.

```
>> FileHash["ExampleData/sunflowers.jpg"]
```
109937059621979839952736809235486742106

```
>> FileHash["ExampleData/sunflowers.jpg", "MD5"]
```
109937059621979839952736809235486742106

```
>> FileHash["ExampleData/sunflowers.jpg", "Adler32"]
```
1607049478

```
>> FileHash["ExampleData/sunflowers.jpg", "SHA256"]
```
111619807552579450300684600241129773909359865098672286468229443390003894913065

21.2.3. FileType

WMA link

`FileType["file"]`
: gives the type of a file, a string. This is typically `File`, `Directory` or `None`.

```
>> FileType["ExampleData/sunflowers.jpg"]
```
File

```
>> FileType["ExampleData"]
```
Directory

```
>> FileType["ExampleData/nonexistent"]
```
None

21.2.4. SetFileDate

WMA link

`SetFileDate["file"]`
: set the file access and modification dates of *file* to the current date.

`SetFileDate["file", date]`
: set the file access and modification dates of *file* to the specified date list.

`SetFileDate["file", date, "type"]`
: set the file date of *file* to the specified date list. The "*type*" can be one of "*Access*", "*Creation*", "*Modification*", or `All`.

Create a temporary file (for example purposes)

```
>> tmpfilename = $TemporaryDirectory <> "/tmp0";
```

```
>> Close[OpenWrite[tmpfilename]];
```

```
>> SetFileDate[tmpfilename, {2002, 1, 1, 0, 0, 0.}, "Access"];
```

21.3. File Utilities

21.3.1. FindList

WMA link

`FindList`[*file*, *text*]
: returns a list of all lines in *file* that contain *text*.

`FindList`[*file*, {$text_1$, $text_2$, ...}]
: returns a list of all lines in *file* that contain any of the specified string.

`FindList`[{$file_1$, $file_2$, ...}, ...]
: returns a list of all lines in any of the *filei* that contain the specified strings.

```
>> stream = FindList["ExampleData/EinsteinSzilLetter.txt", "uranium"];
```

```
>> Length[stream]
```

7

```
>> FindList["ExampleData/EinsteinSzilLetter.txt", "uranium", 1]
```

{in manuscript, leads me to expect that the element uranium may be turned into}

22. Forms of Input and Output

A *Form* format specifies the way Mathics Expression input is read or output written.

The variable $OutputForms' 22.2.1 has a list of Forms defined.

See also WMA link.

Contents

22.1. Form Functions

22.1.1. BaseForm

WMA link

`BaseForm`[*expr*, *n*]
  prints numbers in *expr* in base *n*.

A binary integer:

>> **`BaseForm[33, 2]`**
SubscriptBox [100001, 2]

A hexadecimal number:

>> **`BaseForm[234, 16]`**
SubscriptBox [ea, 16]

A binary real number:

```
>> BaseForm[12.3, 2]
```
SubscriptBox [1100.01001100110011001, 2]

```
>> BaseForm[-42, 16]
```
SubscriptBox [-2a, 16]

```
>> BaseForm[x, 2]
```
x

```
>> BaseForm[12, 3] // FullForm
```
BaseForm [12, 3]

Bases must be between 2 and 36:

```
>> BaseForm[12, -3]
```

```
>> BaseForm[12, 100]
```

22.1.2. FullForm

WMA link

> `FullForm[`*expr*`]`
> displays the underlying form of *expr*.

```
>> FullForm[a + b * c]
```
Plus [a, Times [b, c]]

```
>> FullForm[2/3]
```
Rational [2, 3]

```
>> FullForm["A string"]
```
“A string”

22.1.3. InputForm

WMA link

> `InputForm[`*expr*`]`
> displays *expr* in an unambiguous form suitable for input.

```
>> InputForm[a + b * c]
```
$a + b * c$

>> `InputForm["A string"]`
"A string"

>> `InputForm[f'[x]]`
Derivative $[1]\left[f\right][x]$

>> `InputForm[Derivative[1, 0][f][x]]`
Derivative $[1,0]\left[f\right][x]$

22.1.4. MathMLForm

WMA link

`MathMLForm[`*expr*`]`
displays *expr* as a MathML expression.

>> `MathMLForm[HoldForm[Sqrt[a^3]]]`
<math display="block"><msqrt> <msup><mi>a</mi>
<mn>3</mn></msup> </msqrt></math>

>> `MathMLForm[\[Mu]]`
<math display="block"><mi>μ</mi></math>

This can causes the TeX to fail # » MathMLForm[Graphics[Text["μ"]]] # = ...

= ...

22.1.5. MatrixForm

WMA link

`MatrixForm[`*m*`]`
displays a matrix *m*, hiding the underlying list structure.

>> `Array[a,{4,3}]//MatrixForm`

$$\begin{pmatrix} a[1,1] & a[1,2] & a[1,3] \\ a[2,1] & a[2,2] & a[2,3] \\ a[3,1] & a[3,2] & a[3,3] \\ a[4,1] & a[4,2] & a[4,3] \end{pmatrix}$$

22.1.6. NumberForm

WMA link

`NumberForm`[*expr*, *n*]
: prints a real number *expr* with *n*-digits of precision.

`NumberForm`[*expr*, {*n*, *f*}]
: prints with *n*-digits and *f* digits to the right of the decimal point.

```
>> NumberForm[N[Pi], 10]
```
3.141592654

```
>> NumberForm[N[Pi], {10, 6}]
```
3.141593

```
>> NumberForm[N[Pi]]
```
3.14159

22.1.7. OutputForm

WMA link

`OutputForm`[*expr*]
: displays *expr* in a plain-text form.

```
>> OutputForm[f'[x]]
```
$f'[x]$

```
>> OutputForm[Derivative[1, 0][f][x]]
```
Derivative $[1,0]\left[f\right][x]$

`OutputForm` is used by default:

```
>> OutputForm[{"A string", a + b}]
```
$\{$A string, $a+b\}$

```
>> {"A string", a + b}
```
$\{$A string, $a+b\}$

>> `OutputForm[Graphics[Rectangle[]]]`

22.1.8. PythonForm

`PythonForm[`*expr*`]`
: returns an approximate equivalent of *expr* in Python, when that is possible. We assume that Python has SymPy imported. No explicit import will be include in the result.

>> `PythonForm[Infinity]`

math.inf

>> `PythonForm[Pi]`

sympy.pi

>> `E // PythonForm`

sympy.E

>> `{1, 2, 3} // PythonForm`

(1, 2, 3)

22.1.9. StandardForm

WMA link

`StandardForm[`*expr*`]`
: displays *expr* in the default form.

>> `StandardForm[a + b * c]`

$a + bc$

>> `StandardForm["A string"]`

A string

>> `f'[x]`

$f'[x]$

22.1.10. SympyForm

`SympyForm[`*expr*`]`
: returns an Sympy *expr* in Python. Sympy is used internally to implement a number of Mathics functions, like Simplify.

>> `SympyForm[Pi^2]`

pi**2

>> `E^2 + 3E // SympyForm`

exp(2) + 3*E

22.1.11. TableForm

WMA link

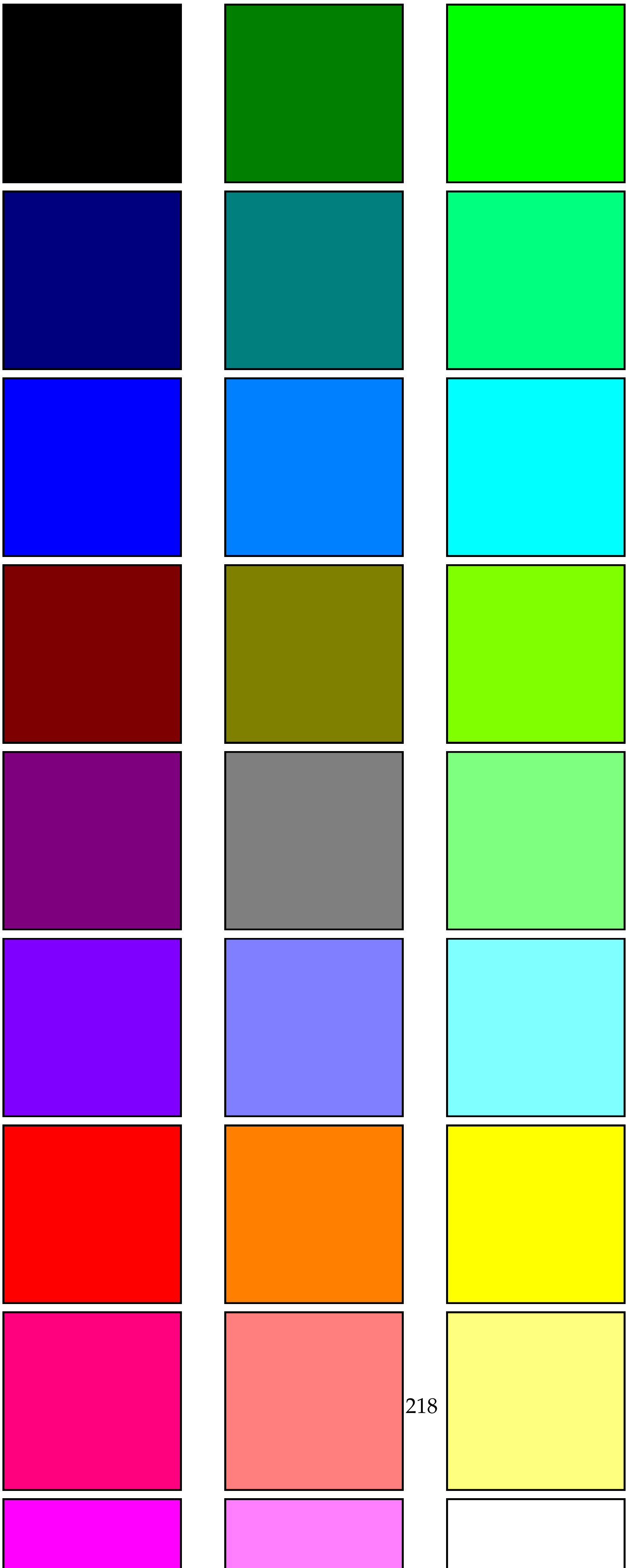
`TableForm[`*expr*`]`
: displays *expr* as a table.

>> `TableForm[Array[a, {3,2}],TableDepth->1]`

$\{a[1,1], a[1,2]\}$
$\{a[2,1], a[2,2]\}$
$\{a[3,1], a[3,2]\}$

A table of Graphics:

```
>>  Table[Style[Graphics[{EdgeForm[{Black}], RGBColor[r,g,b], Rectangle
    []}], ImageSizeMultipliers->{0.2, 1}], {r,0,1,1/2}, {g,0,1,1/2}, {b
    ,0,1,1/2}] // TableForm
```



22.1.12. TeXForm

WMA link

`TeXForm[`*expr*`]`
: displays *expr* using TeX math mode commands.

```
>> TeXForm[HoldForm[Sqrt[a^3]]]
```

\sqrt{a^3}

22.1.13. TraditionalForm

WMA link

`TraditionalForm[`*expr*`]`
: displays *expr* in a format similar to the traditional mathematical notation, where function evaluations are represented by brackets instead of square brackets.

22.2. Form Variables

22.2.1. $OutputForms

`$OutputForms`
: contains the list of all output forms. It is updated automatically when new `OutputForms` are defined by setting format values.

```
>> $OutputForms
```

{TableForm, FullForm, TeXForm, SympyForm, TraditionalForm, BaseForm, PythonForm, StandardForm, MatrixFor

22.2.2. $PrintForms

`$PrintForms`
: contains the list of basic print forms. It is updated automatically when new `PrintForms` are defined by setting format values.

```
>> $PrintForms
```

{FullForm, TeXForm, SympyForm, TraditionalForm, PythonForm, StandardForm, InputForm, MathMLForm, Outpu

Suppose now that we want to add a new format `MyForm`. Initially, it does not belong to `$PrintForms`:

```
>> MemberQ[$PrintForms, MyForm]
   True
```

Now, let's define a format rule:

```
>> Format[F[x_], MyForm] := "F<<" <> ToString[x] <> ">>"
```

Now, the new format belongs to the `$PrintForms` list

```
>> MemberQ[$PrintForms, MyForm]
   True
```

22.3. Forms which are not in `$OutputForms`

22.3.1. SequenceForm

WMA link

`SequenceForm`[*expr*$_1$, *expr*$_2$, ..]
: format the textual concatenation of the printed forms of *expi*.

`SequenceForm`

has been superseded by Row 34.12 and `Text` (which is not implemented yet).

```
>> SequenceForm["[", "x = ", 56, "]"]
   [x = 56]
```

22.3.2. StringForm

WMA link

`StringForm`[*str*, *expr*$_1$, *expr*$_2$, ...]
: displays the string *str*, replacing placeholders in *str* with the corresponding expressions.

```
>> StringForm["`1` bla `2` blub `` bla `2`", a, b, c]
   a bla b blub c bla b
```

23. Functional Programming

Functional programming is a programming paradigm where programs are constructed by applying and composing functions.

It is made richer by expressions like $f[x]$ being treating as symbolic data.

This is term is often used in contrast to Procedural programming.

Contents

23.1. Applying Functions to Lists

Many computations can be conveniently specified in terms of applying functions in parallel to many elements in a list.

Many mathematical functions are automatically taken to be "listable", so that they are always applied to every element in a list.

23.1.1. Apply (`@@`)

WMA link

`Apply`[f, $expr$]
`$f$ @@ $expr$`
    replaces the head of $expr$ with f.
`Apply`[f, $expr$, $levelspec$]
    applies f on the parts specified by $levelspec$.

```
>> f @@ {1, 2, 3}
```
$f[1,2,3]$

```
>> Plus @@ {1, 2, 3}
```
6

The head of *expr* need not be `List`:

```
>> f @@ (a + b + c)
```
$f[a,b,c]$

Apply on level 1:

```
>> Apply[f, {a + b, g[c, d, e * f], 3}, {1}]
```
$\{f[a,b], f[c,d,ef], 3\}$

The default level is 0:

```
>> Apply[f, {a, b, c}, {0}]
```
$f[a,b,c]$

Range of levels, including negative level (counting from bottom):

```
>> Apply[f, {{{{{a}}}}}, {2, -3}]
```
$\{\{f[f[\{a\}]]\}\}$

Convert all operations to lists:

```
>> Apply[List, a + b * c ^ e * f[g], {0, Infinity}]
```
$\{a,\{b,\{g\},\{c,e\}\}\}$

23.1.2. Map (/@)

WMA link

`Map`[*f*, *expr*] or `$f$ /@ $expr$`
: applies *f* to each part on the first level of *expr*.

`Map`[*f*, *expr*, *levelspec*]
: applies *f* to each level specified by *levelspec* of *expr*.

```
>> f /@ {1, 2, 3}
```
$\{f[1], f[2], f[3]\}$

```
>> #^2& /@ {1, 2, 3, 4}
```
$\{1,4,9,16\}$

Map f on the second level:

```
>> Map[f, {{a, b}, {c, d, e}}, {2}]
```
$\{\{f[a], f[b]\}, \{f[c], f[d], f[e]\}\}$

Include heads:

```
>> Map[f, a + b + c, Heads->True]
```
$f[\text{Plus}]\left[f[a], f[b], f[c]\right]$

23.1.3. MapAt

WMA link

`MapAt`[f, $expr$, n]
: applies f to the element at position n in $expr$. If n is negative, the position is counted from the end.

`MapAt`[f, $expr$, {i, j ...}]
: applies f to the part of $expr$ at position {i, j, ...}.

`MapAt`[f, pos]
: represents an operator form of `MapAt` that can be applied to an expression.

Map function f to the second element of an simple flat list:

```
>> MapAt[f, {a, b, c}, 2]
```
$\{a, f[b], c\}$

Above, we specified a simple integer value 2. In general, the expression can be an arbitrary vector.

Using `MapAt` with `Function[0]`, we can zero a value or values in a vector:

```
>> MapAt[0&, {{1, 1}, {1, 1}}, {2, 1}]
```
$\{\{1,1\}, \{0,1\}\}$

When the dimension of the replacement expression is less than the vector, that element's dimension changes:

```
>> MapAt[0&, {{0, 1}, {1, 0}}, 2]
```
$\{\{0,1\}, 0\}$

So now compare what happen when using {{2}, {1}} instead of {2, 1} above:

```
>> MapAt[0&, {{0, 1}, {1, 0}}, {{2}, {1}}]
```
$\{0,0\}$

Map f onto the last element of a list:

```
>> MapAt[f, {a, b, c}, -1]
```
$\{a, b, f[c]\}$

Same as above, but use the operator form of `MapAt`:

```
>> MapAt[f, -1][{a, b, c}]
```
$\{a, b, f[c]\}$

Map f onto at the second position of an association:

```
>> MapAt[f, <|"a" -> 1, "b" -> 2, "c" -> 3, "d" -> 4|>, 2]
```
$\{\mathrm{a}->1, \mathrm{b}->f[2], \mathrm{c}->3, \mathrm{d}->4\}$

Same as above, but select the second-from-the-end position:

```
>> MapAt[f, <|"a" -> 1, "b" -> 2, "c" -> 3, "d" -> 4|>, -2]
```
$\{\mathrm{a}->1, \mathrm{b}->2, \mathrm{c}->f[3], \mathrm{d}->4\}$

23.1.4. MapIndexed

WMA link

`MapIndexed`[f, $expr$]
: applies f to each part on the first level of $expr$, including the part positions in the call to f.

`MapIndexed`[f, $expr$, $levelspec$]
: applies f to each level specified by $levelspec$ of $expr$.

```
>> MapIndexed[f, {a, b, c}]
```
$\{f[a, \{1\}], f[b, \{2\}], f[c, \{3\}]\}$

Include heads (index 0):

```
>> MapIndexed[f, {a, b, c}, Heads->True]
```
$f[\mathrm{List}, \{0\}][f[a, \{1\}], f[b, \{2\}], f[c, \{3\}]]$

Map on levels 0 through 1 (outer expression gets index `{}`):

```
>> MapIndexed[f, a + b + c * d, {0, 1}]
```
$f[f[a, \{1\}] + f[b, \{2\}] + f[cd, \{3\}], \{\}]$

Get the positions of atoms in an expression (convert operations to `List` first to disable `Listable` functions):

```
>> expr = a + b * f[g] * c ^ e;
```

```
>> listified = Apply[List, expr, {0, Infinity}];
```

```
>> MapIndexed[#2 &, listified, {-1}]
```

$\{\{1\},\{\{2,1\},\{\{2,2,1\}\},\{\{2,3,1\},\{2,3,2\}\}\}\}$

Replace the heads with their positions, too:

```
>> MapIndexed[#2 &, listified, {-1}, Heads -> True]
```

$\{0\}[\{1\},\{2,0\}[\{2,1\},\{2,2,0\}[\{2,2,1\}],\{2,3,0\}[\{2,3,1\},\{2,3,2\}]]]$

The positions are given in the same format as used by `Extract`. Thus, mapping `Extract` on the indices given by `MapIndexed` re-constructs the original expression:

```
>> MapIndexed[Extract[expr, #2] &, listified, {-1}, Heads -> True]
```

$a + bf[g]c^e$

23.1.5. MapThread

WMA link

'MapThread[f, $\{\{a_1, a_2, ...\}, \{b_1, b_2, ...\}, ...\}$]
: returns `{$f$[$a_1$, $b_1$, ...], $f$[$a_2$, $b_2$, ...], ...}.`

`MapThread`[f, $\{expr_1, expr_2, ...\}$, n]
: applies f at level n.

```
>> MapThread[f, {{a, b, c}, {1, 2, 3}}]
```

$\{f[a,1],f[b,2],f[c,3]\}$

```
>> MapThread[f, {{{a, b}, {c, d}}, {{e, f}, {g, h}}}, 2]
```

$\{\{f[a,e],f[b,f]\},\{f[c,g],f[d,h]\}\}$

23.1.6. Scan

WMA link

`Scan`[f, $expr$]
: applies f to each element of $expr$ and returns `Null`.

`Scan`[f, $expr$, $levelspec$]
: applies f to each level specified by $levelspec$ of $expr$.

```
>> Scan[Print, {1, 2, 3}]
```

1
2
3

23.1.7. Thread

WMA link

`Thread[$f$[`*args*`]]`
: threads f over any lists that appear in *args*.

`Thread[$f$[`*args*`], `*h*`]`
: threads over any parts with head *h*.

```
>> Thread[f[{a, b, c}]]
```
$\{f[a], f[b], f[c]\}$

```
>> Thread[f[{a, b, c}, t]]
```
$\{f[a,t], f[b,t], f[c,t]\}$

```
>> Thread[f[a + b + c], Plus]
```
$f[a] + f[b] + f[c]$

Functions with attribute `Listable` are automatically threaded over lists:

```
>> {a, b, c} + {d, e, f} + g
```
$\{a + d + g, b + e + g, c + f + g\}$

23.2. Function Application

23.2.1. Function (&)

WMA link

`Function[`*body*`]`
`$body$ &`
: represents a pure function with parameters `#1, #2`, etc.

`Function[`$\{x_1, x_2, ...\}$, *body*`]`
: represents a pure function with parameters x_1, x_2, etc.

`Function[`$\{x_1, x_2, ...\}$, *body*, *attr*`]`
: assume that the function has the attributes *attr*.

```
>> f := # ^ 2 &
```

```
>> f[3]
```
9

```
>> #^3& /@ {1, 2, 3}
```
$\{1, 8, 27\}$

```
>> #1+#2&[4, 5]
```
9

You can use `Function` with named parameters:

```
>> Function[{x, y}, x * y][2, 3]
```
6

Parameters are renamed, when necessary, to avoid confusion:

```
>> Function[{x}, Function[{y}, f[x, y]]][y]
```
Function $[\{y\$\}, f[y, y\$]]$

```
>> Function[{y}, f[x, y]] /. x->y
```
Function $[\{y\}, f[y, y]]$

```
>> Function[y, Function[x, y^x]][x][y]
```
x^y

```
>> Function[x, Function[y, x^y]][x][y]
```
x^y

Slots in inner functions are not affected by outer function application:

```
>> g[#] & [h[#]] & [5]
```
$g[h[5]]$

In the evaluation process, the attributes associated with an Expression are determined by its Head. If the Head is also a non-atomic Expression, in general, no Attribute is assumed. In particular, it is what happens when the head of the expression has the form:

"Function[*body*]" or: "Function[*vars*, *body*]"

```
>> h := Function[{x}, Hold[1+x]]
```

```
>> h[1 + 1]
```
Hold $[1 + 2]$

Notice that *Hold* in the body prevents the evaluation of $1 + x$, but not the evaluation of $1 + 1$. To avoid that evaluation, of its arguments, the Head should have the attribute `HoldAll`. This behavior can be obtained by using the three arguments form version of this expression:

```
>> h:= Function[{x}, Hold[1+x], HoldAll]
```

```
>> h[1+1]
```
Hold $[1 + (1 + 1)]$

In this case, the attribute `HoldAll` is assumed, preventing the evaluation of the argument $1 + 1$ before passing it to the function body.

23.2.2. Slot

WMA link

#n
    represents the n-th argument to a pure function.
#
    is short-hand for #1.
#0
    represents the pure function itself.

```
>> #
```
#1

Unused arguments are simply ignored:

```
>> {#1, #2, #3}&[1, 2, 3, 4, 5]
```
$\{1, 2, 3\}$

Recursive pure functions can be written using #0:

```
>> If[#1<=1, 1, #1 #0[#1-1]]& [10]
```
3628800

23.2.3. SlotSequence

WMA link

##
    is the sequence of arguments supplied to a pure function.
##n
    starts with the n-th argument.

```
>> Plus[##]& [1, 2, 3]
```
6

```
>> Plus[##2]& [1, 2, 3]
```
5

```
>> FullForm[##]
```
SlotSequence [1]

23.3. Functional Composition and Operator Forms

Functional Composition is a way to combine simple functions to build more complicated ones. Like the usual composition of functions in mathematics, the result of each function is passed as the argument of the next, and the result of the last one is the result of the whole.

The symbolic structure of Mathics3 makes it easy to create "operators" that can be composed and manipulated symbolically—forming "pipelines" of operations—and then applied to arguments.

Some built-in functions also directly support a "curried" form, in which they can immediately be given as symbolic operators.

23.3.1. Composition

WMA link

`Composition`[f, g]
    returns the composition of two functions f and g.

```
>> Composition[f, g][x]
```
$f\left[g\left[x\right]\right]$

```
>> Composition[f, g, h][x, y, z]
```
$f\left[g\left[h\left[x,y,z\right]\right]\right]$

```
>> Composition[]
```
Identity

```
>> Composition[][x]
```
x

```
>> Attributes[Composition]
```
{Flat, OneIdentity, Protected}

```
>> Composition[f, Composition[g, h]]
```
Composition $\left[f,g,h\right]$

23.3.2. Identity

WMA link

`Identity`[x]
    is the identity function, which returns x unchanged.

```
>> Identity[x]
```
x

```
>> Identity[x, y]
```
Identity $\left[x,y\right]$

23.4. Iteratively Applying Functions

Functional iteration is an elegant way to represent repeated operations that is used a lot.

23.4.1. FixedPoint

WMA link

`FixedPoint`[f, $expr$]
: starting with $expr$, iteratively applies f until the result no longer changes.

`FixedPoint`[f, $expr$, n]
: performs at most n iterations. The same that using $MaxIterations- > n$

```
>> FixedPoint[Cos, 1.0]
```
0.739085

```
>> FixedPoint[#+1 &, 1, 20]
```
21

23.4.2. FixedPointList

WMA link

`FixedPointList`[f, $expr$]
: starting with $expr$, iteratively applies f until the result no longer changes, and returns a list of all intermediate results.

`FixedPointList`[f, $expr$, n]
: performs at most n iterations.

```
>> FixedPointList[Cos, 1.0, 4]
```
$\{1., 0.540302, 0.857553, 0.65429, 0.79348\}$

Observe the convergence of Newton's method for approximating square roots:

```
>> newton[n_] := FixedPointList[.5(# + n/#)&, 1.];
```

```
>> newton[9]
```
$\{1., 5., 3.4, 3.02353, 3.00009, 3., 3., 3.\}$

Compute the Hailstone Number: for 14:

```
>> collatz[1] := 1;
```

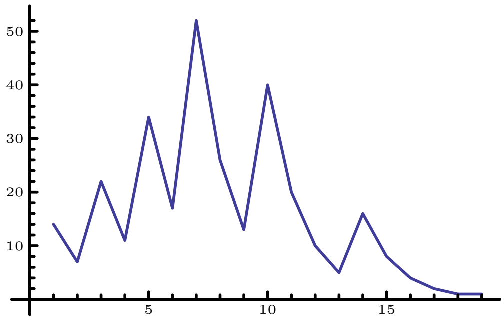
```
>> collatz[x_ ? EvenQ] := x / 2;
```

```
>> collatz[x_] := 3 x + 1;
```

```
>> list = FixedPointList[collatz, 14]
```
$\{14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 1\}$

Plot this:

>> `ListLinePlot[list]`



23.4.3. Fold

WMA link

`Fold[`f, x, *list*`]`
: returns the result of iteratively applying the binary operator f to each element of *list*, starting with x.

`Fold[`f, *list*`]`
: is equivalent to `Fold[$f$, First[$list$], Rest[$list$]]`.

>> `Fold[Plus, 5, {1, 1, 1}]`

8

>> `Fold[f, 5, {1, 2, 3}]`

$f\left[f\left[f[5,1],2\right],3\right]$

23.4.4. FoldList

WMA link

`FoldList[`f, x, *list*`]`
: returns a list starting with x, where each element is the result of applying the binary operator f to the previous result and the next element of *list*.

`FoldList[`f, *list*`]`
: is equivalent to `FoldList[$f$, First[$list$], Rest[$list$]]`.

>> `FoldList[f, x, {1, 2, 3}]`

$\left\{x,f[x,1],f\left[f[x,1],2\right],f\left[f\left[f[x,1],2\right],3\right]\right\}$

>> `FoldList[Times, {1, 2, 3}]`

$\{1,2,6\}$

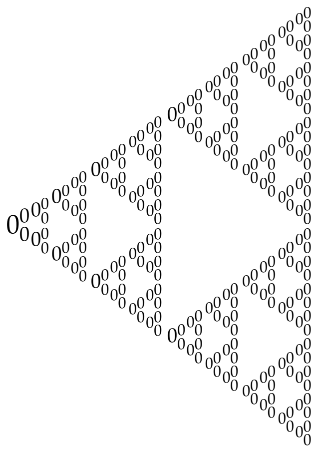
23.4.5. Nest

WMA link

`Nest[`f, $expr$, n`]`
: starting with $expr$, iteratively applies f n times and returns the final result.

```
>> Nest[f, x, 3]
```
$f[f[f[x]]]$

```
>> Nest[(1+#)^ 2 &, x, 2]
```
$\left(1+(1+x)^2\right)^2$

```
>> Nest[Subsuperscript[#,#,#]&,0,5]
```


23.4.6. NestList

WMA link

`NestList[`f, $expr$, n`]`
: starting with $expr$, iteratively applies f n times and returns a list of all intermediate results.

```
>> NestList[f, x, 3]
```
$\{x, f[x], f[f[x]], f[f[f[x]]]\}$

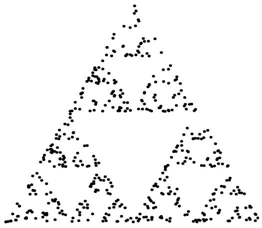
```
>> NestList[2 # &, 1, 8]
```
$\{1,2,4,8,16,32,64,128,256\}$

Chaos game rendition of the Sierpinski triangle:

```
>> vertices = {{0,0}, {1,0}, {.5, .5 Sqrt[3]}};
```

```
>> points = NestList[.5(vertices[[ RandomInteger[{1,3}] ]] + #)&,
   {0.,0.}, 500];
```

```
>> Graphics[Point[points], ImageSize->Small]
```



23.4.7. NestWhile

WMA link

`NestWhile[`f, $expr$, $test$`]`
: applies a function f repeatedly on an expression $expr$, until applying $test$ on the result no longer yields `True`.

`NestWhile[`f, $expr$, $test$, m`]`
: supplies the last m results to $test$ (default value: 1).

`NestWhile[`f, $expr$, $test$, All`]`
: supplies all results gained so far to $test$.

Divide by 2 until the result is no longer an integer:

```
>> NestWhile[#/2&, 10000, IntegerQ]
```

$$\frac{625}{2}$$

Calculate the sum of third powers of the digits of a number until the same result appears twice:

```
>> NestWhile[Total[IntegerDigits[#]^3] &, 5, UnsameQ, All]
```

371

Print the intermediate results:

```
>> NestWhile[Total[IntegerDigits[#]^3] &, 5, (Print[{##}]; UnsameQ[##])
   &, All]
```

```
{5}
{5, 125}
{5, 125, 134}
{5, 125, 134, 92}
{5, 125, 134, 92, 737}
{5, 125, 134, 92, 737, 713}
{5, 125, 134, 92, 737, 713, 371}
{5, 125, 134, 92, 737, 713, 371, 371}
```

371

24. Functions used in Quantum Mechanics

Contents

24.1. Angular Momentum

Angular momentum in physics is the rotational analog of linear momentum. It is an important quantity in physics because it is a conserved quantity the total angular momentum of a closed system remains constant.

24.1.1. ClebschGordan

Clebsch-Gordan coefficients matrices (SymPy, WMA)

`ClebschGordan[`$\{j_1, m_1\}, \{j_2, m_2\}, \{j\ m\}$`]`
returns the Clebsch-Gordan coefficient for the decomposition of $|j, m\rangle$ in terms of $|j_1, m\rangle$, $|j_2, m2\rangle$.

```
>> ClebschGordan[{3 / 2, 3 / 2}, {1 / 2, -1 / 2}, {1, 1}]
```
$\frac{\sqrt{3}}{2}$

`ClebschGordan` works with integer and half-integer arguments:

```
>> ClebschGordan[{1/2, -1/2}, {1/2, -1/2}, {1, -1}]
```
1

```
>> ClebschGordan[{1/2, -1/2}, {1, 0}, {1/2, -1/2}]
```
$-\frac{\sqrt{3}}{3}$

Compare with WMA example:

```
>> ClebschGordan[{5, 0}, {4, 0}, {1, 0}] == Sqrt[5 / 33]
```
True

24.1.2. PauliMatrix

Pauli matrices (SymPy, WMA)

`PauliMatrix`[k]
  returns the k-th Pauli spin matrix).

>> `Table[PauliMatrix[i], {i, 1, 3}]`
$\{\{\{0,1\},\{1,0\}\},\{\{0,-I\},\{I,0\}\},\{\{1,0\},\{0,-1\}\}\}$

>> `PauliMatrix[1] . PauliMatrix[2] == I PauliMatrix[3]`
True

>> `MatrixExp[I \[Phi]/2 PauliMatrix[3]]`
$$\left\{\left\{E^{\frac{I}{2}\phi},0\right\},\left\{0,E^{\left(-\frac{I}{2}\right)\phi}\right\}\right\}$$

>> `% /. \[Phi] -> 2 Pi`
$\{\{-1,0\},\{0,-1\}\}$

24.1.3. SixJSymbol

6-j symbol (SymPy, WMA)

`SixJSymbol`[{j_1, j_2, j_3}, {j_4, j_5, j_6}]
  returns the values of the Wigner 6-j symbol.

>> `SixJSymbol[{1, 2, 3}, {1, 2, 3}]`
$$\frac{1}{105}$$

`SixJSymbol` is symmetric under permutations:

>> `% == SixJSymbol[{3, 2, 1}, {3, 2, 1}]`
True

>> `SixJSymbol[{1, 2, 3}, {1, 2, 3}] == SixJSymbol[{2, 1, 3}, {2, 1, 3}]`
True

`SixJSymbol` works with integer and half-integer arguments:

>> `SixJSymbol[{1/2, 1/2, 1}, {5/2, 7/2, 3}]`
$$-\frac{\sqrt{21}}{21}$$

Compare with WMA example:

```
>> SixJSymbol[{1, 2, 3}, {2, 1, 2}] == 1 / (5 Sqrt[21])
```
True

Result 0 returned for unphysical cases:

```
>> SixJSymbol[{1, 2, 3}, {4, 5, 12}]
```
0

Arguments must be integer or half integer values:

```
>> SixJSymbol[{0.5, 0.5, 1.1},{0.5, 0.5, 1.1}]
SixJSymbol values {0.5, 0.5, 1.1} {0.5, 0.5, 1.1} must be integer or
half integer and fulfill the triangle relation
```
SixJSymbol [{0.5, 0.5, 1.1}, {0.5, 0.5, 1.1}]

24.1.4. ThreeJSymbol

3-j symbol (SymPy, WMA)

`ThreeJSymbol[{`j_1, m_1`}, {`j_2, m_2`}, {`j_3, m_3`}]`
  returns the values of the Wigner 3-j symbol.

Compare with SymPy examples:

```
>> ThreeJSymbol[{2, 0}, {6, 0}, {4, 0}]
```
$\frac{\sqrt{715}}{143}$

`ThreeJSymbol` is symmetric under permutations:

```
>> % == ThreeJSymbol[{2, 0}, {4, 0}, {6, 0}] == ThreeJSymbol[{4, 0}, {2,
   0}, {6, 0}]
```
True

```
>> ThreeJSymbol[{2, 0}, {6, 0}, {4, 1}]
```
0

Compare with WMA examples:

```
>> ThreeJSymbol[{6, 0}, {4, 0}, {2, 0}] == Sqrt[5 / 143]
```
True

```
>> ThreeJSymbol[{2, 1}, {2, 2}, {4, -3}] == -(1 / (3 Sqrt[2]))
```
True

```
>> ThreeJSymbol[{1/2, -1/2}, {1/2, -1/2}, {1, 1}]
```
$-\frac{\sqrt{3}}{3}$

Result 0 returned for unphysical cases:

```
>> ThreeJSymbol[{1, 2}, {3, 4}, {5, 12}]
```

0

Arguments must be integer or half integer values:

```
>> ThreeJSymbol[{2.1, 6}, {4, 0}, {0, 0}]
ThreeJSymbol values {2.1, 6}, {4, 0}, {0, 0} must be integer or half
integer
```

ThreeJSymbol $\left[\{2.1, 6\}, \{4, 0\}, \{0, 0\}\right]$

25. Global System Information

Contents

25.1. $CommandLine

WMA link

`$CommandLine`
is a list of strings passed on the command line to launch the Mathics3 session.

>> **`$CommandLine`**
{docpipeline.py, –output, –keep-going, –load-module, pymathics.icu,pymathics.graph,pymathics.natlang}

25.2. $Machine

WMA link

`$Machine`
returns a string describing the type of computer system on which the Mathics3 is being run.

```
>> $Machine
linux
```

25.3. $MachineName

WMA link

> `$MachineName`
>   is a string that gives the assigned name of the computer on which Mathics3 is being run, if such a name is defined.

```
>> $MachineName
milton
```

25.4. $MaxLengthIntStringConversion

Python 3.11 Integer string conversion length limitation

> `$MaxLengthIntStringConversion`
>   A positive system integer that fixes the largest size of the string that can appear when converting an `Integer` value into a `String`. When the string value is too large, then the middle of the integer contains an indication of the number of digits elided inside « ».
>
> If `$MaxLengthIntStringConversion` is set to 0, there is no bound. Aside from 0, 640 is the smallest value allowed.
> The initial value can be set via environment variable `DEFAULT_MAX_STR_DIGITS`. If that is not set, the default value is 7000.

Although Mathics3 can represent integers of arbitrary size, when it formats the value for display, there can be nonlinear behavior in printing the decimal string or converting it to a `String`.

Python, in version 3.11 and up, puts a default limit on the size of the number of digits allows when converting a large integer into a string.

Show the default value of `$MaxLengthIntStringConversion`:

```
>> $MaxLengthIntStringConversion
640
```

500! is a 1135-digit number:

```
>> 500! //ToString//StringLength
639
```

We first set `$MaxLengthIntStringConversion` to the smallest value allowed, so that we can see the truncation of digits in the middle:

```
>> $MaxLengthIntStringConversion = 640
```
640

Note that setting `$MaxLengthIntStringConversion` has an effect only on Python 3.11 and later; Pyston 2.x however ignores this.

Now when we print the string value of 500! and Pyston 2.x is not used, the middle digits are removed:

```
>> 500!
```
12201368259911100687012387854230469262535743428031928421924135883858453731538819976054964475022032818

<< 501 >> 229 913 340 169 552 363 850 942 885 592 018 727 433 795 173 014 ~
~586 357 570 828 355 780 158 735 432 768 888 680 120 399 882 384 702 151 ~
~467 605 445 407 663 535 984 174 430 480 128 938 313 896 881 639 487 469 ~
~658 817 504 506 926 365 338 175 055 478 128 640 000 000 000 000 000 000 ~
~000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 ~
~000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000

To see this easier, manipulate the result as `String`:

```
>> bigFactorial = ToString[500!]; StringTake[bigFactorial, {310, 330}]
```
787849 «501» 229913

The «501» indicates that 501 digits have been omitted in the string conversion.

Other than 0, an `Integer` value less than 640 is not accepted:

```
>> $MaxLengthIntStringConversion = 10
10 is not 0 or an Integer value greater than 640.
```
640

25.5. $Packages

WMA link

`$Packages`
: returns a list of the contexts corresponding to all packages which have been loaded into Mathics.

```
>> $Packages
```
{ImportExport‘, XML‘, Internal‘, System‘, Global‘}

25.6. $ParentProcessID

WMA link

`$ParentProcesID`
: gives the ID assigned to the process which invokes Mathics3 by the operating system under which it is run.

>> **`$ParentProcessID`**
1106503

25.7. $ProcessID

WMA link

`$ProcessID`
: gives the ID assigned to the Mathics3 process by the operating system under which it is run.

>> **`$ProcessID`**
1106504

25.8. $ProcessorType

WMA link

`$ProcessorType`
: gives a string giving the architecture of the processor on which Mathics3 is being run.

>> **`$ProcessorType`**
x86_64

25.9. $PythonImplementation

`$PythonImplementation`
: gives a string indication the Python implementation used to run Mathics3.

>> **`$PythonImplementation`**
CPython 3.13.7.final.0

25.10. $ScriptCommandLine

WMA link

`$ScriptCommandLine`
: is a list of string arguments when running the kernel is script mode.

>> **`$ScriptCommandLine`**

{}

25.11. $SessionID

WMA link

`$SessionID`
: is a number which is unique to a particular *Mathics3* System session.

>> **`$SessionID`**

123702811663232

25.12. $SystemID

WMA link

`$SystemID`
: is a short string that identifies the type of computer system on which the *Mathics3* is being run.

>> **`$SystemID`**

linux

25.13. $SystemMemory

WMA link

`$SystemMemory`
: Returns the total amount of physical memory.

>> `$SystemMemory`

50240962560

25.14. $SystemWordLength

WMA link

`$SystemWordLength`
: gives the effective number of bits in raw machine words on the computer system where Mathics3 is running.

>> **`$SystemWordLength`**

64

25.15. $UserName

WMA link

$UserName
: returns the login name, according to the operative system, of the user that started the current *Mathics3* session.

>> **`$UserName`**

rocky

25.16. $Version

WMA link

`$Version`
: returns a string with the current Mathics version and the versions of relevant libraries.

>> **`$Version`**

Mathics3 9.0.0 on CPython 3.13.7 (main, Aug 17 2025, 17:14:11) [GCC 13.3.0] using SymPy 1.13.3, mpmath 1.3.0, numpy 2.3.2, cython 3.1.3

25.17. $VersionNumber

WMA link

`$VersionNumber`
: is a real number which gives the current Wolfram Language version that *Mathics3* tries to be compatible with.

```
>> $VersionNumber
   10.
```

25.18. Breakpoint

Python breakpoint()

`Breakpoint[]`
: Invoke a Python breakpoint.

This can be used for debugging the Mathics3 implementation, but if you are familiar with Python, it might assist in debugging a Mathics3 programs as well.
By default, the Python debugger (`pdb`) is loaded. For loading other debuggers, change the environment variable `PYTHONBREAKPOINT`.

Mathics3 code includes a breakpoint handler function, `mathics.disabled_breakpoint` which reports whether `Breakpoint[]` was encountered in Mathics3, or `breakpoint()` was encountered in the Mathics3 source code. In contrast to `pdb`, `trepan3k` and other handlers, this breakpoint handler does not stop inside, it just reports.

Here is how to use `mathics.disabled_breakpoint`:

```
>> SetEnvironment["PYTHONBREAKPOINT" -> "mathics.disabled_breakpoint"];
```

```
>> Breakpoint[]
   Breakpoint[]
```

The environment variable `PYTHONBREAKPOINT` can be changed at runtime to switch `breakpoint()` and `Breakpoint[]` behavior.

25.19. Environment

WMA link

`Environment[`*var*`]`
: gives the value of an operating system environment variable.

```
>> Environment["HOME"]
```

/home/rocky

See also 'GetEnvironment' 25.20 and 'SetEnvironment' 25.25.

25.20. GetEnvironment

WMA link

`GetEnvironment[`"*var*"`]`
: gives the setting corresponding to the variable "var" in the operating system environment.

`GetEnvironment[`{"var_1", "var_2", ...}`]`
: gives a list rules for each of the environment variables listed.

`GetEnvironment[]`
: gives a list rules for all environment variables.

On POSIX systems, the following gets the users HOME directory:

```
>> GetEnvironment["HOME"]
```

HOME− > /home/rocky

We can get both the HOME directory and the user name in one go:

```
>> GetEnvironment[{"HOME", "USER"}]
```

{HOME− > /home/rocky, USER− > rocky}

Arguments however must be strings:

```
>> GetEnvironment[HOME]
HOME is not ALL or a string or a list of strings.
```

GetEnvironment [HOME]

See also 'Environment' 25.19 and 'SetEnvironment' 25.25.

25.21. MathicsVersion

`MathicsVersion`
: this string is the version of Mathics we are running.

```
>> MathicsVersion
```

9.0.0

25.22. MemoryAvailable

WMA link

`MemoryAvailable`
: Returns the amount of the available physical memory.

>> **`MemoryAvailable[]`**
17702006784

The relationship between $SystemMemory, MemoryAvailable, and MemoryInUse:

>> **`$SystemMemory > MemoryAvailable[] > MemoryInUse[]`**
True

25.23. MemoryInUse

WMA link

`MemoryInUse[]`
: Returns the amount of memory used by all of the definitions objects if we can determine that; -1 otherwise.

>> **`MemoryInUse[]`**
22478288

25.24. Run

WMA link

`Run[`*command*`]`
: runs command as an external operating system command, returning the exit code returned from running the system command.

>> **`Run["date"]`**
0

25.25. SetEnvironment

WMA link

`SetEnvironment["var" -> "value"]`
: sets the value of an operating system environment variable.

`SetEnvironment[{"var" -> "value", ...}]`
: sets more than one environment variable.

Set a single environment variable:

```
>> SetEnvironment["FOO" -> "bar"]
```

See that the environment variable has changed:

```
>> GetEnvironment["FOO"]
```

FOO$-$ > bar

Set two environment variables:

```
>> SetEnvironment[{"FOO" -> "baz", "A" -> "B"}]
```

See that the environment variable has changed:

```
>> GetEnvironment["FOO"]
```

FOO$-$ > baz

Environment values must be strings:

```
>> SetEnvironment["FOO" -> 5]
5 must be a string or None.
```

$Failed

```
>> GetEnvironment["FOO"]
```

FOO$-$ > baz

If the environment name is not a string, the evaluation fails without a message.

```
>> SetEnvironment[1 -> "bar"]
```

SetEnvironment [1$-$ > bar]

See also 'Environment' 25.19 and 'GeEnvironment' 25.20.

25.26. Share

WMA link

`Share[]`
: release memory forcing Python to do garbage collection. If Python package `psutil` installed is the amount of released memoryis returned. Otherwise returns 0. This function differs from WMA which tries to reduce the amount of memory required to store definitions, by reducing duplicated definitions.

`Share[Symbol]`
: Does the same thing as `Share[]`; Note: this function differs from WMA which tries to reduce the amount of memory required to store definitions associated to *Symbol*.

```
>> Share[]
1548288
```

26. Graphics and Drawing

Showing something visually can be done in a number of ways:

- Starting with complete images and modifying them using the `Image` Built-in function.
- Use pre-defined 2D or 3D objects like 'Circle' 17.4 and 'Cuboid' 26.4.2 and place them in a coordinate space.
- Compute the points of the space using a function. This is done using functions like 'Plot' 26.2.15 and 'ListPlot' 26.2.9.

Contents

26.1. Drawing Options and Option Values

The various common Plot and Graphics options, along with the meaning of specific option values are described here.

26.1.1. Automatic

WMA link

`Automatic`
: is used to specify an automatically computed option value.

`Automatic` is the default for `PlotRange`, `ImageSize`, and other graphical options:

```
>> Cases[Options[Plot], HoldPattern[_ :> Automatic]]
```

{Background:>Automatic, Exclusions:>Automatic, ImageSize:>Automatic, MaxRecursion:>Automatic, PlotRange:>

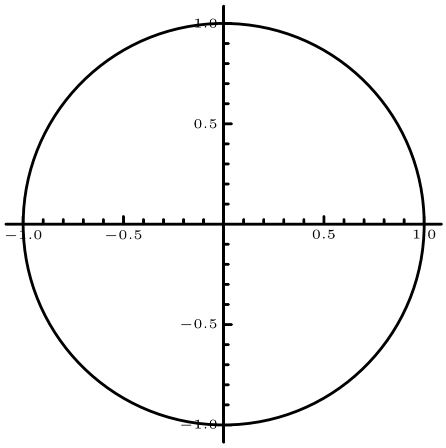
26.1.2. Axes

WMA link

`Axes`
: is an option for charting and graphics functions that specifies whether axes should be drawn.

- `Axes->True` draws all axes.
- `Axes->False` draws no axes.
- `Axes->{False,True}` draws an axis y but no x axis in two dimensions.

```
>> Graphics[Circle[], Axes -> True]
```

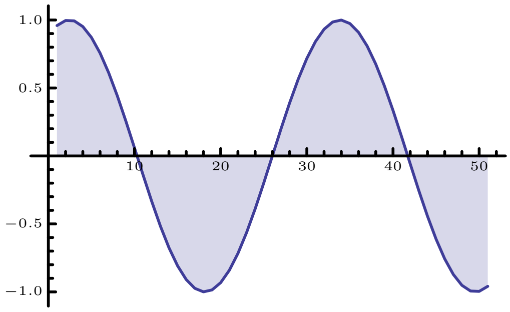


26.1.3. Axis

WMA link

`Axis`
: is a possible value for the `Filling` option.

```
>> ListLinePlot[Table[Sin[x], {x, -5, 5, 0.2}], Filling->Axis]
```



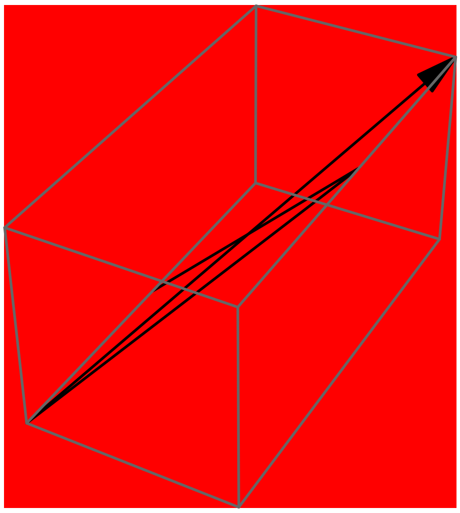
26.1.4. Background

WMA link

`Background`
: is an option that specifies the color of the background.

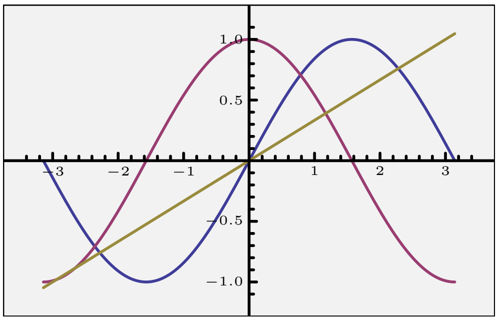
The specification must be a Color specification or `Automatic`:

```
>> Graphics3D[{Arrow[{{0,0,0},{1,0,1},{0,-1,0},{1,1,1}}]}, Background ->
   Red]
```



Notice that opacity cannot be specified by passing a `List` containing `Opacity` together with a color specification like `{Red, Opacity[.1]}`. Use a color directive with an alpha channel instead:

```
>> Plot[{Sin[x], Cos[x], x / 3}, {x, -Pi, Pi}, Background -> RGBColor
   [0.5, .5, .5, 0.1]]
```

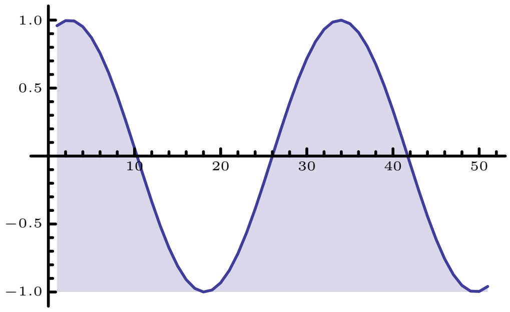


26.1.5. Bottom

WMA link

`Bottom`
    is a possible value for the `Filling` option.

```
>> ListLinePlot[Table[Sin[x], {x, -5, 5, 0.2}], Filling->Bottom]
```

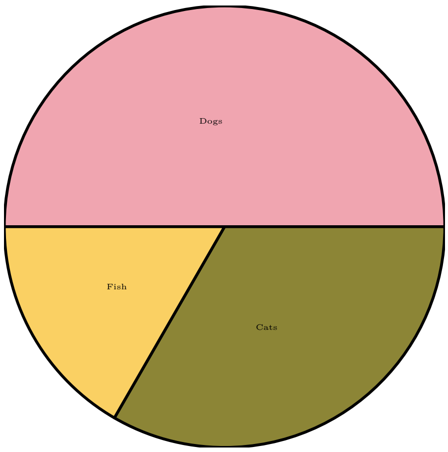


26.1.6. ChartLabels

WMA link

`ChartLabels`
: is a charting option that specifies what labels should be used for chart elements.

```
>> PieChart[{30, 20, 10}, ChartLabels -> {Dogs, Cats, Fish}]
```



26.1.7. ChartLegends

WMA link

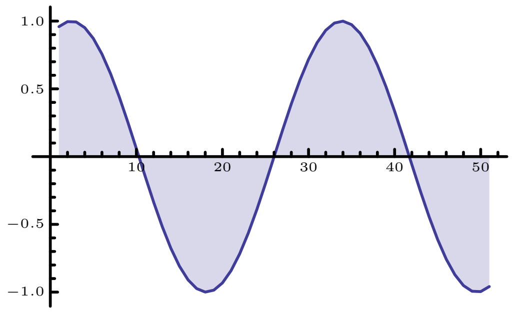
`ChartLegends`
: is an option for charting functions that specifies the legends to be used for chart elements.

26.1.8. Filling

WMA link

Filling -> [Top | Bottom| Axis]
: Filling is a an option to ListPlot, Plot or Plot3D, and related functions that indicates what filling to add under point, curves, and surfaces.

```
>> ListLinePlot[Table[Sin[x], {x, -5, 5, 0.2}], Filling->Axis]
```



26.1.9. Full

WMA link

Full
: is a possible value for the Mesh and PlotRange options.

26.1.10. ImageSize

WMA link

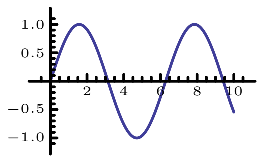
ImageSize
: is an option that specifies the overall size of an image to display.

Specifications for both width and height can be any of the following:

Automatic
: determined by location or other dimension (default)

Tiny, Small, Medium, Large
: pre defined absolute sizes

```
>> Plot[Sin[x], {x, 0, 10}, ImageSize -> Small]
```

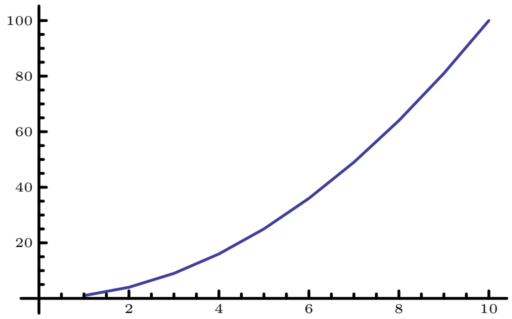


26.1.11. Joined

WMA link

`Joined $boolean$`
: is an option for `Plot` that gives whether to join points to make lines.

```
>> ListPlot[Table[n ^ 2, {n, 10}], Joined->True]
```



26.1.12. MaxRecursion

WMA link

`MaxRecursion`
: is an option for functions like NIntegrate and Plot that specifies how many recursive subdivisions can be made.

```
>> NIntegrate[Exp[-10^8 x^2], {x, -1, 1}, Method->"Internal",
   MaxRecursion -> 3]
```

0.0777778

```
>> NIntegrate[Exp[-10^8 x^2], {x, -1, 1}, Method->"Internal",
   MaxRecursion -> 6]
```

0.00972222

26.1.13. Mesh

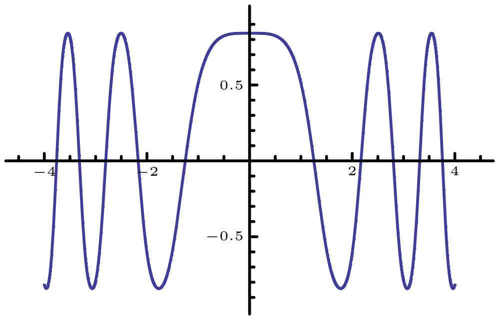
WMA link

`Mesh`
: is a charting option, such as for `Plot`, `BarChart`, `PieChart`, etc. that specifies the mesh to be drawn. The default is `Mesh->None`.

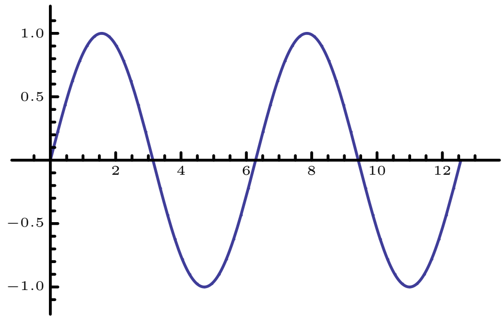
Options include:

- None: No mesh is drawn
- All: mesh divisions between elements
- Full: mesh divisions between regular datapoints

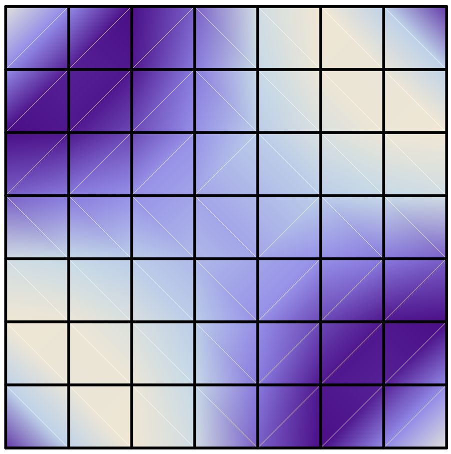
```
>> Plot[Sin[Cos[x^2]],{x,-4,4},Mesh->All]
```



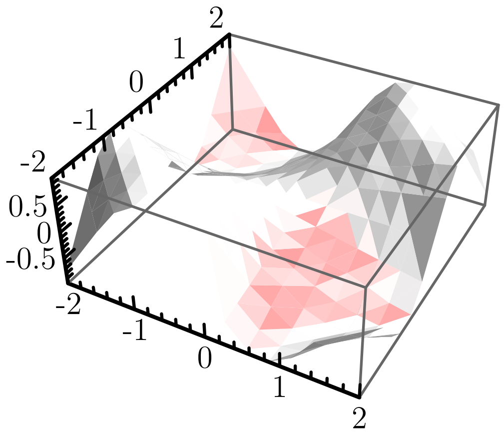

```
>> Plot[Sin[x], {x,0,4 Pi}, Mesh->Full]
```




```
>> DensityPlot[Sin[x y], {x, -2, 2}, {y, -2, 2}, Mesh->Full]
```



```
>> Plot3D[Sin[x y], {x, -2, 2}, {y, -2, 2}, Mesh->Full]
```

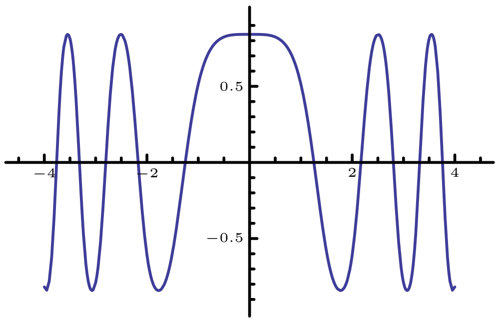


26.1.14. PlotPoints

WMA link

`PlotPoints $n$`
: A number specifies how many initial sample points to use.

```
>> Plot[Sin[Cos[x^2]],{x,-4,4}, PlotPoints->22]
```



26.1.15. PlotRange

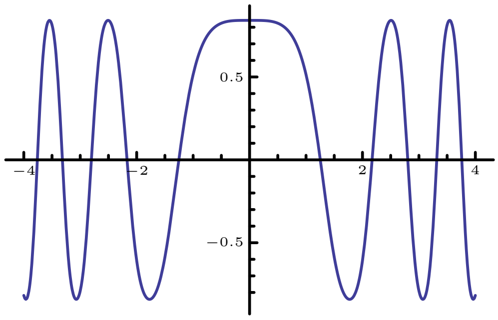
WMA link

`PlotRange`
: is a charting option, such as for `Plot`, `BarChart`, `PieChart`, etc. that gives the range of coordinates to include in a plot.

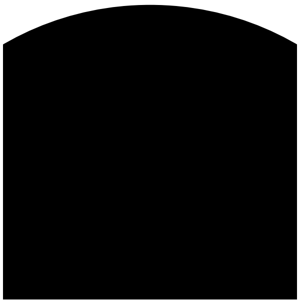
- All all points are included.
- Automatic - outlying points are dropped.

- max - explicit limit for each function.
- $\{min, max\}$ - explicit limits for y (2D), z (3D), or array values.
- $\{\{x_{min}, x_{max}\}, \{\{y_{min}\}, \{y_{max}\}\}$ - explicit limits for x and y.

```
>> Plot[Sin[Cos[x^2]],{x,-4,4}, PlotRange -> All]
```




```
>> Graphics[Disk[], PlotRange -> {{-.5, .5}, {0, 1.5}}]
```



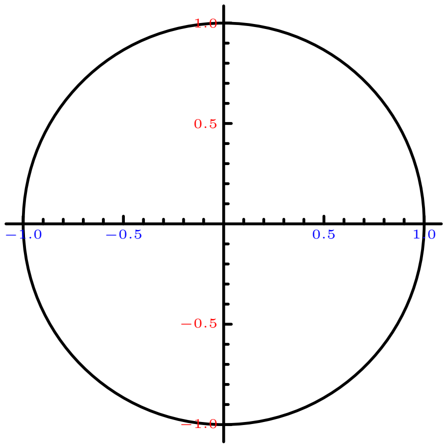
26.1.16. TicksStyle

WMA link

`TicksStyle`
: is an option for graphics functions which specifies how ticks should be rendered.

- TicksStyle gives styles for both tick marks and tick labels.
- TicksStyle can be used in both two and three-dimensional graphics.
- TicksStyle->*list* specifies the colors of each of the axes.

```
>>  Graphics[Circle[], Axes-> True, TicksStyle -> {Blue, Red}]
```

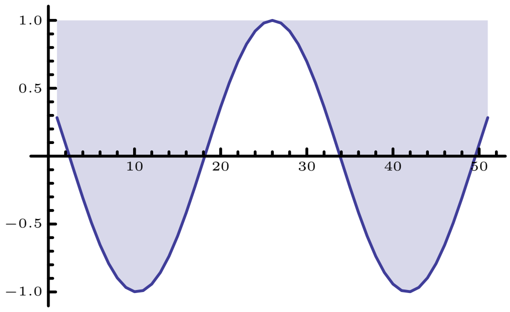


26.1.17. Top

WMA link

`Top`
is a possible value for the `Filling` option.

```
>>  ListLinePlot[Table[Cos[x], {x, -5, 5, 0.2}], Filling->Top]
```



26.2. Plotting Data

Plotting functions take a function as a parameter and data, often a range of points, as another parameter, and plot or show the function applied to the data.

26.2.1. BarChart

WMA link

`BarChart[{`b_1, b_2 `...}]`
    makes a bar chart with lengths b_1, b_2, ...

Drawing options include - Charting:

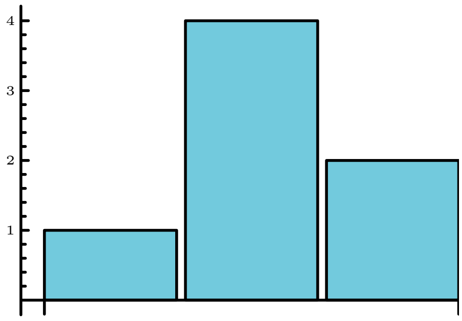
- Mesh
- PlotRange
- ChartLabels
- ChartLegends
- ChartStyle

BarChart specific:

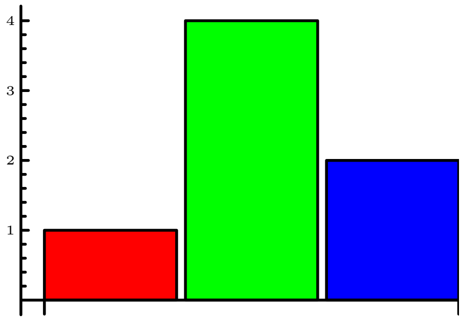
- Axes (default {False, True})
- AspectRatio: (default 1 / GoldenRatio)

A bar chart of a list of heights:

```
>> BarChart[{1, 4, 2}]
```




```
>> BarChart[{1, 4, 2}, ChartStyle -> {Red, Green, Blue}]
```



```
>> BarChart[{{1, 2, 3}, {2, 3, 4}}]
```

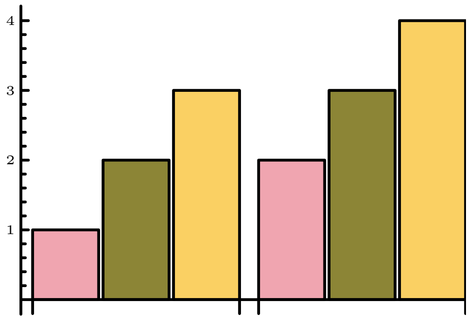
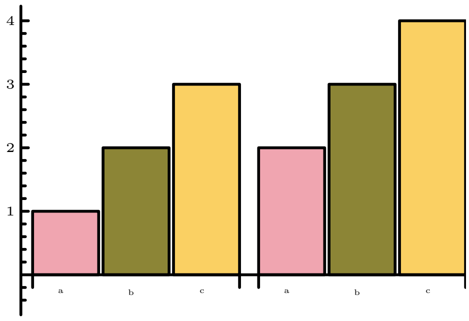
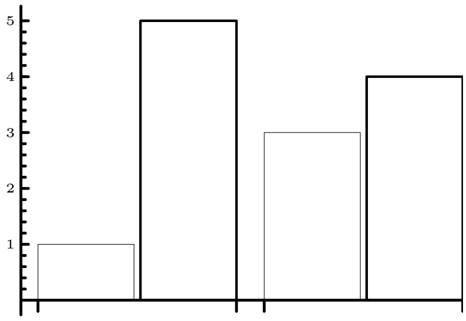


Chart several datasets with categorical labels:

```
>> BarChart[{{1, 2, 3}, {2, 3, 4}}, ChartLabels -> {"a", "b", "c"}]
```




```
>> BarChart[{{1, 5}, {3, 4}}, ChartStyle -> {{EdgeForm[Thin], White}, {
   EdgeForm[Thick], White}}]
```



26.2.2. ColorData

WMA link

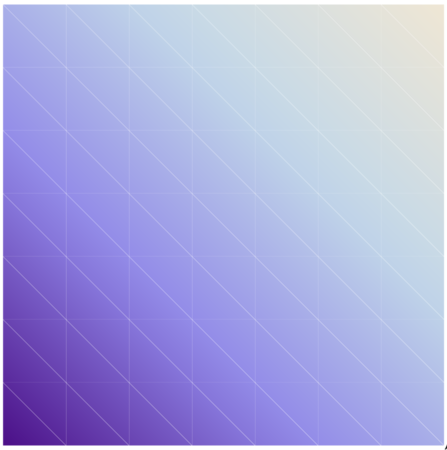
`ColorData["name"]`
: returns a color function with the given *name*.

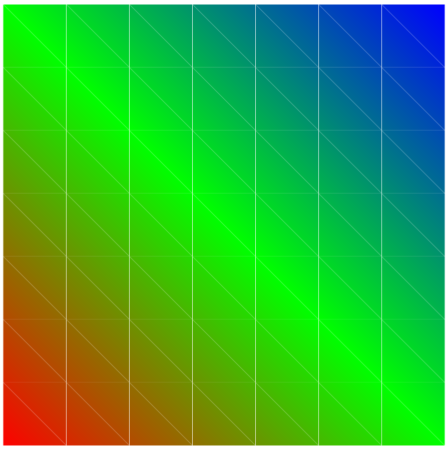
Define a user-defined color function:

```
>> Unprotect[ColorData]; ColorData["test"] := ColorDataFunction["test",
   "Gradients", {0, 1}, Blend[{Red, Green, Blue}, #1] &]; Protect[
   ColorData]
```

Compare it to the default color function, `LakeColors`:

```
>> {DensityPlot[x + y, {x, -1, 1}, {y, -1, 1}], DensityPlot[x + y, {x,
   -1, 1}, {y, -1, 1}, ColorFunction->"test"]}
```





26.2.3. ColorDataFunction

WMA link

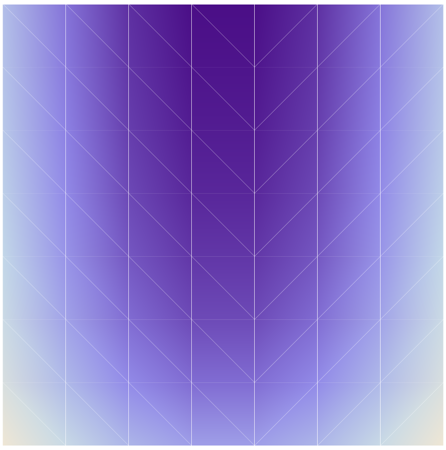
`ColorDataFunction[range, ...]`
  is a function that represents a color scheme.

26.2.4. DensityPlot

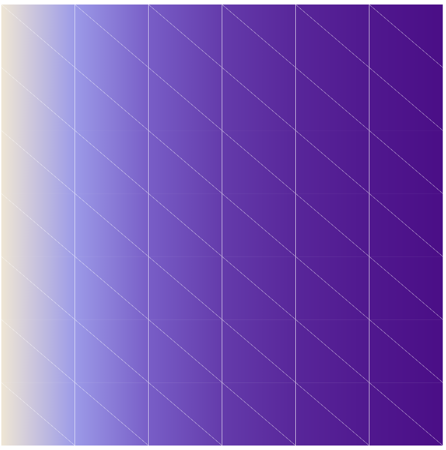
WMA link

`DensityPlot[`f`, {`x`, `x_{min}`, `x_{max}`}, {`y`, `y_{min}`, `y_{max}`}]`
plots a density plot of f with x ranging from x_{min} to x_{max} and y ranging from y_{min} to y_{max}.

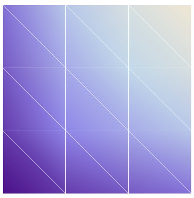
```
>> DensityPlot[x ^ 2 + 1 / y, {x, -1, 1}, {y, 1, 4}]
```

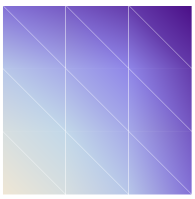


```
>> DensityPlot[1 / x, {x, 0, 1}, {y, 0, 1}]
```

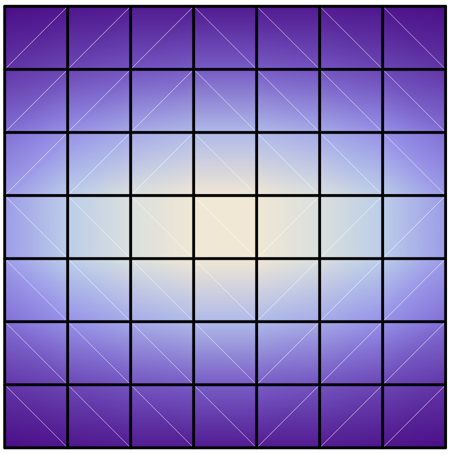


```
>> DensityPlot[Sqrt[x * y], {x, -1, 1}, {y, -1, 1}]
```

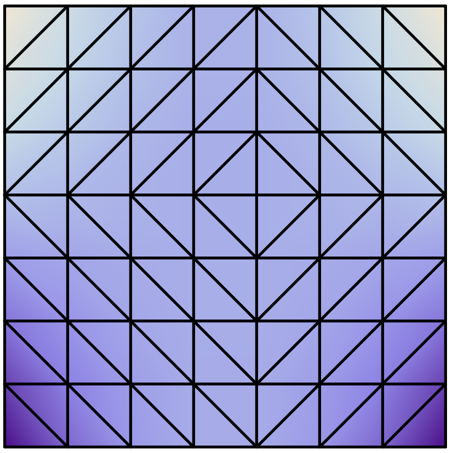




```
>> DensityPlot[1/(x^2 + y^2 + 1), {x, -1, 1}, {y, -2,2}, Mesh->Full]
```



```
>> DensityPlot[x^2 y, {x, -1, 1}, {y, -1, 1}, Mesh->All]
```



26.2.5. DiscretePlot

WMA link

`DiscretePlot`[$expr$, {x, n_{max}}]
: plots $expr$ with x ranging from 1 to n_{max}.

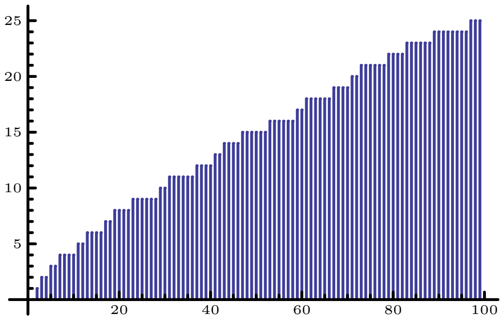
`DiscretePlot`[$expr$, {x, n_{min}, n_{max}}]
: plots $expr$ with x ranging from n_{min} to n_{max}.

`DiscretePlot`[$expr$, {x, n_{min}, n_{max}, dn}]
: plots $expr$ with x ranging from n_{min} to n_{max} usings steps dn.

`DiscretePlot`[{$expr_1$, $expr_2$, ...}, ...]
: plots the values of all $expri$.

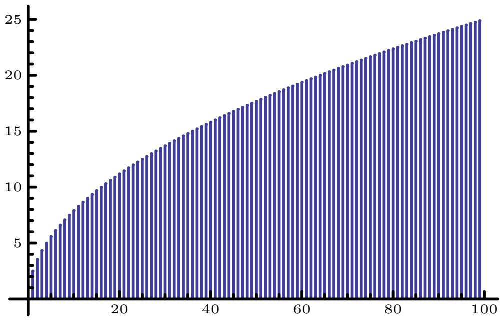
The number of primes for a number k:

>> `DiscretePlot[PrimePi[k], {k, 1, 100}]`



is about the same as `Sqrt[k] * 2.5`:

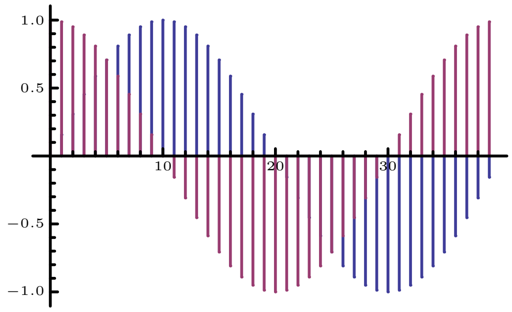
>> `DiscretePlot[2.5 Sqrt[k], {k, 100}]`



Notice in the above that when the starting value, n_{min}, is 1, we can omit it.

A plot can contain several functions, using the same parameter, here x:

>> `DiscretePlot[{Sin[Pi x/20], Cos[Pi x/20]}, {x, 0, 40}]`



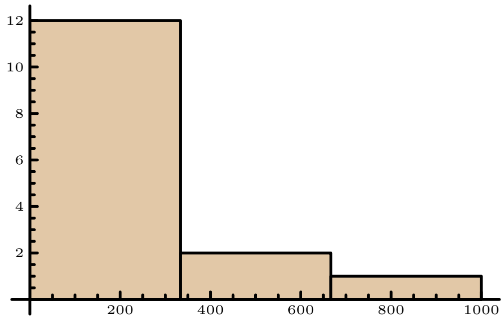
Compare with 'Plot' 26.2.15.

26.2.6. Histogram

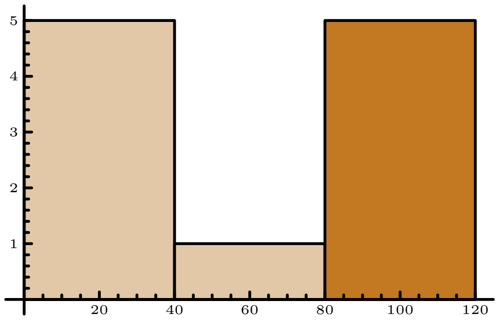
Histogram (WMA link)

`Histogram[{`x_1, x_2 ...`}]`
  plots a histogram using the values x_1, x_2, ...

```
>> Histogram[{3, 8, 10, 100, 1000, 500, 300, 200, 10, 20, 200, 100, 200,
    300, 500}]
```




```
>> Histogram[{{1, 2, 10, 5, 50, 20}, {90, 100, 101, 120, 80}}]
```



26.2.7. ListLinePlot

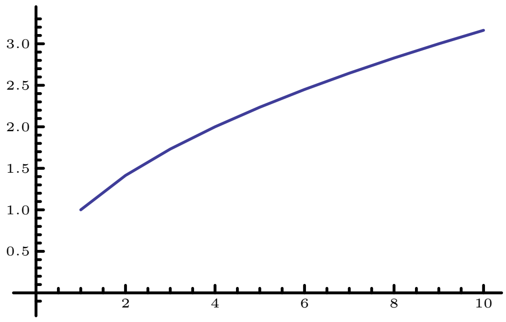
WMA link

`ListLinePlot`[{y_1, y_2, ...}]
: plots a line through a list of y-values, assuming integer x-values 1, 2, 3, ...

`ListLinePlot`[{{x_1, y_1}, {x_2, y_2}, ...}]
: plots a line through a list of x, y pairs.

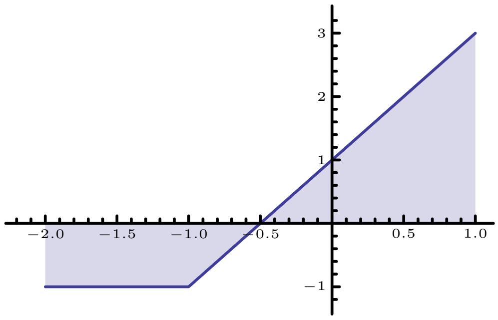
`ListLinePlot`[{$list_1$, $list_2$, ...}]
: plots several lines.

```
>> ListLinePlot[Table[{n, n ^ 0.5}, {n, 10}]]
```



ListPlot accepts a superset of the Graphics options.

```
>> ListLinePlot[{{-2, -1}, {-1, -1}, {1, 3}}, Filling->Axis]
```



26.2.8. ListLogPlot

WMA link

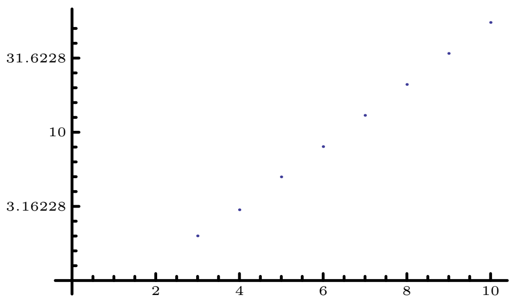
`ListLogPlot`[{y_1, y_2, ...}]
: log plots a list of y-values, assuming integer x-values 1, 2, 3, ...

`ListLogPlot`[{{x_1, y_1}, {x_2, y_2}, ...}]
: log plots a list of x, y pairs.

`ListLogPlot`[{$list_1$, $list_2$, ...}]
: log plots several lists of points.

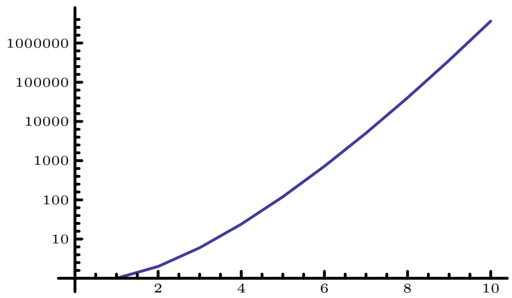
Plotting table of Fibonacci numbers:

```
>> ListLogPlot[Table[Fibonacci[n], {n, 10}]]
```



we see that Fibonacci numbers grow exponentially. So when plotted using on a log scale the result fits points of a sloped line.

```
>> ListLogPlot[Table[n!, {n, 10}], Joined -> True]
```



26.2.9. ListPlot

WMA link

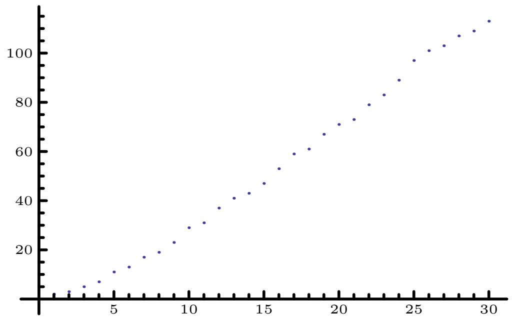
`ListPlot`[{y_1, y_2, ...}]
: plots a list of y-values, assuming integer x-values 1, 2, 3, ...

`ListPlot`[{{x_1, y_1}, {x_2, y_2}, ...}]
: plots a list of x, y pairs.

`ListPlot`[{$list_1$, $list_2$, ...}]
: plots several lists of points.

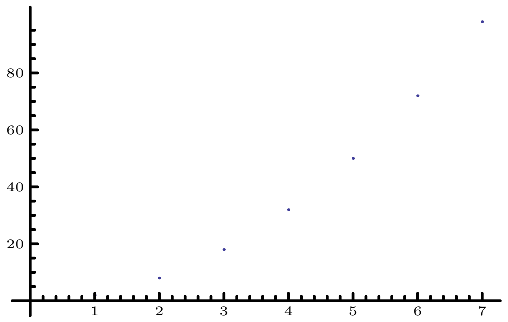
The frequency of Primes:

```
>> ListPlot[Prime[Range[30]]]
```



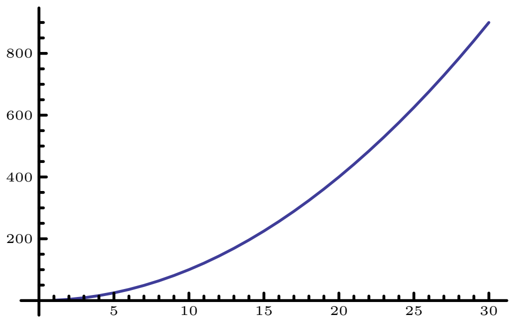
seems very roughly to fit a table of quadratic numbers:

```
>> ListPlot[Table[n ^ 2 / 8, {n, 30}]]
```



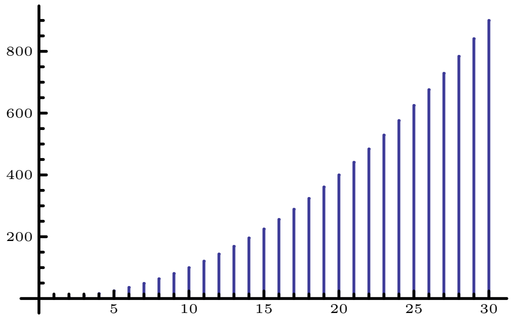
ListPlot accepts some Graphics options:

```
>> ListPlot[Table[n ^ 2, {n, 30}], Joined->True]
```



Compare with 'Plot' 26.2.15.

```
>> ListPlot[Table[n ^ 2, {n, 30}], Filling->Axis]
```



Compare with 'Plot' 26.2.15.

26.2.10. ListStepPlot

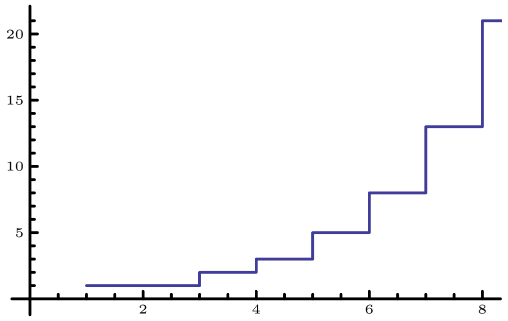
WMA link

`ListStepPlot[{`y_1, y_2, ...`}]`
: plots a line through a list of y-values, assuming integer x-values 1, 2, 3, ...

`ListStepPlot[{{`x_1, y_1`}, {`x_2, y_2`}, ...}]`
: plots a line through a list of x, y pairs.

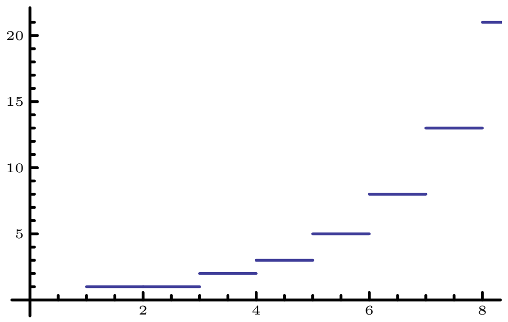
`ListStepPlot[{`$list_1$, $list_2$, ...`}]`
: plots several lines.

```
>> ListStepPlot[{1, 1, 2, 3, 5, 8, 13, 21}]
```



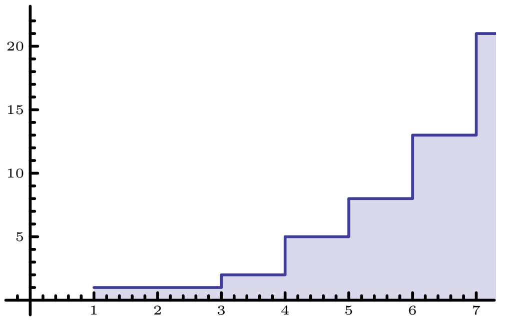
`ListStepPlot` accepts a superset of the Graphics options. By default, `ListStepPlots` are joined, but that can be disabled.

>> `ListStepPlot[{1, 1, 2, 3, 5, 8, 13, 21}, Joined->False]`



The same as the first example but using a list of point as data, and filling the plot to the x axis.

>> `ListStepPlot[{{1, 1}, {3, 2}, {4, 5}, {5, 8}, {6, 13}, {7, 21}}, Filling->Axis]`

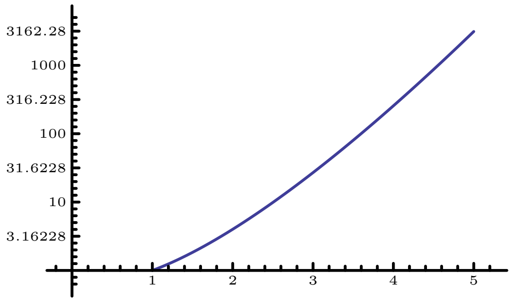


26.2.11. LogPlot

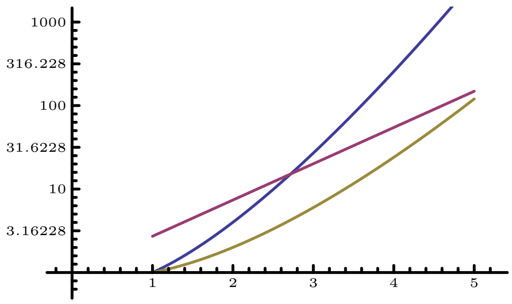
Semi-log plot (WMA link)

`LogPlot`$[f, \{x, x_{min}, x_{max}\}]$
  log plots f with x ranging from x_{min} to x_{max}.
`Plot`$[\{f_1, f_2, ...\}, \{x, x_{min}, x_{max}\}]$
  log plots several functions $f_1, f_2, ...$

>> `LogPlot[x^x, {x, 1, 5}]`



```
>> LogPlot[{x^x, Exp[x], x!}, {x, 1, 5}]
```

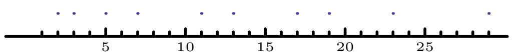


26.2.12. NumberLinePlot

WMA link

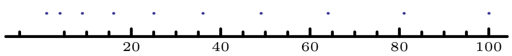
`NumberLinePlot[{`v_1, v_2, ...`}]`
    plots a list of values along a line.

```
>> NumberLinePlot[Prime[Range[10]]]
```



Compare with:

```
>> NumberLinePlot[Table[x^2, {x, 10}]]
```



26.2.13. ParametricPlot

WMA link

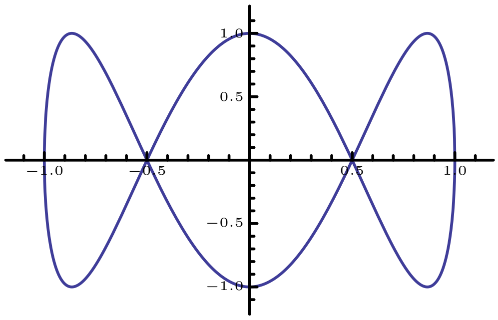
`ParametricPlot[{`f_x, f_y`}, {`u, u_{min}, u_{max}`}]`
    plots a parametric function f with the parameter u ranging from u_{min} to u_{max}.

`ParametricPlot[{{`f_x, f_y`}, {`g_x, g_y`}, ...}, {`u, u_{min}, u_{max}`}]`
    plots several parametric functions f, g, ...

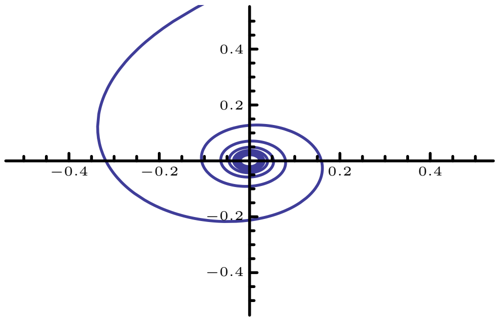
`ParametricPlot[{`f_x, f_y`}, {`u, u_{min}, u_{max}`}, {`v, v_{min}, v_{max}`}]`
    plots a parametric area.

`ParametricPlot[{{`f_x, f_y`}, {`g_x, g_y`}, ...}, {`u, u_{min}, u_{max}`}, {`v, v_{min}, v_{max}`}]`
    plots several parametric areas.

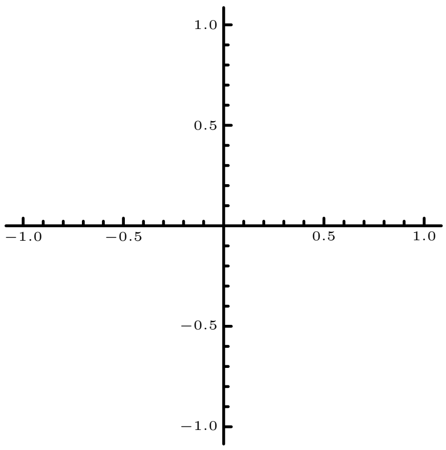
```
>> ParametricPlot[{Sin[u], Cos[3 u]}, {u, 0, 2 Pi}]
```



```
>> ParametricPlot[{Cos[u] / u, Sin[u] / u}, {u, 0, 50}, PlotRange->0.5]
```



```
>> ParametricPlot[{{Sin[u], Cos[u]},{0.6 Sin[u], 0.6 Cos[u]}, {0.2 Sin[u
   ], 0.2 Cos[u]}}, {u, 0, 2 Pi}, PlotRange->1, AspectRatio->1]
```



26.2.14. PieChart

Pie Chart (WMA link)

`PieChart[{`a_1, a_2 ...`}]`
: draws a pie chart with sector angles proportional to a_1, a_2, ...

Drawing options include - Charting:

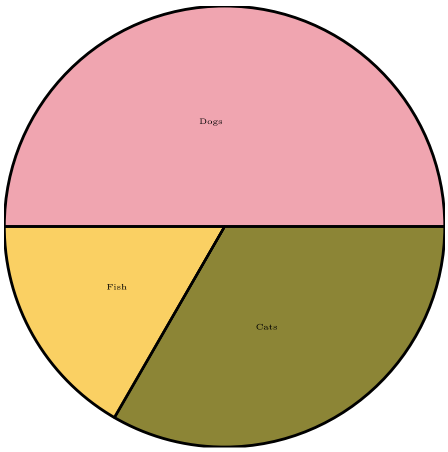
- Mesh
- PlotRange
- ChartLabels
- ChartLegends
- ChartStyle

PieChart specific:

- Axes (default: False, False)
- AspectRatio (default 1)
- SectorOrigin: (default {Automatic, 0})
- SectorSpacing" (default Automatic)

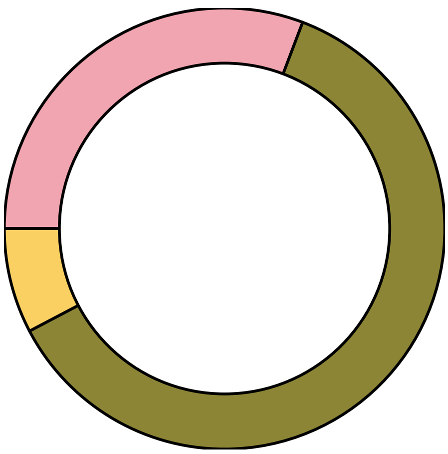
A hypothetical comparison between types of pets owned:

```
>> PieChart[{30, 20, 10}, ChartLabels -> {Dogs, Cats, Fish}]
```



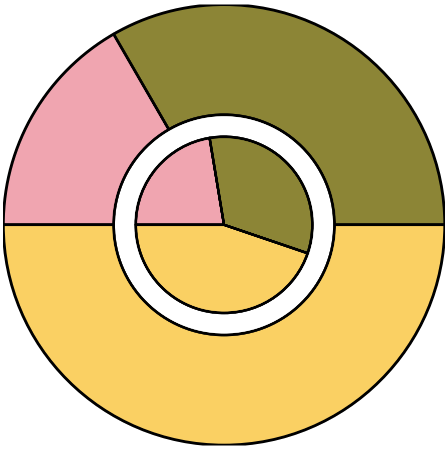
A doughnut chart for a list of values:

```
>> PieChart[{8, 16, 2}, SectorOrigin -> {Automatic, 1.5}]
```



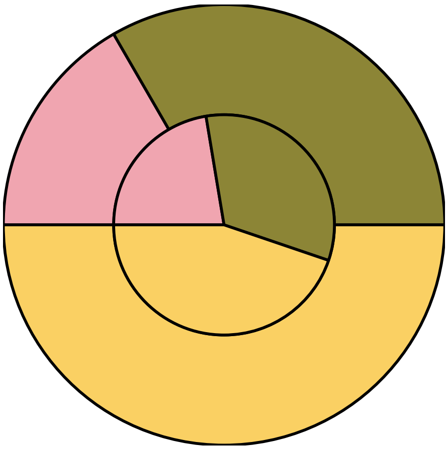
A Pie chart with multiple datasets:

```
>>  PieChart[{{10, 20, 30}, {15, 22, 30}}]
```



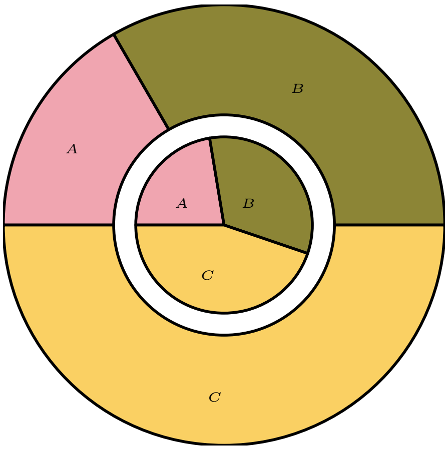
Same as the above, but without gaps between the groups of data:

```
>>  PieChart[{{10, 20, 30}, {15, 22, 30}}, SectorSpacing -> None]
```



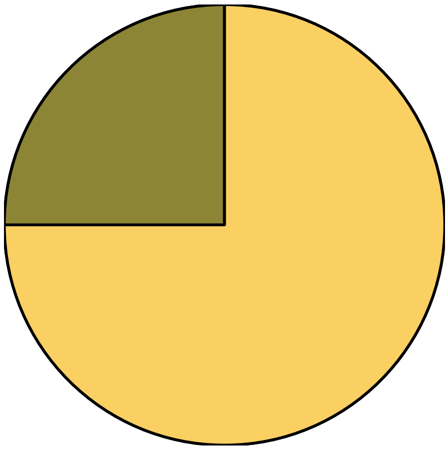
The doughnut chart above with labels on each of the 3 pieces:

```
>>  PieChart[{{10, 20, 30}, {15, 22, 30}}, ChartLabels -> {A, B, C}]
```



Negative values are removed, the data below is the same as {1, 3}:

```
>> PieChart[{1, -1, 3}]
```



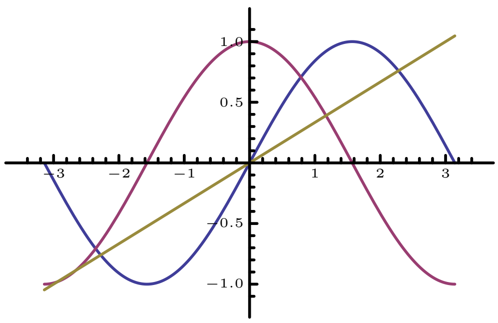
26.2.15. Plot

WMA link

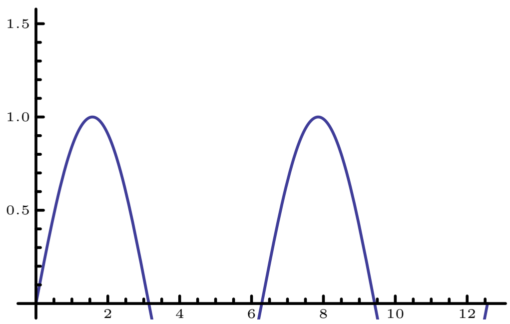
`Plot`[f, {x, x_{min}, x_{max}}]
: plots f with x ranging from x_{min} to x_{max}.

`Plot`[{f_1, f_2, ...}, {x, x_{min}, x_{max}}]
: plots several functions f_1, f_2, ...

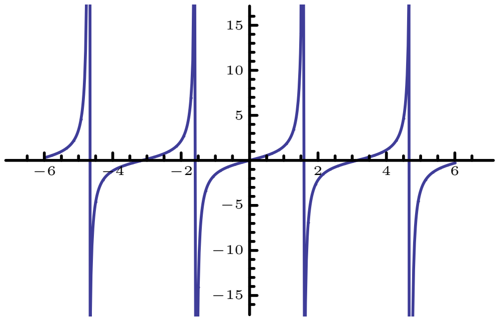
```
>> Plot[{Sin[x], Cos[x], x / 3}, {x, -Pi, Pi}]
```



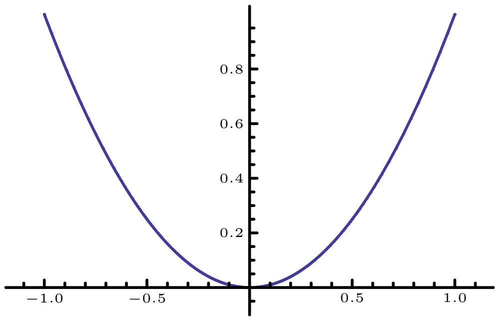

```
>> Plot[Sin[x], {x, 0, 4 Pi}, PlotRange->{{0, 4 Pi}, {0, 1.5}}]
```



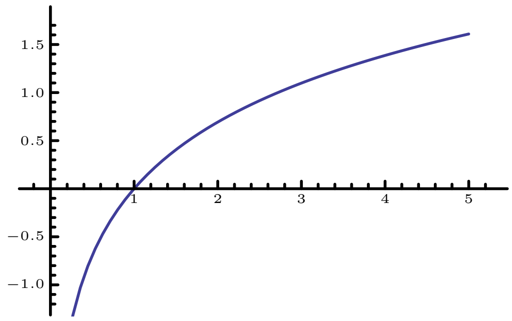
```
>> Plot[Tan[x], {x, -6, 6}, Mesh->Full]
```



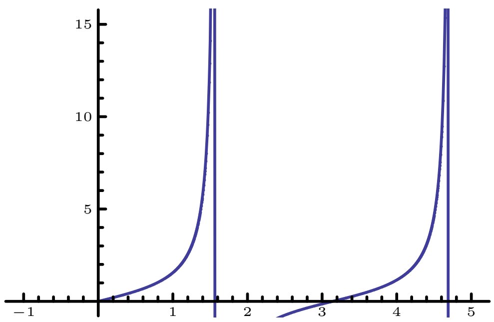
```
>> Plot[x^2, {x, -1, 1}, MaxRecursion->5, Mesh->All]
```



```
>> Plot[Log[x], {x, 0, 5}, MaxRecursion->0]
```

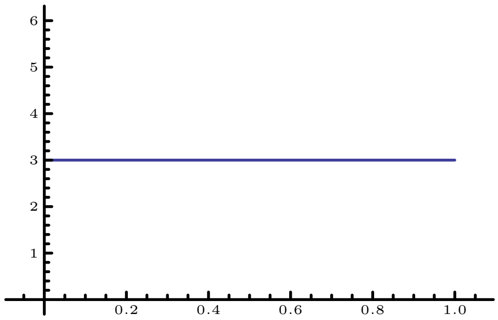


```
>> Plot[Tan[x], {x, 0, 6}, Mesh->All, PlotRange->{{-1, 5}, {0, 15}},
   MaxRecursion->10]
```



A constant function:

>> `Plot[3, {x, 0, 1}]`

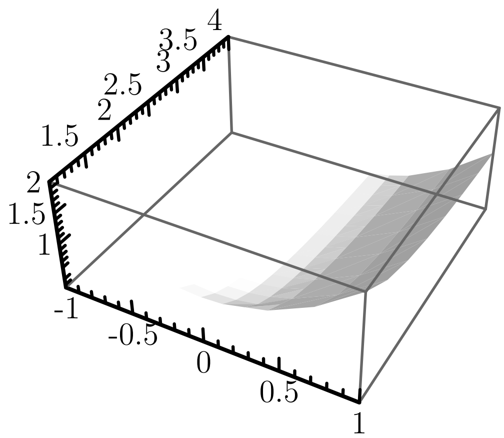


26.2.16. Plot3D

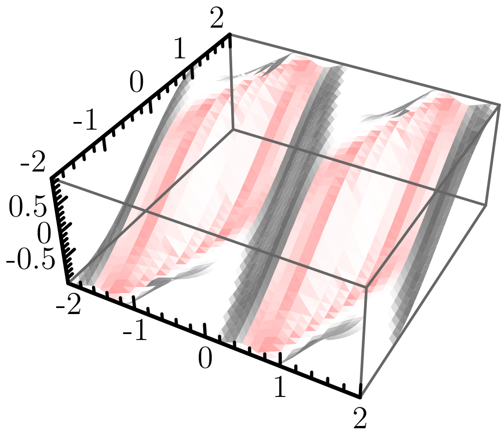
WMA link

`Plot3D`[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}]
creates a three-dimensional plot of f with x ranging from x_{min} to x_{max} and y ranging from y_{min} to y_{max}.
See Drawing Option and Option Values 26.1 for a list of Plot options.

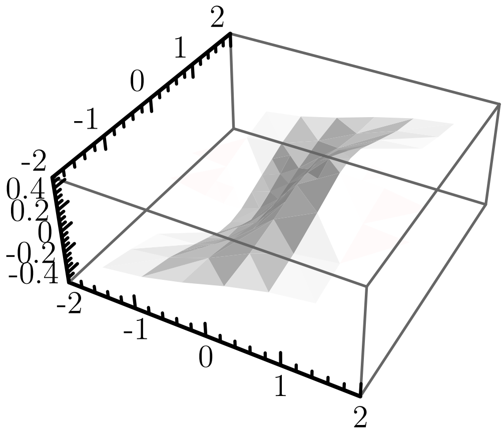
>> `Plot3D[x ^ 2 + 1 / y, {x, -1, 1}, {y, 1, 4}]`



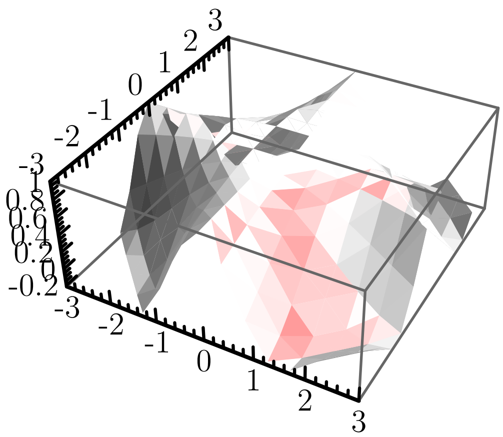
```
>> Plot3D[Sin[y + Sin[3 x]], {x, -2, 2}, {y, -2, 2}, PlotPoints->20]
```



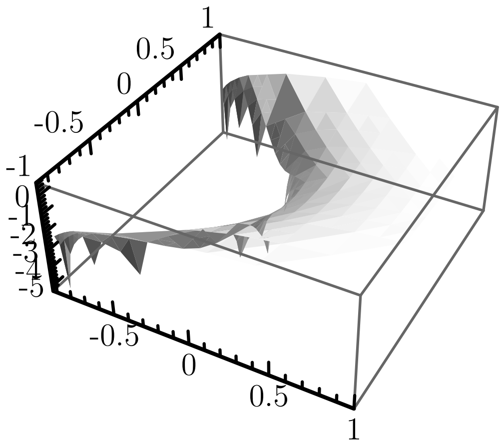

```
>> Plot3D[x / (x ^ 2 + y ^ 2 + 1), {x, -2, 2}, {y, -2, 2}, Mesh->None]
```




```
>> Plot3D[Sin[x y] /(x y), {x, -3, 3}, {y, -3, 3}, Mesh->All]
```



\>> `Plot3D[Log[x + y^2], {x, -1, 1}, {y, -1, 1}]`



26.2.17. PolarPlot

WMA link

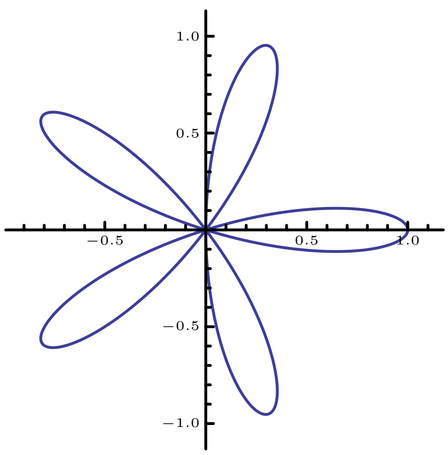
`PolarPlot[`r, $\{t, t_{min}, t_{max}\}$`]`
: creates a polar plot of curve with radius r as a function of angle t ranging from t_{min} to t_{max}.

In a Polar Plot, a polar coordinate system is used.

A polar coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a reference point and an angle from a reference direction.

Here is a 5-blade propeller, or maybe a flower, using `PolarPlot`:

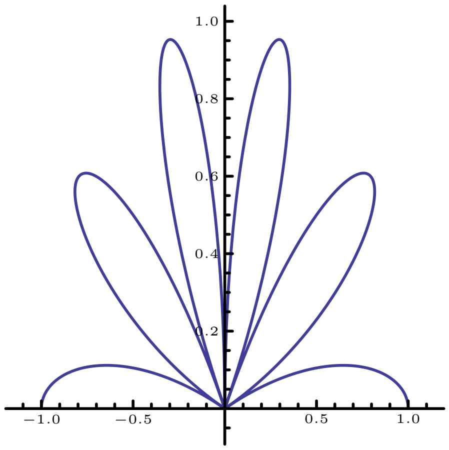
\>> `PolarPlot[Cos[5t], {t, 0, Pi}]`



The number of blades and be change by adjusting the t multiplier.

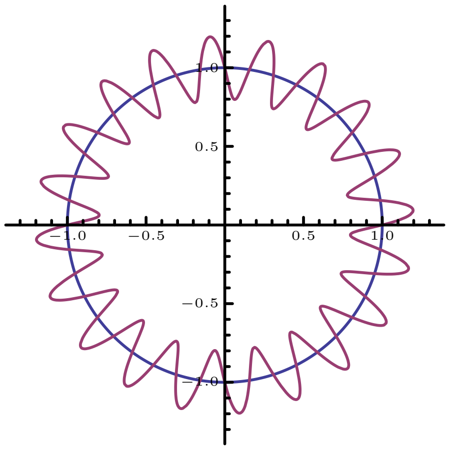
A slight change adding `Abs` turns this a clump of grass:

```
>>  PolarPlot[Abs[Cos[5t]], {t, 0, Pi}]
```



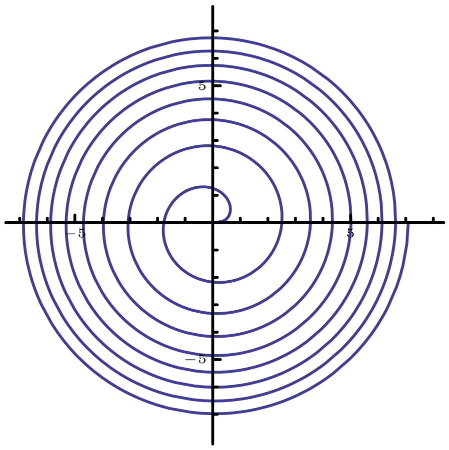
Coils around a ring:

```
>>  PolarPlot[{1, 1 + Sin[20 t] / 5}, {t, 0, 2 Pi}]
```



A spring having 16 turns:

```
>>  PolarPlot[Sqrt[t], {t, 0, 16 Pi}]
```



26.3. Splines

A Spline is a mathematical function used for interpolation or smoothing. Splines are used both in graphics and computations

26.3.1. BernsteinBasis

Bernstein polynomial basis (SciPy :WMA:

A Bernstein is a polynomial that is a linear combination of Bernstein basis polynomials. With the advent of computer graphics, Bernstein polynomials, restricted to the interval [0, 1], became important in the form of Bézier curves. `BernsteinBasis[d,n,x]` equals `Binomial[d, n] x^n (1-x)^(d-n)` in the interval [0, 1] and zero elsewhere.

`BernsteinBasis`[d,n,x]
: returns the nth Bernstein basis of degree d at x.

```
>> BernsteinBasis[4, 3, 0.5]
   0.25
```

26.3.2. BezierCurve

WMA link

`BezierCurve`[{pt_1, pt_2 ...}]
: represents a Bézier curve with control points p_i.
The result is a curve by combining the Bézier curves when points are taken triples at a time.

Option:

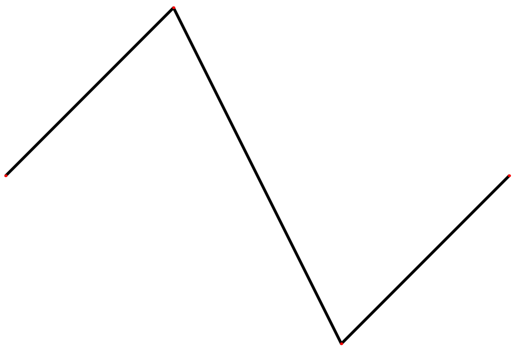
- `SplineDegree->$d$` specifies that the underlying polynomial basis should have maximal degree d.

Set up some points to form a simple zig-zag...

```
>> pts = {{0, 0}, {1, 1}, {2, -1}, {3, 0}};
```

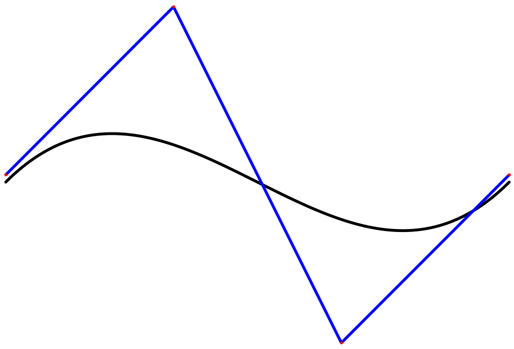
=

```
>> Graphics[{Line[pts], Red, Point[pts]}]
```



A composite Bézier curve, shown in blue, smooths the zig zag. Control points are shown in red:

```
>> Graphics[{BezierCurve[pts], Blue, Line[pts], Red, Point[pts]}]
```



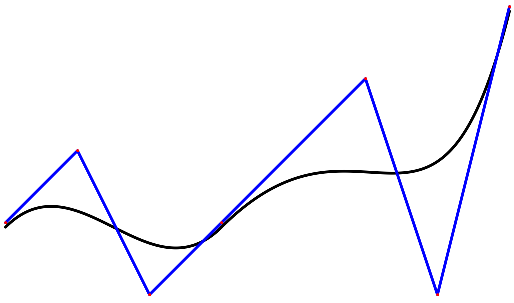
Extend points...

```
>> pts = {{0, 0}, {1, 1}, {2, -1}, {3, 0}, {5, 2}, {6, -1}, {7, 3}};
```

=

A longer composite Bézier curve and its control points:

```
>> Graphics[{BezierCurve[pts], Blue, Line[pts], Red, Point[pts]}]
```



Notice how the curve from the first to third point is not changed by any points outside the interval. The same is true for points three to five, and so on.

26.3.3. BezierFunction

WMA link

`BezierFunction[{`pt_1, pt_2, ...`}]`
: returns a Bézier function for the curve defined by points pt_i. The embedding dimension for the curve represented by `BezierFunction[{$pt_1$,$pt_2$,...}]` is given by the length of the lists pt_i.

```
>> f = BezierFunction[{{0, 0}, {1, 1}, {2, 0}, {3, 2}}];
```

=

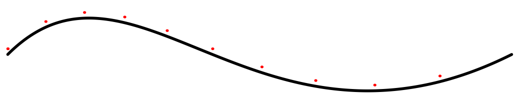
```
>> f[.5]
```

$\{1.5, 0.625\}$

=

Plotting the Bézier Function accoss a Bézier curve:

```
>> Module[{p={{0, 0},{1, 1},{2, -1},{4, 0}}}, Graphics[{BezierCurve[p],
   Red, Point[Table[BezierFunction[p][x], {x, 0, 1, 0.1}]]}]]
```



26.4. Three-Dimensional Graphics

Functions for working with 3D graphics.

26.4.1. Cone

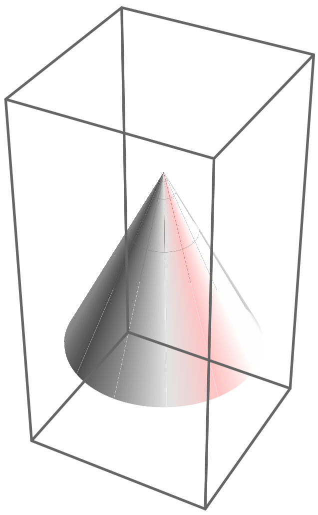
Cone (WMA)

`Cone[]`
: is a cone of radius 1 and height 2 oriented in the upward z direction.

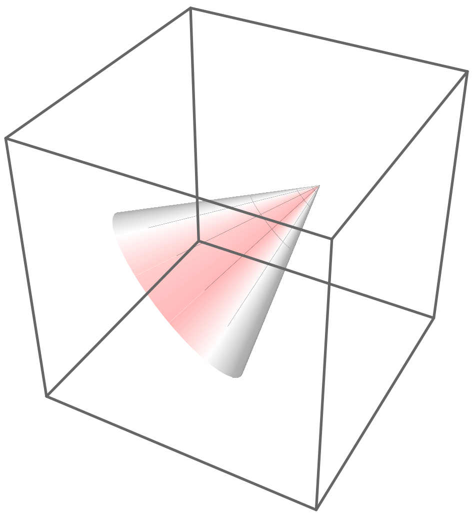
`Cone[{{`x_1, y_1, z_1`}, {`x_2, y_2, z_2`}}, `r`]`
: is a cone of radius r starting at (x_1, y_1, z_1) and ending at (x_2, y_2, z_2).

`Cone[{{`x_1, y_1, z_1`}, {`x_2, y_2, z_2`}, ... }, `r`]`
: is a collection cones of radius r.

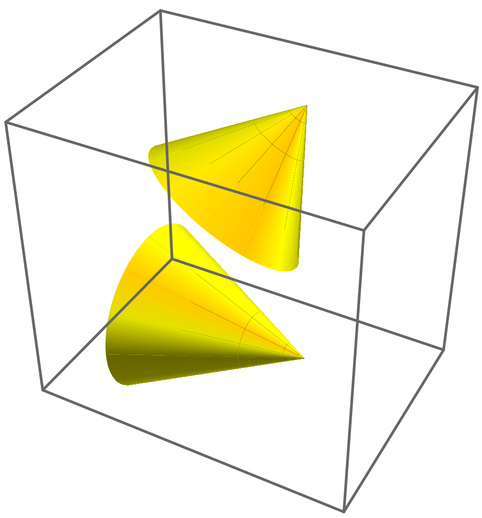
```
>>  Graphics3D[Cone[]]
```



```
>>  Graphics3D[Cone[{{0, 0, 0}, {1, 1, 1}}, 1]]
```



```
>>  Graphics3D[{Yellow, Cone[{{-1, 0, 0}, {1, 0, 0}, {0, 0, Sqrt[3]}, {1,
     1, Sqrt[3]}}, 1]}]
```



26.4.2. Cuboid

Cuboid (WMA)

Cuboid also known as interval, rectangle, square, cube, rectangular parallelepiped, tesseract, orthotope, and box.

`Cuboid`[p_{min}]
: is a unit cube/square with its lower corner at point p_{min}.

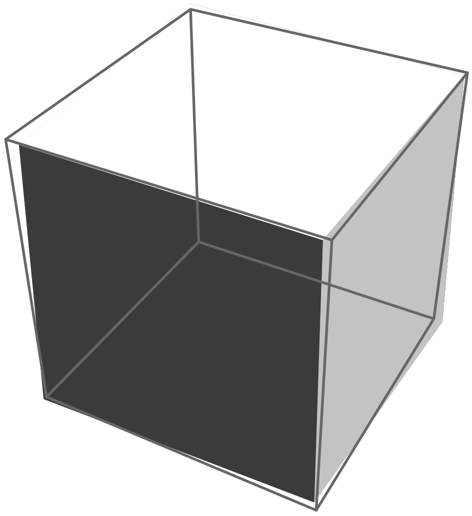
`Cuboid`[p_{min}, p_{max}]
: is a 2d square with with lower corner p_{min} and upper corner p_{max}.

`Cuboid`[{p_{min}, p_{max}}]
: is a cuboid with lower corner p_{min} and upper corner p_{max}.

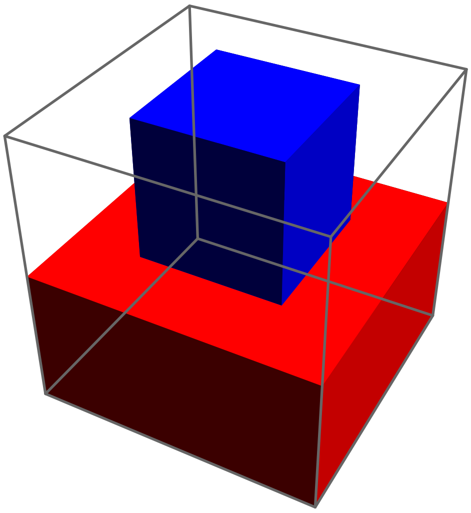
`Cuboid`[{$p1_{min}$, $p1_{max}$, ...}]
: is a collection of cuboids.

`Cuboid[]` is equivalent to `Cuboid`[{0,0,0}].

```
>> Graphics3D[Cuboid[{0, 0, 1}]]
```



```
>> Graphics3D[{Red, Cuboid[{{0, 0, 0}, {1, 1, 0.5}}], Blue, Cuboid
   [{{0.25, 0.25, 0.5}, {0.75, 0.75, 1}}]}]
```



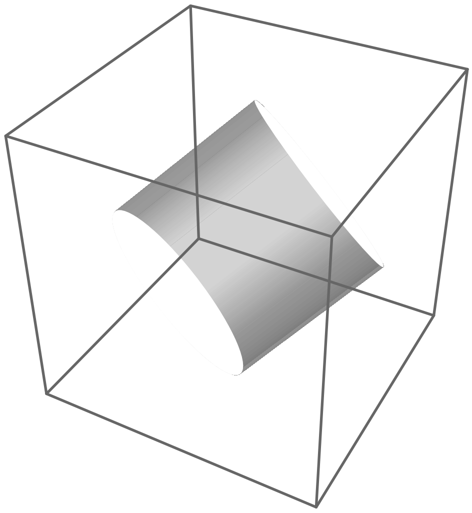
```
>> Graphics[Cuboid[{0, 0}]]
```

26.4.3. Cylinder

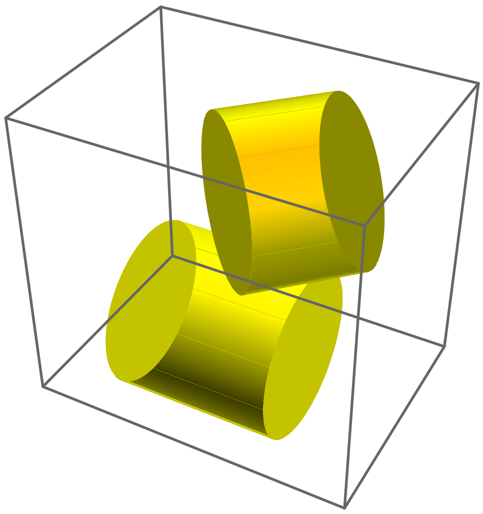
Cylinder (WMA)

`Cylinder`[{{x_1, y_1, z_1}, {x_2, y_2, z_2}}]
    represents a cylinder of radius 1.
`Cylinder`[{{x_1, y_1, z_1}, {x_2, y_2, z_2}}, r]
    represents a cylinder of radius r starting at (x_1, y_1, z_1) and ending at (x_2, y_2, z_2).
`Cylinder`[{{x_1, y_1, z_1}, {x_2, y_2, z_2}, ... }, r]
    represents is a collection cylinders of radius r.

```
>> Graphics3D[Cylinder[{{0, 0, 0}, {1, 1, 1}}, 1]]
```



```
>> Graphics3D[{Yellow, Cylinder[{{-1, 0, 0}, {1, 0, 0}, {0, 0, Sqrt[3]},
    {1, 1, Sqrt[3]}}, 1]}]
```

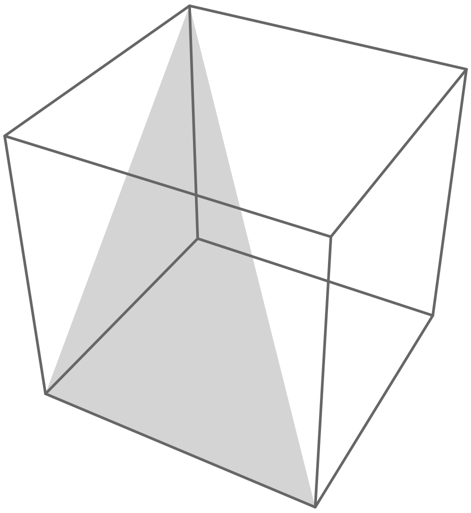


26.4.4. Graphics3D

WMA link

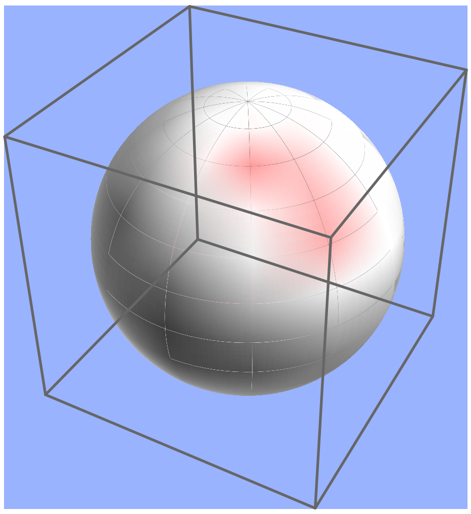
`Graphics3D[`*primitives*, *options*`]`
    represents a three-dimensional graphic.
See Drawing Option and Option Values 26.1 for a list of Plot options.

```
>> Graphics3D[Polygon[{{0,0,0}, {0,1,1}, {1,0,0}}]]
```



The Background option allows to set the color of the background:

```
>> Graphics3D[Sphere[], Background->RGBColor[.6, .7, 1.]]
```



In TeXForm, Graphics3D creates Asymptote figures:

>> **Graphics3D[Sphere[]] // TeXForm**

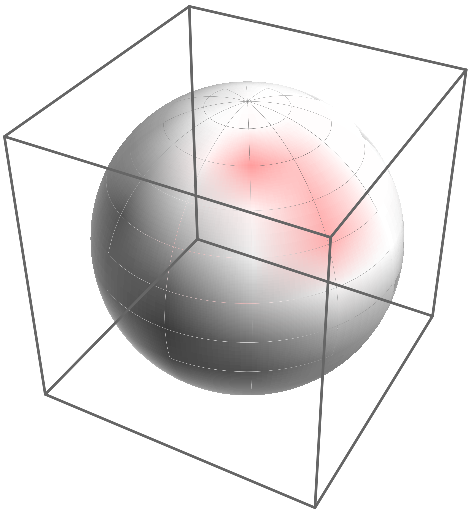
```
\begin{asy}
import three;
import solids;
import tube;
size(6.6667cm, 6.6667cm);
currentprojection=perspective(2.6,-4.8,4.0);
currentlight=light(rgb(0.5,0.5,0.5), specular=red, (2,0,2), (2,2,2), (0,2,2));
// Sphere3DBox
draw(surface(sphere((0, 0, 0), 1)), rgb(1,1,1)+opacity(1));
draw(((-1,-1,-1)--(1,-1,-1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((-1,1,-1)--(1,1,-1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((-1,-1,1)--(1,-1,1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((-1,1,1)--(1,1,1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((-1,-1,-1)--(-1,1,-1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((1,-1,-1)--(1,1,-1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((-1,-1,1)--(-1,1,1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((1,-1,1)--(1,1,1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((-1,-1,-1)--(-1,-1,1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((1,-1,-1)--(1,-1,1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((-1,1,-1)--(-1,1,1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
draw(((1,1,-1)--(1,1,1)), rgb(0.4, 0.4, 0.4)+linewidth(1));
\end{asy}
```

26.4.5. Sphere

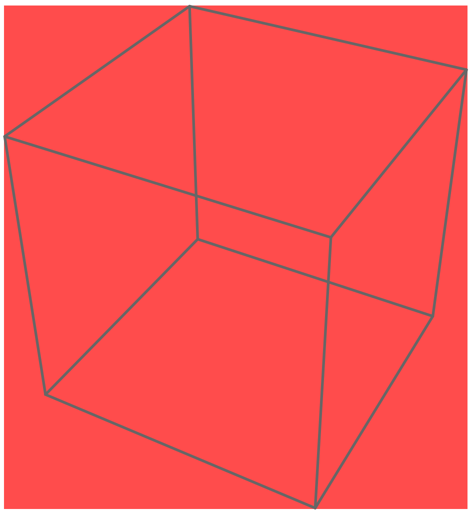
WMA link

`Sphere`[{x, y, z}]
    is a sphere of radius 1 centered at the point {x, y, z}.
`Sphere`[{x, y, z}, r]
    is a sphere of radius r centered at the point {x, y, z}.
`Sphere`[{{x_1, y_1, z_1}, {x_2, y_2, z_2}, ... }, r]
    is a collection spheres of radius r centered at the points {x_1, y_2, z_2}, {x_2, y_2, z_2}, ...

```
>> Graphics3D[Sphere[{0, 0, 0}, 1]]
```



```
>> Graphics3D[{Yellow, Sphere[{{-1, 0, 0}, {1, 0, 0}, {0, 0, Sqrt[3.]}},
   1]}]
```



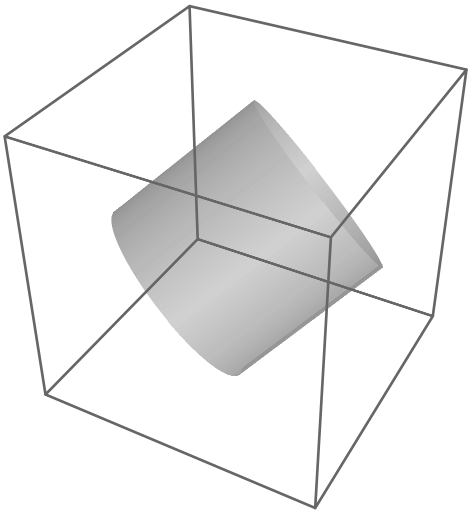
26.4.6. Tube

WMA link

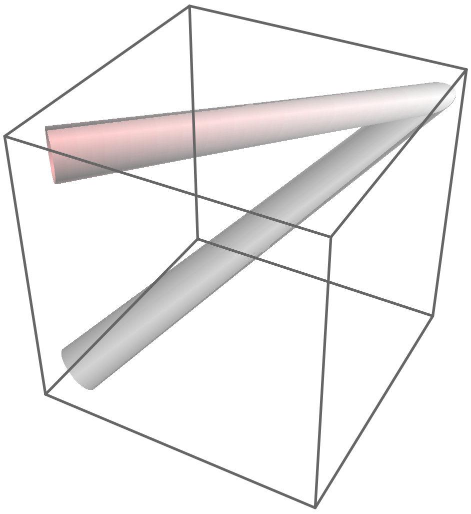
`Tube[{`p_1, p_2, ...`}]`
: represents a tube passing through p_1, p_2, ... with radius 1.

`Tube[{`p_1, p_2, ...`}, `r`]`
: represents a tube with radius r.

```
>> Graphics3D[Tube[{{0,0,0}, {1,1,1}}]]
```



```
>> Graphics3D[Tube[{{0,0,0}, {1,1,1}, {0, 0, 1}}, 0.1]]
```



26.5. Uniform Polyhedra

Uniform polyhedra is the grouping of platonic solids, Archimedean solids,and regular star polyhedra.

26.5.1. Cube

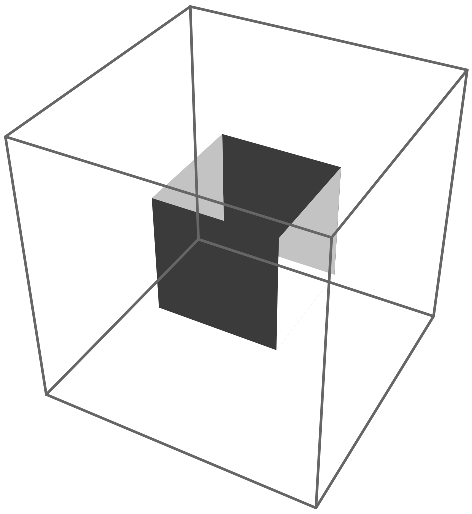
Cube (WMA)

`Cube[]`
: represents a regular cube centered at the origin with unit edge length.

`Cube[`l`]`
: represents a cube centered at the origin with edge length l.

`Cube[`$\{x, y, z\}$`, ...]`
: represents a cube centered at $\{x\ y, z\}$.

>> **`Graphics3D[Cube[]]`**



26.5.2. Dodecahedron

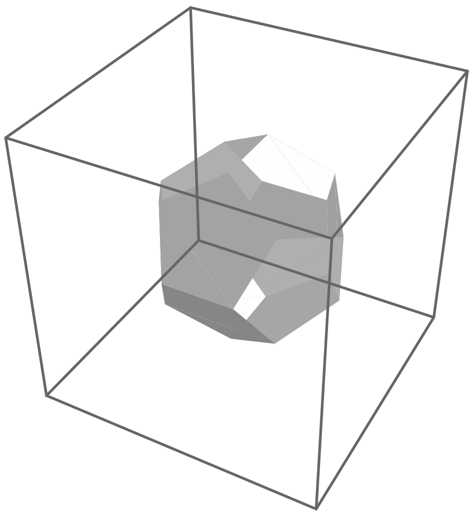
Dodecahedron (WMA)

`Dodecahedron[]`
: a regular dodecahedron centered at the origin with unit edge length.

`Dodecahedron[`l`]`
: a regular dodecahedron centered at the origin with edge length l.

`Dodecahedron[`$\{x, y, z\}$`, ...]`
: a regular dodecahedron centered at $\{x\ y, z\}$.

>> `Graphics3D[Dodecahedron[]]`



26.5.3. Icosahedron

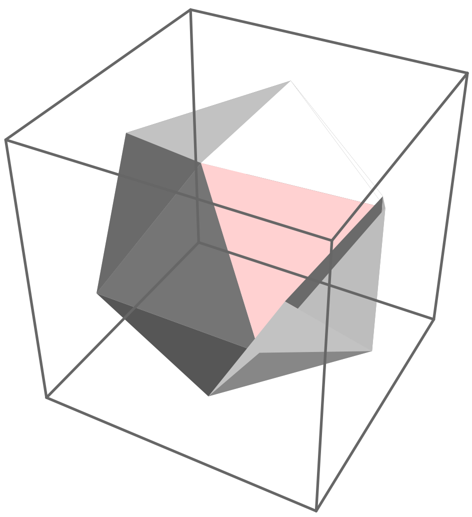
Icosahedron (WMA)

`Icosahedron[]`
: a regular Icosahedron centered at the origin with unit edge length.

`Icosahedron[`l`]`
: a regular icosahedron centered at the origin with edge length l.

`Icosahedron[`$\{x, y, z\}$`, ...]`
: a regular icosahedron centered at $\{x\ y, z\}$.

>> `Graphics3D[Icosahedron[]]`



26.5.4. Octahedron

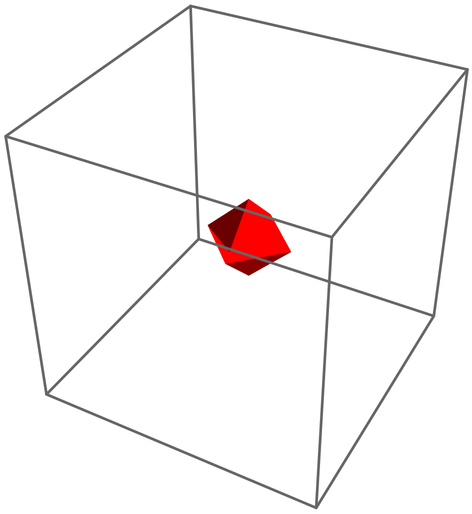
Octahedron (WMA)

`Octahedron[]`
: a regular octahedron centered at the origin with unit edge length.

`Octahedron[$l$]`
: a regular octahedron centered at the origin with edge length l.

`Octahedron[{$x$, $y$, $z$}, ...]`
: a regular octahedron centered at $\{x\ y, z\}$.

```
>> Graphics3D[{Red, Octahedron[]}]
```



26.5.5. Tetrahedron

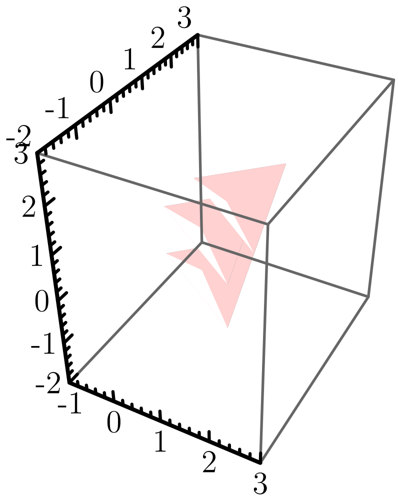
Tetrahedron (WMA)

`Tetrahedron[]`
: a regular tetrahedron centered at the origin with unit edge length.

`Tetrahedron[`l`]`
: a regular tetrahedron centered at the origin with edge length l.

`Tetrahedron[`$\{x, y, z\}$, ...`]`
: a regular tetrahedron centered at $\{x\ y, z\}$.

>> **`Graphics3D[Tetrahedron[{{1,0,0}, {1,0,1}, {1, 1, 1}}, 2], Axes->True]`**

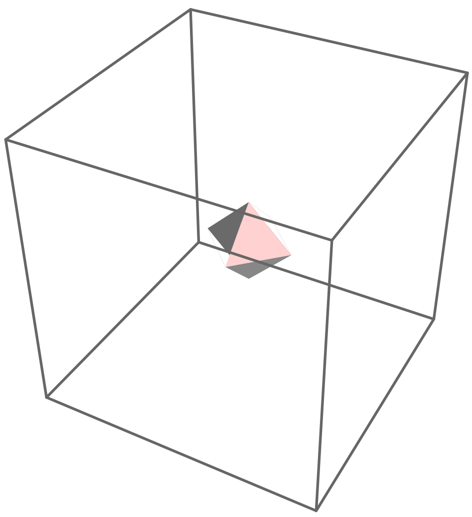


26.5.6. UniformPolyhedron

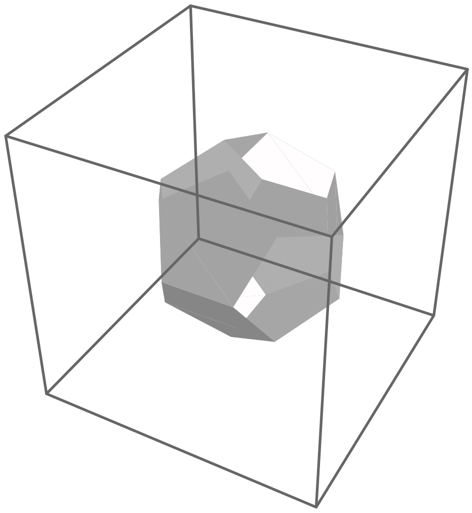
Uniform polyhedron (WMA link)

> `UniformPolyhedron`[“*name*”]
> return a uniform polyhedron with the given name.
> Names are “*tetrahedron*”, “*octahedron*”, “*dodecahedron*”, or “*icosahedron*”.

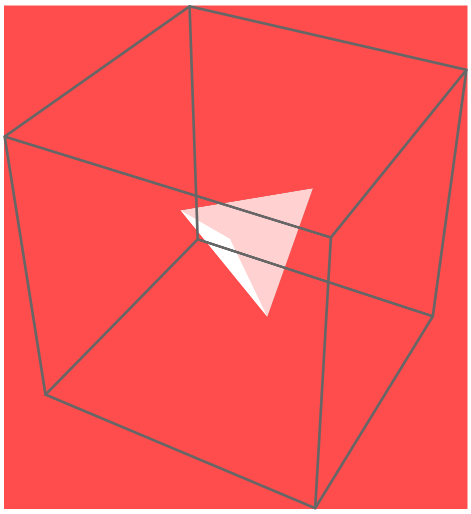
>> **`Graphics3D[UniformPolyhedron["octahedron"]]`**



```
>> Graphics3D[UniformPolyhedron["dodecahedron"]]
```



```
>> Graphics3D[{"Brown", UniformPolyhedron["tetrahedron"]}]
```



27. Image Manipulation

For the full compliment of functions, you need to have scikit-image installed.

Contents

27.1. Basic Image Processing

27.1.1. Blur

WMA link

`Blur`[*image*]
: gives a blurred version of *image*.

`Blur`[*image*, *r*]
: blurs *image* with a kernel of size *r*.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> Blur[hedy]
```

```
>> Blur[hedy, 5]
```

27.1.2. ImageAdjust

WMA link

`ImageAdjust`[*image*]
: adjusts the levels in *image*.

`ImageAdjust`[*image*, *c*]
: adjusts the contrast in *image* by *c*.

`ImageAdjust`[*image*, {*c*, *b*}]
: adjusts the contrast *c*, and brightness *b* in *image*.

`ImageAdjust`[*image*, {*c*, *b*, *g*}]
: adjusts the contrast *c*, brightness *b*, and gamma *g* in *image*.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> ImageAdjust[hedy]
```

27.1.3. ImagePartition

WMA link

`ImagePartition`[*image*, *s*]
: Partitions an image into an array of *s* x *s* pixel subimages.

`ImagePartition`[*image*, {*w*, *h*}]
: Partitions an image into an array of *w* x *h* pixel subimages.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> ImageDimensions[hedy]
```
{646, 800}

```
>> ImagePartition[hedy, 256]
```

```
>> ImagePartition[hedy, {512, 128}]
```

27.1.4. Sharpen

WMA link

Sharpen[*image*]
    gives a sharpened version of *image*.
Sharpen[*image*, *r*]
    sharpens *image* with a kernel of size *r*.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> Sharpen[hedy]
```

```
>> Sharpen[hedy, 5]
```

27.1.5. Threshold

WMA link

`Threshold[`*image*`]`
: gives a value suitable for binarizing *image*.

The option “Method” may be “Cluster” (use Otsu’s threshold), “Median”, or “Mean”.

```
>> img = Import["ExampleData/hedy.tif"];
```

```
>> Threshold[img]
```

0.408203

```
>> Binarize[img, %]
```

```
>> Threshold[img, Method -> "Mean"]
```

0.22086

```
>> Threshold[img, Method -> "Median"]
```

0.0593961

27.2. Geometric Operations

27.2.1. ImageReflect

WMA link

`ImageReflect[`*image*`]`
: Flips *image* top to bottom.

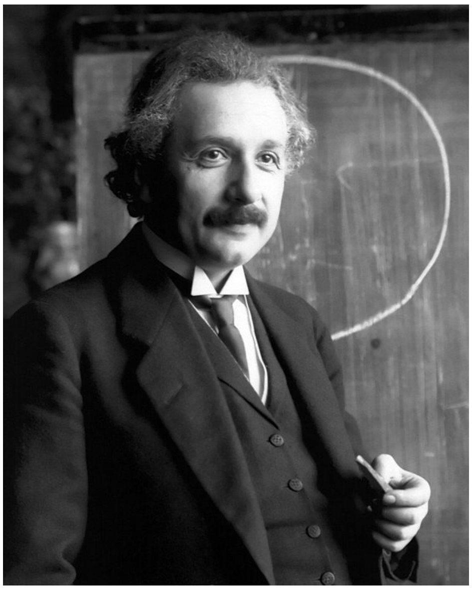
`ImageReflect[`*image*, *side*`]`
: Flips *image* so that *side* is interchanged with its opposite.

`ImageReflect[`*image*, $side_1$ -> $side_2$`]`
: Flips *image* so that $side_1$ is interchanged with $side_2$.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

`>>` `ImageReflect[ein]`

`>>` `ImageReflect[ein, Left]`



`>>` `ImageReflect[ein, Left -> Top]`

27.2.2. ImageResize

WMA link

> `ImageResize`[*image*, *width*]
> `ImageResize`[*image*, {*width*, *height*}]

The Resampling option can be used to specify how to resample the image. Options are:

- Automatic
- Bicubic
- Bilinear
- Box
- Hamming
- Lanczos
- Nearest

See Pillow Filters for a description of these.

```
>> alice = Import["ExampleData/MadTeaParty.gif"]
```

```
>> shape = ImageDimensions[alice]
```

$\{640, 487\}$

```
>> ImageResize[alice, shape / 2]
```

The default sampling method is "Bicubic" which has pretty good upscaling and downscaling quality. However "Box" is the fastest:

```
>> ImageResize[alice, shape / 2, Resampling -> "Box"]
```

27.2.3. ImageRotate

WMA link

`ImageRotate[`*image*`]`
: Rotates *image* 90 degrees counterclockwise.

`ImageRotate[`*image*, *theta*`]`
: Rotates *image* by a given angle *theta*

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> ImageRotate[ein]
```

```
>> ImageRotate[ein, 45 Degree]
```

```
>> ImageRotate[ein, Pi / 4]
```

27.3. Image Colors

27.3.1. Binarize

WMA link

`Binarize[`*image*`]`
: gives a binarized version of *image*, in which each pixel is either 0 or 1.

`Binarize[`*image*, *t*`]`
: map values $x > t$ to 1, and values $x <= t$ to 0.

`Binarize[`*image*, $\{t_1, t_2\}$`]`
: map $t_1 < x < t_2$ to 1, and all other values to 0.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> Binarize[hedy]
```

```
>> Binarize[hedy, 0.7]
```

```
>> Binarize[hedy, {0.2, 0.6}]
```

27.3.2. ColorQuantize

WMA link

`ColorQuantize[`*image*, *n*`]`
    gives a version of *image* using only *n* colors.

```
>> img = Import["ExampleData/hedy.tif"];
```

```
>> ColorQuantize[img, 6]
```

27.3.3. ColorSeparate

WMA link

`ColorSeparate[`*image*`]`
: Gives each channel of *image* as a separate grayscale image.

```
>> img = Import["ExampleData/hedy.tif"];
```

```
>> ColorSeparate[img]
```

27.3.4. Colorize

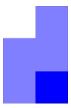
WMA link

`Colorize[`*values*`]`
: returns an image where each number in the rectangular matrix *values* is a pixel and each occurrence of the same number is displayed in the same unique color, which is different from the colors of all non-identical numbers.

`Colorize[`*image*`]`
: gives a colorized version of *image*.

```
>> Colorize[{{1.3, 2.1, 1.5}, {1.3, 1.3, 2.1}, {1.3, 2.1, 1.5}}]
```

```
>> Colorize[{{1, 2}, {2, 2}, {2, 3}}, ColorFunction -> (Blend[{White,
   Blue}, #]&)]
```



27.3.5. ImageColorSpace

WMA link

`ImageColorSpace[`*image*`]`
: gives *image*'s color space, e.g. "RGB" or "CMYK".

```
>> img = Import["ExampleData/MadTeaParty.gif"];
```

```
>> ImageColorSpace[img]
```

Grayscale

```
>> img = Import["ExampleData/sunflowers.jpg"];
```

```
>> ImageColorSpace[img]
```

RGB

27.4. Image Compositions

27.4.1. ImageAdd

WMA link

`ImageAdd`[*image*, $expr_1$, $expr_2$, ...]
    adds all $expr_i$ to *image* where each $expr_i$ must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1, 0.9}, {1.0, 0.1, 0.3, 0.8, 0.6}}];
```

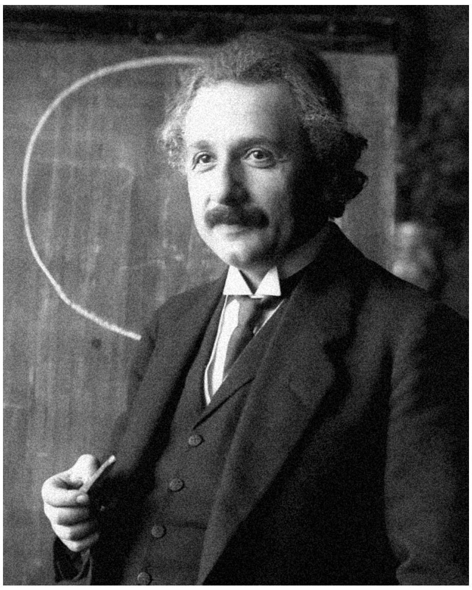
```
>> ImageAdd[i, 0.5]
```

```
>> ImageAdd[i, i]
```

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> noise = RandomImage[{-0.1, 0.1}, ImageDimensions[ein]];
```

```
>> ImageAdd[noise, ein]
```



```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> noise = RandomImage[{-0.2, 0.2}, ImageDimensions[hedy], ColorSpace ->
    "RGB"];
```

```
>> ImageAdd[noise, hedy]
```

27.4.2. ImageMultiply

WMA link

`ImageMultiply`[*image*, $expr_1$, $expr_2$, ...]
multiplies all $expr_i$ with *image* where each $expr_i$ must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1, 0.9}, {1.0, 0.1, 0.3, 0.8, 0.6}}];
```

```
>> ImageMultiply[i, 0.2]
```

```
>> ImageMultiply[i, i]
```

27.4.3. ImageSubtract

WMA link

`ImageSubtract`[*image*, $expr_1$, $expr_2$, ...]
subtracts all $expr_i$ from *image* where each $expr_i$ must be an image or a real number.

```
>> i = Image[{{0, 0.5, 0.2, 0.1, 0.9}, {1.0, 0.1, 0.3, 0.8, 0.6}}];

>> ImageSubtract[i, 0.2]
```

```
>> ImageSubtract[i, i]
```

27.4.4. WordCloud

WMA link

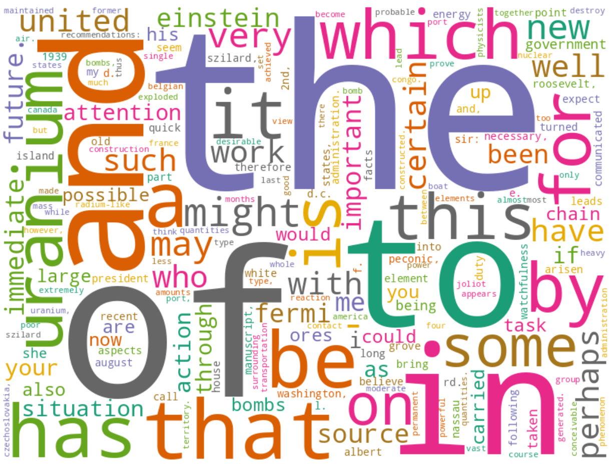
`WordCloud`[{*word*$_1$, *word*$_2$, ...}]
Gives a word cloud with the given list of words.

`WordCloud`[{*weight*$_1$ -> *word*$_1$, *weight*$_2$ -> *word*$_2$, ...}]
Gives a word cloud with the words weighted using the given weights.

`WordCloud`[{*weight*$_1$, *weight*$_2$, ...} -> {*word*$_1$, *word*$_2$, ...}]
Also gives a word cloud with the words weighted using the given weights.

`WordCloud`[{{*word*$_1$, *weight*$_1$}, {*word*$_2$, *weight*$_2$}, ...}]
Gives a word cloud with the words weighted using the given weights.

```
>> WordCloud[StringSplit[Import["ExampleData/EinsteinSzilLetter.txt",
   CharacterEncoding->"UTF8"]]]
```



```
>>  WordCloud[Range[50] -> ToString /@ Range[50]]
```

27.5. Image Filters

27.5.1. GaussianFilter

WMA link

`GaussianFilter[`*image*, *r*`]`
: blurs *image* using a Gaussian blur filter of radius *r*.

```
>>  hedy = Import["ExampleData/hedy.tif"];
```

```
>>  GaussianFilter[hedy, 2.5]
```

27.5.2. ImageConvolve

WMA link

`ImageConvolve[`*image*, *kernel*`]`
: Computes the convolution of *image* using *kernel*.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> ImageConvolve[hedy, DiamondMatrix[5] / 61]
```

```
>> ImageConvolve[hedy, DiskMatrix[5] / 97]
```

```
>> ImageConvolve[hedy, BoxMatrix[5] / 121]
```

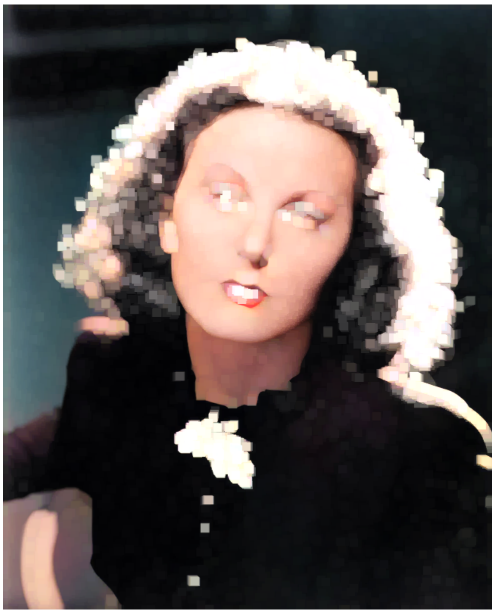
27.5.3. MaxFilter

WMA link

`MaxFilter`[*image*, *r*]
: gives *image* with a maximum filter of radius *r* applied on it. This always picks the largest value in the filter's area.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> MaxFilter[hedy, 5]
```



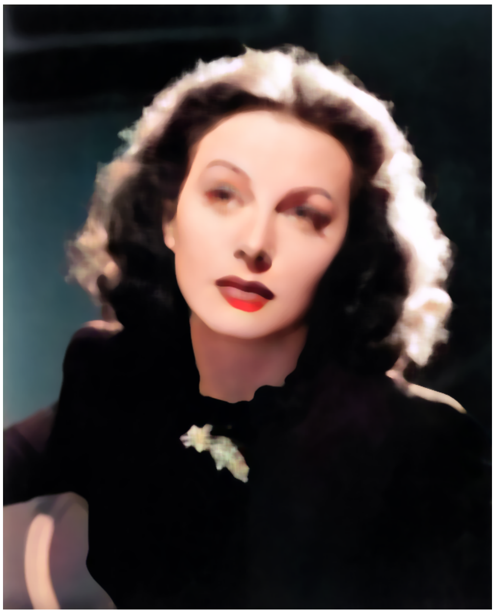
27.5.4. MedianFilter

WMA link

`MedianFilter[`*image*, *r*`]`
: gives *image* with a median filter of radius *r* applied on it. This always picks the median value in the filter's area.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> MedianFilter[hedy, 5]
```



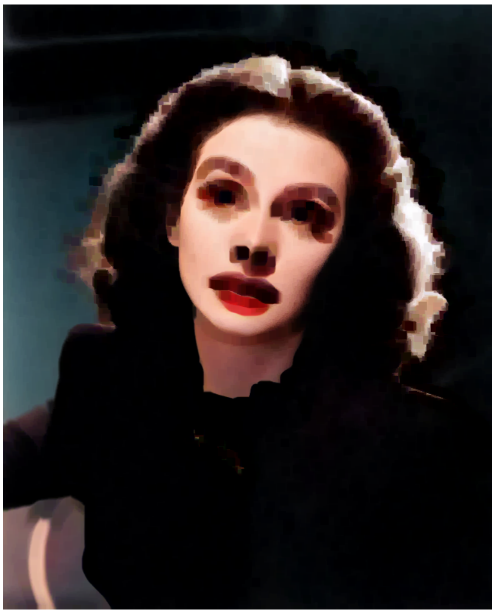
27.5.5. MinFilter

WMA link

`MinFilter`[*image*, *r*]
: gives *image* with a minimum filter of radius *r* applied on it. This always picks the smallest value in the filter's area.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> MinFilter[hedy, 5]
```



27.6. Image Properties

27.6.1. ImageAspectRatio

WMA link

`ImageAspectRatio[`*image*`]`
: gives the aspect ratio of *image*.

```
>> img = Import["ExampleData/hedy.tif"];
```

```
>> ImageAspectRatio[img]
```

$$\frac{400}{323}$$

```
>> ImageAspectRatio[Image[{{0, 1}, {1, 0}, {1, 1}}]]
```

$$\frac{3}{2}$$

27.6.2. ImageChannels

WMA link

`ImageChannels[`*image*`]`
: gives the number of channels in *image*.

```
>> ImageChannels[Image[{{0, 1}, {1, 0}}]]
```
1

```
>> img = Import["ExampleData/hedy.tif"];
```

```
>> ImageChannels[img]
```
3

27.6.3. ImageData

WMA link

`ImageData`[*image*]
gives a list of all color values of *image* as a matrix.
`ImageData`[*image*, *stype*]
gives a list of color values in type *stype*.

```
>> img = Image[{{0.2, 0.4}, {0.9, 0.6}, {0.5, 0.8}}];
```

```
>> ImageData[img]
```
$\{\{0.2, 0.4\}, \{0.9, 0.6\}, \{0.5, 0.8\}\}$

```
>> ImageData[img, "Byte"]
```
$\{\{51, 102\}, \{229, 153\}, \{127, 204\}\}$

```
>> ImageData[Image[{{0, 1}, {1, 0}, {1, 1}}], "Bit"]
```
$\{\{0, 1\}, \{1, 0\}, \{1, 1\}\}$

27.6.4. ImageDimensions

WMA link

`ImageDimensions`[*image*]
Returns the dimensions {*width*, *height*} of *image* in pixels.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> ImageDimensions[hedy]
```
$\{646, 800\}$

```
>> ImageDimensions[RandomImage[1, {50, 70}]]
```
$\{50, 70\}$

27.6.5. ImageType

WMA link

`ImageType[`*image*`]`
: gives the interval storage type of *image*, e.g. “Real”, “Bit32”, or “Bit”.

```
>> img = Import["ExampleData/hedy.tif"];
```

```
>> ImageType[img]
```
Byte

```
>> ImageType[Image[{{0, 1}, {1, 0}}]]
```
Real

```
>> ImageType[Binarize[img]]
```
Bit

27.7. Image testing

27.7.1. BinaryImageQ

WMA link

`BinaryImageQ[`*image*`]`
: returns `True` if the pixels of *image* are binary bit values, and `False` otherwise.

```
>> img = Import["ExampleData/hedy.tif"];
```

```
>> BinaryImageQ[img]
```
False

```
>> BinaryImageQ[Binarize[img]]
```
True

27.7.2. ImageQ

WMA link

`ImageQ[Image[`*pixels*`]]`
: returns `True` if *pixels* has dimensions from which an Image can be constructed, and `False` otherwise.

```
>> ImageQ[Image[{{0, 1}, {1, 0}}]]
```
True

```
>> ImageQ[Image[{{{0, 0, 0}, {0, 1, 0}}, {{0, 1, 0}, {0, 1, 1}}}]]
```
True

```
>> ImageQ[Image[{{{0, 0, 0}, {0, 1}}, {{0, 1, 0}, {0, 1, 1}}}]]
```
False

```
>> ImageQ[Image[{1, 0, 1}]]
```
False

```
>> ImageQ["abc"]
```
False

27.8. Miscellaneous image-related functions

27.8.1. EdgeDetect

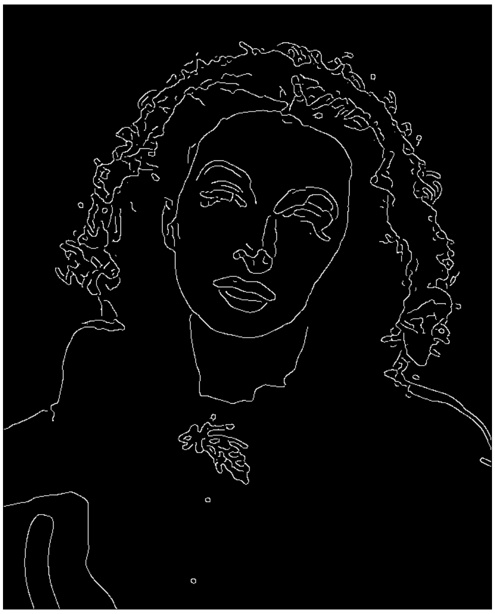
WMA link

`EdgeDetect[`*image*`]`
: returns an image showing the edges in *image*.

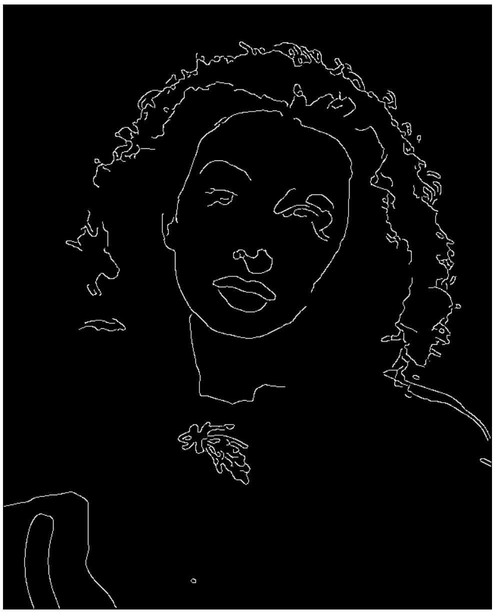
```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> EdgeDetect[hedy]
```

```
>> EdgeDetect[hedy, 5]
```



```
>> EdgeDetect[hedy, 4, 0.5]
```



27.8.2. RandomImage

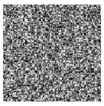
WMA link

RandomImage[*max*]
: creates an image of random pixels with values 0 to *max*.

RandomImage[{*min*, *max*}]
: creates an image of random pixels with values *min* to *max*.

RandomImage[..., *size*]
: creates an image of the given *size*.

```
>> RandomImage[1, {100, 100}]
```

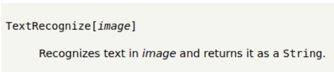


27.8.3. TextRecognize

WMA link

TextRecognize[*image*]
: Recognizes text in *image* and returns it as a `String`.

```
>> textimage = Import["ExampleData/TextRecognize.png"]
```




```
>> TextRecognize[textimage]
```

TextRecognize[image]

Recognizes text in image and returns it as a String.

27.9. Morphological Image Processing

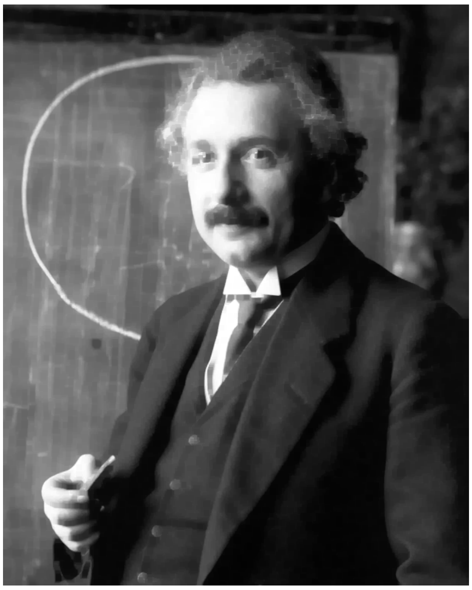
27.9.1. Closing

WMA link

Closing[*image*, *ker*]
: Gives the morphological closing of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> Closing[ein, 2.5]
```



27.9.2. Dilation

WMA link

`Dilation[`*image*, *ker*`]`
: Gives the morphological dilation of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> Dilation[ein, 2.5]
```

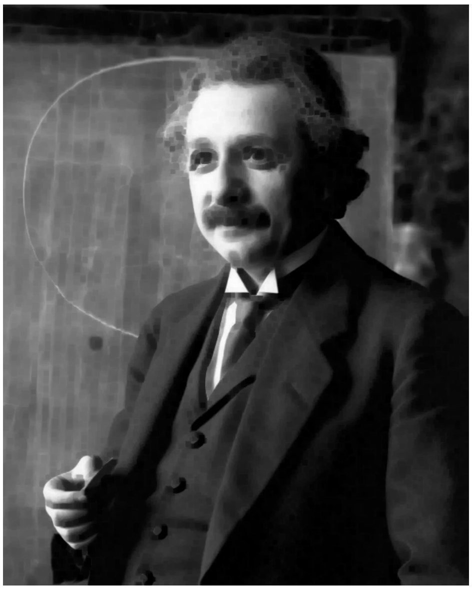
27.9.3. Erosion

WMA link

`Erosion`[*image*, *ker*]
: Gives the morphological erosion of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> Erosion[ein, 2.5]
```



27.9.4. MorphologicalComponents

WMA link

`MorphologicalComponents`[*image*]
: Builds a 2-D array in which each pixel of *image* is replaced by an integer index representing the connected foreground image component in which the pixel lies.

`MorphologicalComponents`[*image*, *threshold*]
: consider any pixel with a value above *threshold* as the foreground.

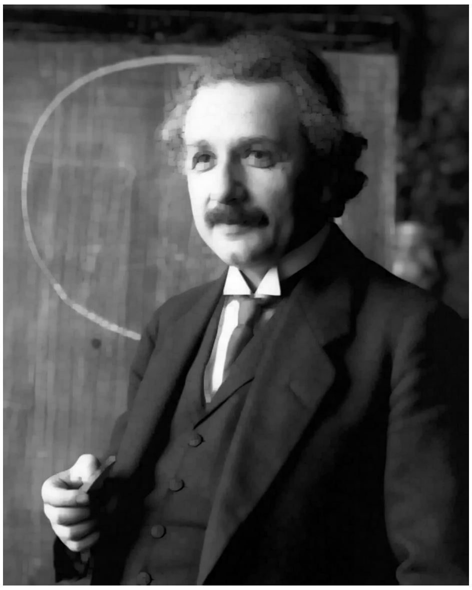
27.9.5. Opening

WMA link

`Opening`[*image*, *ker*]
: Gives the morphological opening of *image* with respect to structuring element *ker*.

```
>> ein = Import["ExampleData/Einstein.jpg"];
```

```
>> Opening[ein, 2.5]
```



27.10. Operations on Image Structure

27.10.1. ImageTake

Extract Image parts WMA link

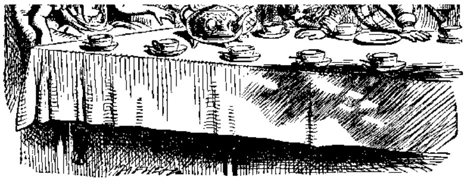
> `ImageTake`[*image*, *n*]
> gives the first *n* rows of *image*.
> `ImageTake`[*image*, -*n*]
> gives the last *n* rows of *image*.
> `ImageTake`[*image*, {r_1, r_2}]
> gives rows r_1, ..., r_2 of *image*.
> `ImageTake`[*image*, {r_1, r_2}, {c_1, c_2}]
> gives a cropped version of *image*.

Crop to the include only the upper half (244 rows) of an image:

```
>> alice = Import["ExampleData/MadTeaParty.gif"]; ImageTake[alice, 244]
```

Now crop to the include the lower half of that image:

```
>> ImageTake[alice, -244]
```



Just the text around the hat:

```
>> ImageTake[alice, {40, 150}, {500, 600}]
```

27.11. Pixel Operations

27.11.1. PixelValue

WMA link

`PixelValue`[*image*, {*x*, *y*}]
gives the value of the pixel at position {*x*, *y*} in *image*.

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> PixelValue[hedy, {1, 1}]
```

$\{0.439216, 0.356863, 0.337255\}$

27.11.2. PixelValuePositions

WMA link

`PixelValuePositions`[*image*, *val*]
gives the positions of all pixels in *image* that have value *val*.

```
>> PixelValuePositions[Image[{{0, 1}, {1, 0}, {1, 1}}], 1]
```

$\{\{1,1\},\{1,2\},\{2,1\},\{2,3\}\}$

```
>> PixelValuePositions[Image[{{0.2, 0.4}, {0.9, 0.6}, {0.3, 0.8}}], 0.5,
    0.15]
```

$\{\{2,2\},\{2,3\}\}$

```
>> hedy = Import["ExampleData/hedy.tif"];
```

```
>> PixelValuePositions[hedy, 1, 0][[1]]
```
$\{101, 491, 1\}$

```
>> PixelValue[hedy, {180, 192}]
```
$\{0.00784314, 0.00784314, 0.0156863\}$

28. Input and Output

Contents

28.1. $Echo

WMA link

`$Echo`
    gives a list of files and pipes to which all input is echoed.

28.2. Print

WMA link

`Print`[*expr*, ...]
    prints each *expr* in string form.

```
>> Print["Hello world!"]
Hello world!
>> Print["The answer is ", 7 * 6, "."]
The answer is 42.
```

29. Input/Output, Files, and Filesystem

Contents

29.1. File and Stream Operations

29.1.1. Character

WMA link

`Character`
: is a data type for `Read`.

29.1.2. Close

WMA link

`Close[`*obj*`]`
: Closes a stream or socket.

obj can be an `InputStream`, or an `OutputStream` object, or a `String`. When *obj* is a string file path, one of the channels associated with it is closed.

```
>> Close[StringToStream["123abc"]]
```
String

```
>> file=Close[OpenWrite[]]
```
/tmp/tmp47f92_kb

Closing a file doesn't delete it from the filesystem.

```
>> DeleteFile[file];
```

If two streams are open with the same file, then a `Close` by file path closes only one of the streams:

```
>> stream1 = OpenRead["ExampleData/numbers.txt"]
```
InputStream [ExampleData/numbers.txt, 15]

```
>> stream2 = OpenRead["ExampleData/numbers.txt"]
```
InputStream [ExampleData/numbers.txt, 16]

```
>> Close["ExampleData/numbers.txt"]
```
ExampleData/numbers.txt

Usually, the most-recent stream is closed, while the earlier-opened stream still persists:

```
>> Read[stream1]
```
8205.79

However, one of the streams *is* closed:

```
>> Read[stream2]
InputStream[ExampleData/numbers.txt, 16] is not open.
$Failed
```

```
>> Close["ExampleData/numbers.txt"]
ExampleData/numbers.txt
```

```
>> Read[stream1]
InputStream[ExampleData/numbers.txt, 15] is not open.
$Failed
```

29.1.3. EndOfFile

WMA link

`EndOfFile`
    is returned by `Read` when the end of an input stream is reached.

29.1.4. Expression

WMA link

`Expression`
    is a data type for `Read`.

For information about underlying data structure Expression (a kind of M-expression) that is central in evaluation, see: AST, M-Expression, General List same thing.

29.1.5. FilePrint

WMA link

`FilePrint`[*file*]
    prints the raw contents of *file*.

29.1.6. Find

WMA link

`Find`[*stream*, *text*]
    find the first line in *stream* that contains *text*.

```
>> stream = OpenRead["ExampleData/EinsteinSzilLetter.txt",
   CharacterEncoding->"UTF8"];
```

```
>> Find[stream, "uranium"]
```
in manuscript, leads me to expect that the element uranium may be turned into

```
>> Find[stream, "uranium"]
```
become possible to set up a nuclear chain reaction in a large mass of uranium,

```
>> stream = OpenRead["ExampleData/EinsteinSzilLetter.txt",
   CharacterEncoding->"UTF8"];
```

```
>> Find[stream, {"energy", "power"} ]
```
a new and important source of energy in the immediate future. Certain aspects

```
>> Find[stream, {"energy", "power"} ]
```
by which vast amounts of power and large quantities of new radium-like

29.1.7. Get (<<)

WMA link

`<<$name$`
    reads a file and evaluates each expression, returning only the last one.
`Get[`*name*, Trace->True]
    Runs Get tracing each line before it is evaluated.
`Settings`$TraceGet` can be also used to trace lines on all `Get[]` calls.

```
>> filename = $TemporaryDirectory <> "/example_file";
```

```
>> Put[x + y, filename]
```

```
>> Get[filename]
```
$x + y$

```
>> filename = $TemporaryDirectory <> "/example_file";
```

```
>> Put[x + y, 2x^2 + 4z!, Cos[x] + I Sin[x], filename]
```

```
>> Get[filename]
```
$\text{Cos}[x] + I\text{Sin}[x]$

```
>> DeleteFile[filename]
```

29.1.8. $InputFileName

WMA link

`$InputFileName`
: is the name of the file from which input is currently being read.

While in interactive mode, `$InputFileName` is "".

```
>> $InputFileName
```
/src/external-vcs/github/Mathics3/mathics-core/mathics

29.1.9. InputStream

WMA link

`InputStream[`*name*, *n*`]`
: represents an input stream for functions such as `Read` or `Find`.

`StringToStream` opens an input stream:

```
>> stream = StringToStream["Mathics is cool!"]
```
InputStream [String, 17]

```
>> Close[stream]
```
String

29.1.10. $Input

WMA link

`$Input`
: is the name of the stream from which input is currently being read.

```
>> $Input
```

29.1.11. Number

WMA link

`Number`
: is a data type for `Read`.

29.1.12. OpenAppend

WMA link

`OpenAppend[``file']'`
: opens a file and returns an OutputStream to which writes are appended.

```
>> OpenAppend[]
```

OutputStream [/tmp/tmpvc8ff3yk, 17]

```
>> DeleteFile[Close[%]];
```

29.1.13. OpenRead

WMA link

`OpenRead[``file']'`
: opens a file and returns an `InputStream`.

```
>> OpenRead["ExampleData/EinsteinSzilLetter.txt", CharacterEncoding->"
   UTF8"]
```

InputStream [ExampleData/EinsteinSzilLetter.txt, 17]

The stream must be closed after using it to release the resource:

```
>> Close[%];
```

29.1.14. OpenWrite

WMA link

`OpenWrite[``file']'`
: opens a file and returns an OutputStream.

```
>> OpenWrite[]
```

OutputStream [/tmp/tmp6dm032sc, 17]

```
>> DeleteFile[Close[%]];
```

29.1.15. OutputStream

WMA link

`OutputStream[`*name*, *n*`]`
: represents an output stream.

By default, the list of Streams normally `OutputStream` entries for `stderr` and `stdout`

```
>> Streams[]
```

{InputStream [stdin, 0], OutputStream [stdout, 1], OutputStream [stderr, 2], InputStream [String, 3], InputStream [ExampleData/numbers.txt, 4], InputStream [ExampleData/numbers.txt, 5], InputStream [ExampleData/numbers.txt, 6], InputStream [ExampleData/strings.txt, 7], InputStream [ExampleData/strings.txt, 8], InputStream [ExampleData/strings.txt, 9], InputStream [ExampleData/strings.txt, 10], InputStream [ExampleData/strings.txt, 11], InputStream [ExampleData/sentences.txt, 12], InputStream [String, 13], InputStream [String, 14], InputStream [ExampleData/EinsteinSzilLetter.txt, 15], InputStream [ExampleData/EinsteinSzilLetter.txt, 16]}

29.1.16. Put (>>)

WMA link

`$expr$ >> $filename$`
: write *expr* to a file.

`Put[`$expr_1$, $expr_2$, ..., "*filename*"`]`
: write a sequence of expressions to a file.

```
>> Put[40!, fortyfactorial]
```

```
fortyfactorial is not string, InputStream[], or OutputStream[]
```

815915283247897734345611269596115894272000000000»fortyfactorial

```
>> filename = $TemporaryDirectory <> "/fortyfactorial";
```

```
>> Put[40!, filename]
```

```
>> FilePrint[filename]
```

```
815915283247897734345611269596115894272000000000
```

```
>> Get[filename]
```

815915283247897734345611269596115894272000000000

```
>> DeleteFile[filename]

>> filename = $TemporaryDirectory <> "/fiftyfactorial";

>> Put[10!, 20!, 30!, filename]

>> FilePrint[filename]
   3628800
   2432902008176640000
   265252859812191058636308480000000

>> DeleteFile[filename]
```

=

```
>> filename = $TemporaryDirectory <> "/example_file";

>> Put[x + y, 2x^2 + 4z!, Cos[x] + I Sin[x], filename]

>> FilePrint[filename]
   x + y
   2*x^2 + 4*z!
   Cos[x] + I*Sin[x]

>> DeleteFile[filename]
```

29.1.17. PutAppend (>>>)

WMA link

`$expr$ >>> $filename$`
: append *expr* to a file.

`PutAppend`[$expr_1$, $expr_2$, ..., "*filename*"]
: write a sequence of expressions to a file.

```
>> Put[50!, "factorials"]

>> FilePrint["factorials"]
   30414093201713378043612608166064768844377641568960512000000000000

>> PutAppend[10!, 20!, 30!, "factorials"]

>> FilePrint["factorials"]
   30414093201713378043612608166064768844377641568960512000000000000
   3628800
   2432902008176640000
   265252859812191058636308480000000

>> 60! >>> "factorials"
```

```
>> FilePrint["factorials"]
   30414093201713378043612608166064768844377641568960512000000000000

   3628800

   2432902008176640000

   265252859812191058636308480000000

   8320987112741390144276341183223364380754172606361245952449277696409600000000000000

>> "string" >>> factorials

>> FilePrint["factorials"]
   30414093201713378043612608166064768844377641568960512000000000000

   3628800

   2432902008176640000

   265252859812191058636308480000000

   8320987112741390144276341183223364380754172606361245952449277696409600000000000000

   "string"

>> DeleteFile["factorials"];
```

29.1.18. Read

WMA link

`Read`[*stream*]
: reads the input stream and returns one expression.

`Read`[*stream*, *type*]
: reads the input stream and returns an object of the given type.

`Read`[*stream*, *type*]
: reads the input stream and returns an object of the given type.

`Read[$stream$, Hold[Expression]]`
: reads the input stream for an Expression and puts it inside `Hold`.

type is one of:

- Byte
- Character
- Expression
- HoldExpression
- Number
- Real
- Record
- String
- Word

```
>> stream = StringToStream["abc123"];
```

```
>> Read[stream, String]
```
abc123

```
>> Read[stream, String]
```
EndOfFile

```
>> stream = StringToStream["abc 123"];
```

```
>> Read[stream, Word]
```
abc

```
>> Read[stream, Word]
```
123

```
>> Read[stream, Word]
```
EndOfFile

```
>> stream = StringToStream["123, 4"];
```

```
>> Read[stream, Number]
```
123

```
>> Read[stream, Number]
```
4

```
>> Read[stream, Number]
```
EndOfFile

```
>> stream = StringToStream["2+2\n2+3"];
```

`Read` with a `Hold[Expression]` returns the expression it reads unevaluated so it can be later inspected and evaluated:

```
>> Read[stream, Hold[Expression]]
```
Hold [2 + 2]

```
>> Read[stream, Expression]
```
5

Reading a comment, a non-expression, will return `Hold[Null]`

```
>> stream = StringToStream["(* ::Package:: *)"];
```

```
>> Read[stream, Hold[Expression]]
```
Hold [Null]

```
>> stream = StringToStream["123 abc"];
```

```
>> Read[stream, {Number, Word}]
```
{123, abc}

```
>> Read[stream, {Number, Word}]
```
EndOfFile

Multiple lines:

```
>> stream = StringToStream["\"Tengo una\nvaca lechera.\""]; Read[stream]
```
Tengo una
  vaca lechera.

29.1.19. ReadList

WMA link

`ReadList["file"]`
    Reads all the expressions until the end of file.
`ReadList["file", type]`
    Reads objects of a specified type until the end of file.
`ReadList["file", {type1, type2, ...}]`
    Reads a sequence of specified types until the end of file.

To read all the numbers in a file and return a list of them:

```
>> ReadList["ExampleData/numbers.txt", Number]
```
$\{11.1, 22.2, 33.3, 44.4, 55.5, 66.6\}$

(Use 'FilePrint[]' 29.1.5 to get the raw data for the examples above and below.)

This does the same, but groups the numbers in to a pairs:

```
>> ReadList["ExampleData/numbers.txt", {Number, Number}]
```
$\{\{11.1, 22.2\}, \{33.3, 44.4\}, \{55.5, 66.6\}\}$

Now let us read and put blocks of 3 numbers in its own list:

```
>> ReadList["ExampleData/numbers.txt", Table[Number, {3}]]
```
$\{\{11.1, 22.2, 33.3\}, \{44.4, 55.5, 66.6\}\}$

Like 'Read[]' 29.1.18, `ReadList` handles types of objects other than numbers. We can read a list of characters in a file putting each character as an item in a list:

```
>> ReadList["ExampleData/strings.txt", Character]
```
{H, e, r, e, , i, s, , t, e, x, t, .,
  , A, n, d, , m, o, r, e, , t, e, x, t, .,
  }

And now, here are the integer codes corresponding to each of the bytes in the file:

```
>> ReadList["ExampleData/strings.txt", Byte]
```
{72, 101, 114, 101, 32, 105, 115, 32, 116, 101, 120, 116, 46, 10, 65, 110, 100, 32, 109, 111, 114, 101, 32, 116, 101, 120, 116, 46,

But the data can also be read by "words":

```
>> ReadList["ExampleData/strings.txt", Word]
```
{Here, is, text., And, more, text.}

The above uses the default value which is space of some sort., However you can set your own value:

```
>> ReadList["ExampleData/strings.txt", Word, WordSeparators -> {"e",
   "."}]
```
{H, r, is t, xt, And mor, t, xt}

See WordSeparators for more information.

Reading by records uses the separators found in

```
>> ReadList["ExampleData/strings.txt", Record]
```
{Here is text., And more text.}

See RecordSeparators works analogously for records as `WordSeparators` does for words.

To allow both periods and newlines as record separators:

```
>> ReadList["ExampleData/sentences.txt", Record, RecordSeparators ->
   {".", "\n"}]
```
{Here is text, And more, And a second line}

See also Reading Textual Data.

29.1.20. Record

WMA link

`Record`
is a data type for `Read`.

29.1.21. SetStreamPosition

WMA link

`SetStreamPosition`[*stream*, *n*]
sets the current position in a stream.

```
>> stream = StringToStream["Mathics is cool!"]
```
InputStream [String, 33]

```
>> SetStreamPosition[stream, 8]
```
8

>> `Read[stream, Word]`
is

>> `SetStreamPosition[stream, Infinity]`
16

29.1.22. Skip

WMA link

`Skip[`*stream*, *type*`]`
skips ahead in an input steream by one object of the specified *type*.
`Skip[`*stream*, *type*, *n*`]`
skips ahead in an input steream by *n* objects of the specified *type*.

>> `stream = StringToStream["a b c d"];`

>> `Read[stream, Word]`
a

>> `Skip[stream, Word]`

>> `Read[stream, Word]`
c

>> `stream = StringToStream["a b c d"];`

>> `Read[stream, Word]`
a

>> `Skip[stream, Word, 2]`

>> `Read[stream, Word]`
d

>> `Skip[stream, Word]`
EndOfFile

29.1.23. StreamPosition

WMA link

`StreamPosition[`*stream*`]`
returns the current position in a stream as an integer.

>> `stream = StringToStream["Mathics is cool!"]`
InputStream [String, 36]

```
>> Read[stream, Word]
```
Mathics

```
>> StreamPosition[stream]
```
7

29.1.24. Streams

WMA link

`Streams[]`
returns a list of all open streams.

```
>> Streams[]
```
{InputStream [stdin, 0] , OutputStream [stdout, 1] , OutputStream [stderr, 2] , InputStream [String, 3] , InputStream [ExampleData/numbers.txt, 4] , InputStream [ExampleData/numbers.txt, 5] , InputStream [ExampleData/numbers.txt, 6] , InputStream [ExampleData/strings.txt, 7] , InputStream [ExampleData/strings.txt, 8] , InputStream [ExampleData/strings.txt, 9] , InputStream [ExampleData/strings.txt, 10] , InputStream [ExampleData/strings.txt, 11] , InputStream [ExampleData/sentences.txt, 12] , InputStream [String, 13] , InputStream [String, 14] , InputStream [ExampleData/EinsteinSzilLetter.txt, 15] , InputStream [ExampleData/EinsteinSzilLetter.txt, 16] , InputStream [String, 17] , InputStream [String, 18] , InputStream [String, 19] , InputStream [String, 20] , InputStream [String, 21] , InputStream [String, 22] , InputStream [String, 23] , InputStream [ExampleData/numbers.txt, 24] , InputStream [ExampleData/numbers.txt, 25] , InputStream [ExampleData/numbers.txt, 26] , InputStream [ExampleData/strings.txt, 27] , InputStream [ExampleData/strings.txt, 28] , InputStream [ExampleData/strings.txt, 29] , InputStream [ExampleData/strings.txt, 30] , InputStream [ExampleData/strings.txt, 31] , InputStream [ExampleData/sentences.txt, 32] , InputStream [String, 33] , InputStream [String, 34] , InputStream [String, 35] , InputStream [String, 36] }

```
>> Streams["stdout"]
```
{OutputStream [stdout, 1]}

29.1.25. StringToStream

WMA link

`StringToStream[`*string*`]`
: converts a *string* to an open input stream.

```
>> strm = StringToStream["abc 123"]
```

InputStream [String, 37]

The stream must be closed after using it, to release the resource:

```
>> Close[strm];
```

29.1.26. Word

WMA link

`Word`
: is a data type for `Read`.

29.1.27. Write

WMA link

`Write[`*channel*, $expr_1$, $expr_2$, ...`]`
: writes the expressions to the output channel followed by a newline.

```
>> stream = OpenWrite[]
```

OutputStream [/tmp/tmp7ljotkx4, 37]

```
>> Write[stream, 10 x + 15 y ^ 2]
```

```
>> Write[stream, 3 Sin[z]]
```

The stream must be closed in order to use the file again:

```
>> Close[stream];
```

```
>> stream = OpenRead[%];
```

```
>> ReadList[stream]
```

$\left\{10x + 15y^2, 3\text{Sin}\left[z\right]\right\}$

```
>> DeleteFile[Close[stream]];
```

29.1.28. WriteString

WMA link

`WriteString`[*stream*, str_1, str_2, ...]
    writes the strings to the output stream.

```
>> stream = OpenWrite[];

>> WriteString[stream, "This is a test 1"]

>> WriteString[stream, "This is also a test 2"]

>> pathname = Close[stream];

>> FilePrint[%]
   This is a test 1This is also a test 2

>> DeleteFile[pathname];

>> stream = OpenWrite[];

>> WriteString[stream, "This is a test 1", "This is also a test 2"]

>> pathname = Close[stream]
   /tmp/tmpbtcurwwy

>> FilePrint[%]
   This is a test 1This is also a test 2

>> DeleteFile[pathname];
```

If stream is the string "stdout" or "stderr", writes to the system standard output/ standard error channel:

```
>> WriteString["stdout", "Hola"]
```

29.2. Filesystem Operations

29.2.1. AbsoluteFileName

WMA link

`AbsoluteFileName`["*name*"]
    returns the absolute version of the given filename.

```
>> AbsoluteFileName["ExampleData/sunflowers.jpg"]
```

/src/external-vcs/github/Mathics3/mathics-core/mathics/data/ExampleData/sunflowers.jpg

29.2.2. CopyDirectory

WMA link

`CopyDirectory`["*dir*$_1$", "*dir*$_2$"]
: copies directory *dir*$_1$ to *dir*$_2$.

29.2.3. CopyFile

WMA link

`CopyFile`["*file*$_1$", "*file*$_2$"]
: copies *file*$_1$ to *file*$_2$.

```
>> CopyFile["ExampleData/sunflowers.jpg", "MathicsSunflowers.jpg"]
```

MathicsSunflowers.jpg

```
>> DeleteFile["MathicsSunflowers.jpg"]
```

29.2.4. CreateFile

WMA link

`CreateFile[``filename']'`
: Creates a file named "filename" temporary file, but do not open it.

`CreateFile[]`
: Creates a temporary file, but do not open it.

29.2.5. CreateTemporary

WMA link

`CreateTemporary[]`
: Creates a temporary file, but do not open it.

29.2.6. DeleteFile

WMA link

`Delete["file"]`
: deletes *file*.

`Delete[{"file1", "file2", ...}]`
: deletes a list of files.

```
>> CopyFile["ExampleData/sunflowers.jpg", "MathicsSunflowers.jpg"];
>> DeleteFile["MathicsSunflowers.jpg"]
>> CopyFile["ExampleData/sunflowers.jpg", "MathicsSunflowers1.jpg"];
>> CopyFile["ExampleData/sunflowers.jpg", "MathicsSunflowers2.jpg"];
>> DeleteFile[{"MathicsSunflowers1.jpg", "MathicsSunflowers2.jpg"}]
```

29.2.7. Directory

WMA link

`Directory[]`
: returns the current working directory.

```
>> Directory[]
```

/home/rocky

29.2.8. DirectoryStack

WMA link

`DirectoryStack[]`
: returns the directory stack.

```
>> DirectoryStack[]
```

{/src/external-vcs/github/Mathics3/mathics-core/mathics, /home/rocky}

29.2.9. ExpandFileName

WMA link

`ExpandFileName["name"]`
    expands *name* to an absolute filename for your system.

```
>> ExpandFileName["ExampleData/sunflowers.jpg"]
```
/home/rocky/ExampleData/sunflowers.jpg

29.2.10. File

WMA link

`File["file"]`
    is a symbolic representation of an element in the local file system.

29.2.11. FileBaseName

WMA link

`FileBaseName["file"]`
    gives the base name for the specified file name.

```
>> FileBaseName["file.txt"]
```
file

```
>> FileBaseName["file.tar.gz"]
```
file.tar

29.2.12. FileByteCount

WMA link

`FileByteCount[file]`
    returns the number of bytes in *file*.

```
>> FileByteCount["ExampleData/sunflowers.jpg"]
```
142286

29.2.13. FileExistsQ

WMA link

> `FileExistsQ`["*file*"]
>   returns `True` if *file* exists and `False` otherwise.

```
>> FileExistsQ["ExampleData/sunflowers.jpg"]
```
True

```
>> FileExistsQ["ExampleData/sunflowers.png"]
```
False

29.2.14. FileExtension

WMA link

> `FileExtension`["*file*"]
>   gives the extension for the specified file name.

```
>> FileExtension["file.txt"]
```
txt

```
>> FileExtension["file.tar.gz"]
```
gz

29.2.15. FileInformation

WMA link

> `FileInformation`["*file*"]
>   returns information about *file*.

This function is totally undocumented in MMA!

```
>> FileInformation["ExampleData/sunflowers.jpg"]
```
{File− > /home/rocky/ExampleData/sunflowers.jpg, FileType
− > File, ByteCount− > 142286, Date− > 3882725439}

29.2.16. FileNameTake

WMA link

`FileNameTake["`*file*`"]`
: returns the last path element in the file name *name*.

`FileNameTake["`*file*`", `n`]`
: returns the first n path elements in the file name *name*.

`FileNameTake["`*file*`", `$-n$`]`
: returns the last n path elements in the file name *name*.

29.2.17. FileNames

WMA link

`FileNames[]`
: Returns a list with the filenames in the current working folder.

`FileNames[`*form*`]`
: Returns a list with the filenames in the current working folder that matches with *form*.

`FileNames[{`$form_1$, $form_2$, ...`}]`
: Returns a list with the filenames in the current working folder that matches with one of $form_1$, $form_2$, ...

`FileNames[{`$form_1$, $form_2$, ...`},{`dir_1, dir_2, ...`}]`
: Looks into the directories dir_1, dir_2, ...

`FileNames[{`$form_1$, $form_2$, ...`},{`dir_1, dir_2, ...`}]`
: Looks into the directories dir_1, dir_2, ...

`FileNames[{`*forms*, *dirs*, n`]`
: Look for files up to the level n.

```
>> SetDirectory[$InstallationDirectory <> "/autoload"];
```

```
>> FileNames["*.m", "formats"]//Length
```
0

```
>> FileNames["*.m", "formats", 3]//Length
```
14

```
>> FileNames["*.m", "formats", Infinity]//Length
```
14

29.2.18. FindFile

WMA link

`FindFile[`*name*`]`
: searches `$Path` for the given filename.

```
>> FindFile["ExampleData/sunflowers.jpg"]
```
/src/external-vcs/github/Mathics3/mathics-core/mathics/data/ExampleData/sunflowers.jpg

```
>> FindFile["VectorAnalysis`"]
```
/src/external-vcs/github/Mathics3/mathics-core/mathics/Packages/VectorAnalysis/Kernel/init.m

```
>> FindFile["VectorAnalysis`VectorAnalysis`"]
```
/src/external-vcs/github/Mathics3/mathics-core/mathics/Packages/VectorAnalysis/VectorAnalysis.m

29.2.19. Needs

WMA link

`Needs["context`"]`
loads the specified context if not already in `$Packages`.

```
>> Needs["VectorAnalysis`"]
```

29.2.20. $OperatingSystem

WMA link

`$OperatingSystem`
gives the type of operating system running Mathics.

```
>> $OperatingSystem
```
Unix

29.2.21. $PathnameSeparator

WMA link

`$PathnameSeparator`
returns a string for the separator in paths.

```
>> $PathnameSeparator
```
/

29.2.22. RenameFile

WMA link

`RenameFile`["$file_1$", "$file_2$"]
: renames $file_1$ to $file_2$.

>> `CopyFile["ExampleData/sunflowers.jpg", "MathicsSunflowers.jpg"]`

MathicsSunflowers.jpg

>> `RenameFile["MathicsSunflowers.jpg", "MathicsSunnyFlowers.jpg"]`

MathicsSunnyFlowers.jpg

>> `DeleteFile["MathicsSunnyFlowers.jpg"]`

29.2.23. ResetDirectory

WMA link

`ResetDirectory[]`
: pops a directory from the directory stack and returns it.

>> `ResetDirectory[]`

/src/external-vcs/github/Mathics3/mathics-core/mathics/autoload

29.2.24. SetDirectory

WMA link

`SetDirectory`[*dir*]
: sets the current working directory to *dir*.

>> `SetDirectory[]`

/home/rocky

29.2.25. ToFileName

WMA link

`ToFileName`[{"dir_1", "dir_2", ...}]
: joins the dir_i together into one path.

`ToFileName` has been superseded by `FileNameJoin`.

```
>> ToFileName[{"dir1", "dir2"}, "file"]
```
dir1/dir2/file

```
>> ToFileName["dir1", "file"]
```
dir1/file

```
>> ToFileName[{"dir1", "dir2", "dir3"}]
```
dir1/dir2/dir3

29.2.26. URLSave

WMA link

`URLSave[``url']'`
: Save "url" in a temporary file.

`URLSave["url", filename]`
: Save "url" in *filename*.

29.3. Importing and Exporting

Many kinds data formats can be read into
emphMathics3. Variable $ExportFormats 29.3.6 contains a list of file formats that are supported by Export 29.3.5, while $ImportFormats 29.3.10 does the corresponding thing for Import 29.3.9.

29.3.1. System'Convert'B64Dump'B64Decode

WMA link

`System`Convert`B64Dump`B64Decode[string]`
: Decode *string* in Base64 coding to an expression.

```
>> System`Convert`B64Dump`B64Decode["R!="]
String "R!=" is not a valid b64 encoded string.
```
$Failed

29.3.2. System'Convert'B64Dump'B64Encode

WMA link

`System`Convert`B64Dump`B64Encode[expr]`
: Encodes *expr* in Base64 coding

```
>> System`Convert`B64Dump`B64Encode["Hello world"]
```
SGVsbG8gd29ybGQ=

```
>> System`Convert`B64Dump`B64Decode[%]
```
Hello world

```
>> System`Convert`B64Dump`B64Encode[Integrate[f[x],{x,0,2}]]
```
SW50ZWdyYXRlW2ZbeF0sIHt4LCAwLCAyfV0=

```
>> System`Convert`B64Dump`B64Decode[%]
```
Integrate[f[x], {x, 0, 2}]

29.3.3. System`ConvertersDump`$ExtensionMappings

`System`ConvertersDump`$ExtensionMappings`
Returns a list of associations between file extensions and file types.

The format associated to the extension "*.jpg"

```
>> "*.jpg"/. System`ConvertersDump`$ExtensionMappings
```
JPEG

29.3.4. System`ConvertersDump`$FormatMappings

`System`ConverterDump$FormatMappings`
Returns a list of associations between file extensions and file types.

The list of MIME types associated to the extension JPEG:

```
>> Select[System`ConvertersDump`$FormatMappings,(#1[[2]]=="JPEG")&][[All
, 1]]
```
{APPLICATION/JPG, APPLICATION/X-JPG, IMAGE/JPEG, IMAGE/JPG, IMAGE/PJPEG, JPEG, JPG}

29.3.5. Export

WMA link

Export["*file.ext*", *expr*]
: exports *expr* to a file, using the extension *ext* to determine the format.

Export["*file*", *expr*, "*format*"]
: exports *expr* to a file in the specified format.

Export["*file*", *exprs*, *elems*]
: exports *exprs* to a file as elements specified by *elems*.

29.3.6. $ExportFormats

WMA link

$ExportFormats
: returns a list of file formats supported by Export.

```
>> $ExportFormats
```
{BMP, Base64, CSV, ExampleFormat1, ExampleFormat2, GIF, JPEG, JPEG2000, PBM, PCX, PGM, PNG, PPM, SVG, T

29.3.7. ExportString

WMA link

ExportString[*expr*, *form*]
: exports *expr* to a string, in the format *form*.

Export["*file*", *exprs*, *elems*]
: exports *exprs* to a string as elements specified by *elems*.

```
>> ExportString[{{1,2,3,4},{3},{2},{4}}, "CSV"]
```
1,2,3,4
3,
2,
4,

```
>> ExportString[{1,2,3,4}, "CSV"]
```
1,
2,
3,
4,

```
>> ExportString[Integrate[f[x],{x,0,2}], "SVG"]//Head
```
String

29.3.8. FileFormat

WMA link

`FileFormat`[“*name*”]
: attempts to determine what format `Import` should use to import specified file.

```
>> FileFormat["ExampleData/sunflowers.jpg"]
```
JPEG

```
>> FileFormat["ExampleData/EinsteinSzilLetter.txt"]
```
Text

```
>> FileFormat["ExampleData/hedy.tif"]
```
TIFF

29.3.9. Import

WMA link

`Import`[“*file*”]
: imports data from a file.

`Import`[“*file*”, “*fmt*”]
: imports file assuming the specified file format.

`Import`[“*file*”, *elements*]
: imports the specified elements from a file.

`Import`[“*file*”, {“*fmt*”, *elements*}]
: imports the specified elements from a file assuming the specified file format.

`Import`[”http://*url*”, ...] and `Import`[”ftp://*url*”, ...]
: imports from a URL.

```
>> Import["ExampleData/ExampleData.txt", "Elements"]
```
{Data, Lines, Plaintext, String, Words}

```
>> Import["ExampleData/ExampleData.txt", "Lines"]
```
{Example File Format, Created by Angus, 0.629452
0.586355, 0.711009 0.687453, 0.246540 0.433973, 0.926871
0.887255, 0.825141 0.940900, 0.847035 0.127464, 0.054348
0.296494, 0.838545 0.247025, 0.838697 0.436220, 0.309496 0.833591}

```
>> Import["ExampleData/colors.json"]
```

{colorsArray− > {{colorName− > black, rgbValue− > (0, 0, 0), hexValue− > #000000} , {colorName− > red, rgbValue− > (255, 0, 0), hexValue− > #FF0000} , {colorName− > green, rgbValue− > (0, 255, 0), hexValue− > #00FF00} , {colorName− > blue, rgbValue− > (0, 0, 255), hexValue− > #0000FF} , {colorName− > yellow, rgbValue− > (255, 255, 0), hexValue− > #FFFF00} , {colorName− > cyan, rgbValue− > (0, 255, 255), hexValue− > #00FFFF} , {colorName− > magenta, rgbValue− > (255, 0, 255), hexValue− > #FF00FF} , {colorName− > white, rgbValue− > (255, 255, 255), hexValue− > #FFFFFF}}}

29.3.10. $ImportFormats

WMA link

`$ImportFormats`
returns a list of file formats supported by Import.

```
>> $ImportFormats
```

{BMP, Base64, CSV, ExampleFormat1, ExampleFormat2, GIF, HTML, ICO, JPEG, JPEG2000, JSON, PBM, PCX, PGM

29.3.11. ImportString

WMA link

`ImportString[`"*data*", "*format*"`]`
imports data in the specified format from a string.
`ImportString[`"*file*", *elements*`]`
imports the specified elements from a string.
`ImportString[`"*data*"`]`
attempts to determine the format of the string from its content.

```
>> str = "Hello!\n This is a testing text\n";
```

```
>> ImportString[str, "Elements"]
```

{Data, Lines, Plaintext, String, Words}

```
>> ImportString[str, "Lines"]
```

{Hello!, This is a testing text}

29.3.12. ImportExport‘RegisterExport

`RegisterExport["format", func]`
register `$func$` as the default function used when exporting from a file of type `` `` `` `$format$''`.

Simple text exporter

```
>> ExampleExporter1[filename_, data_, opts___] := Module[{strm =
   OpenWrite[filename], char = data}, WriteString[strm, char]; Close[
   strm]]
```

```
>> ImportExport`RegisterExport["ExampleFormat1", ExampleExporter1]
```

```
>> Export["sample.txt", "Encode this string!", "ExampleFormat1"];
```

```
>> FilePrint["sample.txt"]
   Encode this string!
```

```
>> DeleteFile["sample.txt"]
```

Very basic encrypted text exporter:

```
>> ExampleExporter2[filename_, data_, opts___] := Module[{strm =
   OpenWrite[filename], char}, (* TODO: Check data *)char =
   FromCharacterCode[Mod[ToCharacterCode[data] - 84, 26] + 97];
   WriteString[strm, char]; Close[strm]]
```

```
>> ImportExport`RegisterExport["ExampleFormat2", ExampleExporter2]
```

```
>> Export["sample.txt", "encodethisstring", "ExampleFormat2"];
```

```
>> FilePrint["sample.txt"]
   rapbqrguvffgevat
```

```
>> DeleteFile["sample.txt"]
```

29.3.13. ImportExport‘RegisterImport

`RegisterImport["format", defaultFunction]`
register `$defaultFunction$` as the default function used when importing from a file of type ``` ``$format$'' ```.

``` RegisterImport[``$format$', {``elem1'' :> conditionalFunction1, ``elem2'' :> conditionalFunction2, ..., defaultFunction}]' ```
registers multiple elements ($elem_1$, ...) and their corresponding converter functions ($conditionalFunction_1$, ...) in addition to the $defaultFunction$.

``` RegisterImport[``$format$', {``conditionalFunctions, defaultFunction, ''elem3" :> postFunction3, ``elem4'' :> postFunction4, ...}]' ```
also registers additional elements ($elem_3$, ...) whose converters ($postFunction_3$, ...) act on output from the low-level functions.

First, define the default function used to import the data.

```
>> ExampleFormat1Import[filename_String] := Module[{stream, head, data},
    stream = OpenRead[filename]; head = ReadList[stream, String, 2];
   data = Partition[ReadList[stream, Number], 2]; Close[stream]; {"
   Header" -> head, "Data" -> data}]
```

`RegisterImport` is then used to register the above function to a new data format.

```
>> ImportExport`RegisterImport["ExampleFormat1", ExampleFormat1Import]
```

```
>> FilePrint["ExampleData/ExampleData.txt"]
   Example File Format
   Created by Angus
   0.629452 0.586355
   0.711009 0.687453
   0.246540 0.433973
   0.926871 0.887255
   0.825141 0.940900
   0.847035 0.127464
   0.054348 0.296494
   0.838545 0.247025
   0.838697 0.436220
   0.309496 0.833591
```

```
>> Import["ExampleData/ExampleData.txt", {"ExampleFormat1", "Elements"}]
```

{Data, Header}

```
>> Import["ExampleData/ExampleData.txt", {"ExampleFormat1", "Header"}]
```

{Example File Format, Created by Angus}

Conditional Importer:

```
>> ExampleFormat2DefaultImport[filename_String] := Module[{stream, head
   }, stream = OpenRead[filename]; head = ReadList[stream, String, 2];
   Close[stream]; {"Header" -> head}]
```

```
>> ExampleFormat2DataImport[filename_String] := Module[{stream, data},
   stream = OpenRead[filename]; Skip[stream, String, 2]; data =
   Partition[ReadList[stream, Number], 2]; Close[stream]; {"Data" ->
   data}]
```

```
>> ImportExport`RegisterImport["ExampleFormat2", {"Data" :>
   ExampleFormat2DataImport, ExampleFormat2DefaultImport}]
```

```
>> Import["ExampleData/ExampleData.txt", {"ExampleFormat2", "Elements"}]
```

{Data, Header}

```
>> Import["ExampleData/ExampleData.txt", {"ExampleFormat2", "Header"}]
```

{Example File Format, Created by Angus}

```
>> Import["ExampleData/ExampleData.txt", {"ExampleFormat2", "Data"}] //
   Grid
```

0.629452	0.586355
0.711009	0.687453
0.24654	0.433973
0.926871	0.887255
0.825141	0.9409
0.847035	0.127464
0.054348	0.296494
0.838545	0.247025
0.838697	0.43622
0.309496	0.833591

29.3.14. URLFetch

WMA link

`URLFetch`[*URL*]
: Returns the content of *URL* as a string.

30. Integer Functions

Integer Functions can work on integers of any size.

Contents

30.1. Combinatorial Functions

Combinatorics is an area of mathematics primarily concerned with counting, both as a means and an end in obtaining results, and certain properties of finite structures.

It is closely related to many other areas of Mathematics and has many applications ranging from logic to statistical physics, from evolutionary biology to computer science, etc.

30.1.1. BellB

Bell number (SymPy, WMA)

`BellB[`n`]`
: Bell number B_n.

`BellB[`n, x`]`
: Bell polynomial $B_n(x)$.

```
>> BellB[10]
```
115975

```
>> BellB[5, x]
```
$x + 15x^2 + 25x^3 + 10x^4 + x^5$

30.1.2. Binomial

Binomial Coefficient (SymPy, WMA)

`Binomial`[n, k]
: gives the binomial coefficient n choose k.

```
>> Binomial[5, 3]
```
10

`Binomial` supports inexact numbers:

```
>> Binomial[10.5,3.2]
```
165.286

Some special cases:

```
>> Binomial[10, -2]
```
0

```
>> Binomial[-10.5, -3.5]
```
0.

30.1.3. CatalanNumber

Catalan Number (SymPy, WMA)

`CatalanNumber`[n]
: gives the n-th Catalan number.

A list of the first five Catalan numbers:

```
>> Table[CatalanNumber[n], {n, 1, 5}]
```
$\{1, 2, 5, 14, 42\}$

30.1.4. DiceDissimilarity

Sørensen–Dice coefficient (Sympy, DiceDissimilarity)

DiceDissimilarity[u, v]
: returns the Dice dissimilarity between the two Boolean 1-D lists u and v. This is defined as $(c_{tf} + c_{ft})$ / $(2 * c_{tt} + c_{ft} + c_{tf})$. n is len(u) and c_{ij} is the number of occurrences of $u[k] = i$ and $v[k] = j$ for $k < n$.

```
>> DiceDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{1}{2}$

30.1.5. EulerE

Euler numbers (SymPy, WMA)

EulerE[n]
: Euler number E_n.

EulerE[n, x]
: Euler polynomial $E_n(x)$.

Odd-index Euler numbers are zero:

```
>> Table[EulerE[k], {k, 1, 9, 2}]
```

$\{0, 0, 0, 0, 0\}$

Even-index Euler numbers alternate in sign:

```
>> Table[EulerE[k], {k, 0, 8, 2}]
```

$\{1, -1, 5, -61, 1385\}$

```
>> EulerE[5, z]
```

$-\frac{1}{2} + \frac{5z^2}{2} - \frac{5z^4}{2} + z^5$

30.1.6. JaccardDissimilarity

Jaccard index (SciPy, WMA)

JaccardDissimilarity[u, v]
: returns the Jaccard-Needham dissimilarity between the two Boolean 1-D lists u and v, which is defined as $(c_{tf} + c_{ft})/(c_{tt} + c_{ft} + c_{tf})$, where n is len(u) and c_{ij} is the number of occurrences of $u[k] = i$ and $v[k] = j$ for $k < n$.

```
>> JaccardDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{2}{3}$

30.1.7. LucasL

Lucas Number (SymPy, WMA)

`LucasL[`n`]`
: gives the nth Lucas number.

`LucasL[`n, x`]`
: gives the nth Lucas polynomial $L_(x)$.

A list of the first five Lucas numbers:

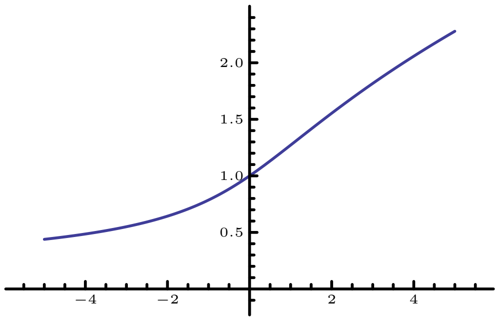
```
>> Table[LucasL[n], {n, 1, 5}]
```

$\{1,3,4,7,11\}$

```
>> Series[LucasL[1/2, x], {x, 0, 5}]
```

$$1+\frac{1}{4}x+\frac{1}{32}x^2+\left(-\frac{1}{128}\right)x^3+\left(-\frac{5}{2048}\right)x^4+\frac{7}{8192}x^5+O[x]^6$$

```
>> Plot[LucasL[1/2, x], {x, -5, 5}]
```



30.1.8. MatchingDissimilarity

WMA link

`MatchingDissimilarity[`u, v`]`
: returns the Matching dissimilarity between the two Boolean 1-D lists u and v, which is defined as $(c_{tf}+c_{ft})/n$, where n is len(u) and c_{ij} is the number of occurrences of $u[k]=i$ and $v[k]=j$ for $k<n$.

```
>> MatchingDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$\frac{4}{7}$

30.1.9. Multinomial

Multinomial distribution (WMA)

`Multinomial`[n_1, n_2, ...]
: gives the multinomial coefficient $(n_1 + n_2 + ...)! / (n_1! \, n_2! \, ...)$.

```
>> Multinomial[2, 3, 4, 5]
```
2522520

```
>> Multinomial[]
```
1

Multinomial is expressed in terms of `Binomial`:

```
>> Multinomial[a, b, c]
```
Binomial $[a, a]$ Binomial $[a + b, b]$ Binomial $[a + b + c, c]$

`Multinomial`[n-k, k] is equivalent to `Binomial`[n, k]:

```
>> Multinomial[2, 3]
```
10

See also 'Binomial' 30.1.2.

30.1.10. PolygonalNumber

Polygonal number (WMA)

`PolygonalNumber`[n]
: gives the n-th triangular number.

`PolygonalNumber`[r, n]
: gives the n-th r-gonal number.

```
>> Table[PolygonalNumber[n], {n, 10}]
```
$\{1, 3, 6, 10, 15, 21, 28, 36, 45, 55\}$

The sum of two consecutive Polygonal numbers is the square of the larger number:

```
>> Table[PolygonalNumber[n-1] + PolygonalNumber[n], {n, 10}]
```
$\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

`PolygonalNumber`[r, n] can be interpreted as the number of points arranged in the form of n-1 polygons of r sides.

List the tenth r-gonal number of regular polygons from 3 to 8:

```
>> Table[PolygonalNumber[r, 10], {r, 3, 8}]
```
$\{55, 100, 145, 190, 235, 280\}$

See also Binomial 30.1.2, and RegularPolygon 17.21.

30.1.11. RogersTanimotoDissimilarity

Rogers Tanimoto coefficient (WMA)

`RogersTanimotoDissimilarity[`u, v`]`
: returns the Rogers-Tanimoto dissimilarity between the two Boolean 1-D lists u and v, which is defined as $R/(c_{tt} + c_{ff} + R)$ where n is $len(u)$, c_{ij} is the number of occurrences of $u[k] = i$, $v[k] = j$ for $k < n$, and $R = 2(c_{tf} + c_{ft})$.

```
>> RogersTanimotoDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0,
   1}]
```

$$\frac{8}{11}$$

30.1.12. RussellRaoDissimilarity

Russel-Rao coefficient (WMA)

`RussellRaoDissimilarity[`u, v`]`
: returns the Russell-Rao dissimilarity between the two Boolean 1-D lists u and v, which is defined as $(n - c_{tt})/c_{tt}$ where n is $len(u)$, c_{ij} is the number of occurrences of $u[k] = i$ and $v[k] = j$ for $k < n$.

```
>> RussellRaoDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```

$$\frac{5}{7}$$

30.1.13. SokalSneathDissimilarity

Snokal-Sneath coefficient (WMA)

`SokalSneathDissimilarity[`u, v`]`
: returns the Sokal-Sneath dissimilarity between the two Boolean 1-D lists u and v, which is defined as $R/(c_{tt} + R)$ where n is $len(u)$, c_{ij} is the number of occurrences of $u[k] = i$, $v[k] = j$ for $k < n$, and $R = 2(c_{tf} + c_{ft})$.

```
>> SokalSneathDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0,
   1}]
```

$$\frac{4}{5}$$

30.1.14. Subsets

Subset (WMA link)

`Subsets`[*list*]
: finds a list of all possible subsets of *list*.

`Subsets`[*list*, *n*]
: finds a list of all possible subsets containing at most *n* elements.

`Subsets`[*list*, {*n*}]
: finds a list of all possible subsets containing exactly *n* elements.

`Subsets`[*list*, {*min*, *max*}]
: finds a list of all possible subsets containing between *min* and *max* elements.

`Subsets`[*list*, *spec*, *n*]
: finds a list of the first *n* possible subsets.

`Subsets`[*list*, *spec*, {*n*}]
: finds the *n*-th possible subset.

All possible subsets (power set):

```
>> Subsets[{a, b, c}]
```

$\{\{\},\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\}$

All possible subsets containing up to 2 elements:

```
>> Subsets[{a, b, c, d}, 2]
```

$\{\{\},\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}\}$

Subsets containing exactly 2 elements:

```
>> Subsets[{a, b, c, d}, {2}]
```

$\{\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}\}$

The first 5 subsets containing 3 elements:

```
>> Subsets[{a, b, c, d, e}, {3}, 5]
```

$\{\{a,b,c\},\{a,b,d\},\{a,b,e\},\{a,c,d\},\{a,c,e\}\}$

All subsets with even length:

```
>> Subsets[{a, b, c, d}, {0, 4, 2}]
```

$\{\{\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c,d\}\}$

The 25th subset:

```
>> Subsets[Range[5], All, {25}]
```

$\{\{2,4,5\}\}$

The odd-numbered subsets of {*a*,*b*,*c*,*d*} in reverse order:

```
>> Subsets[{a, b, c, d}, All, {15, 1, -2}]
```
$\{\{b,c,d\},\{a,b,d\},\{c,d\},\{b,c\},\{a,c\},\{d\},\{b\},\{\}\}$

30.1.15. YuleDissimilarity

WMA link

`YuleDissimilarity[`u, v`]`
: returns the Yule dissimilarity between the two Boolean 1-D lists u and v, which is defined as $R/(c_{tt}c_{ff} + R/2)$ where n is $len(u)$, c_{ij} is the number of occurrences of $u[k] = i$, $v[k] = j$ for $k < n$, and $R = 2c_{tf}c_{ft}$.

```
>> YuleDissimilarity[{1, 0, 1, 1, 0, 1, 1}, {0, 1, 1, 0, 0, 0, 1}]
```
$\frac{6}{5}$

30.2. Division-Related Functions

30.2.1. CompositeQ

WMA link

`CompositeQ[`n`]`
: returns `True` if n is a composite number

- A composite number is a positive number that is the product of two integers other than 1.
- For negative integer n, `CompositeQ[$n$]` is effectively equivalent to `CompositeQ[-$n$]`.

```
>> Table[CompositeQ[n], {n, 0, 10}]
```
{False, False, False, False, True, False, True, False, True, True, True}

30.2.2. Divisible

WMA link

`Divisible[`n, m`]`
: returns `True` if n is divisible by m, and `False` otherwise.

 n is divisible by m if n is the product of m by an integer. `Divisible[$n$,$m$]` is effectively equivalent to `Mod[$n$,$m$]==0`.

Test whether the number 10 is divisible by 2

```
>> Divisible[10, 2]
```
True

But the other way around is False: 2 is not divisible by 10:

```
>> Divisible[2, 10]
```
False

30.2.3. GCD

WMA link

`GCD`[n_1, n_2, ...]
: computes the greatest common divisor of the given integers.

```
>> GCD[20, 30]
```
10

```
>> GCD[10, y]
```
$\text{GCD}\left[10, y\right]$

`GCD` is `Listable`:

```
>> GCD[4, {10, 11, 12, 13, 14}]
```
$\{2, 1, 4, 1, 2\}$

`GCD` does not work for rational numbers and Gaussian integers yet.

30.2.4. LCM

WMA link

`LCM`[n_1, n_2, ...]
: computes the least common multiple of the given integers.

```
>> LCM[15, 20]
```
60

```
>> LCM[20, 30, 40, 50]
```
600

30.2.5. Mod

WMA link

`Mod[`x, m`]`
: returns x modulo m.

```
>> Mod[14, 6]
```
2

```
>> Mod[-3, 4]
```
1

```
>> Mod[-3, -4]
```
-3

```
>> Mod[5, 0]
The argument 0 should be nonzero.
```
Mod [5, 0]

30.2.6. ModularInverse

Modular multiplicative inverse (SymPy, WMA)

`ModularInverse[`k, n`]`
: returns the modular inverse k^(-1) mod n.

`ModularInverse[$k$,$n$]` gives the smallest positive integer r where the remainder of the division of r x k by n is equal to 1.

```
>> ModularInverse[2, 3]
```
2

The following is be True for all values n, k which have a modular inverse:

```
>> k = 2; n = 3; Mod[ModularInverse[k, n] * k, n] == 1
```
True

Some modular inverses just do not exists. For example when k is a multiple of n:

```
>> ModularInverse[k, k]
```
ModularInverse [2, 2]

30.2.7. PowerMod

Modular exponentiation. See `https://en.wikipedia.org/wiki/Modular_exponentiation`.

`PowerMod[`x, y, m`]`
: computes x^y modulo m.

```
>> PowerMod[2, 10000000, 3]
```
1

```
>> PowerMod[3, -2, 10]
```
9

```
>> PowerMod[0, -1, 2]
0 is not invertible modulo 2.
```
PowerMod $[0, -1, 2]$

```
>> PowerMod[5, 2, 0]
The argument 0 should be nonzero.
```
PowerMod $[5, 2, 0]$

`PowerMod` does not support rational coefficients (roots) yet.

30.2.8. Quotient

WMA link

`Quotient[m, n]`
: computes the integer quotient of m and n.

```
>> Quotient[23, 7]
```
3

30.2.9. QuotientRemainder

WMA link

`QuotientRemainder[m, n]`
: computes a list of the quotient and remainder from division of m by n.

```
>> QuotientRemainder[23, 7]
```
$\{3, 2\}$

30.3. Miscelanea of Integer Functions

30.3.1. BernoulliB

WMA link

`BernoulliB`[n]
  represents the Bernoulli number B_n.
`BernouilliB`[n, x]
  represents the Bernoulli polynomial $B_n(x)$.

```
>> BernoulliB[42]
```
$$\frac{1520097643918070802691}{1806}$$

First five Bernoulli numbers:

```
>> Table[BernoulliB[k], {k, 0, 5}]
```
$$\left\{1, \frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0\right\}$$

First five Bernoulli polynomials:

```
>> Table[BernoulliB[k, z], {k, 0, 3}]
```
$$\left\{1, -\frac{1}{2} + z, \frac{1}{6} - z + z^2, \frac{z}{2} - \frac{3z^2}{2} + z^3\right\}$$

30.4. Recurrence and Sum Functions

A recurrence relation is an equation that recursively defines a sequence or multidimensional array of values, once one or more initial terms are given; each further term of the sequence or array is defined as a function of the preceding terms.

30.4.1. Fibonacci

Fibonacci Sequence, (`:WMAlink:https://reference.wolfram.com/language/ref/Fibonacci.html`)

`Fibonacci`[n]
  computes the n-th Fibonacci number.
`Fibonacci`[n, x]
  computes the Fibonacci polynomial $F_n(x)$.

```
>> Fibonacci[0]
```
0

>> `Fibonacci[1]`
1

>> `Fibonacci[10]`
55

>> `Fibonacci[200]`
280571172992510140037611932413038677189525

>> `Fibonacci[7, x]`
$1 + 6x^2 + 5x^4 + x^6$

See also LinearRecurrence 30.4.3.

30.4.2. HarmonicNumber

Harmonic Number (WMA link)

`HarmonicNumber[n]`
returns the n-th harmonic number.

>> `Table[HarmonicNumber[n], {n, 8}]`
$\left\{1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, \frac{761}{280}\right\}$

>> `HarmonicNumber[3.8]`
2.03806

30.4.3. LinearRecurrence

Linear recurrence with constant coefficients, WMA link

`LinearRecurrence`[ker, $init$, n]
computes n terms of the linear recurrence with kernel ker and initial values $init$.
`LinearRecurrence`[ker, $init$, {n}]
computes the n-th term.
`LinearRecurrence`[ker, $init$, {n_{min}, n_{max}}]
computes n terms of the linear recurrence with kernel ker and initial values $init$.

Generate first 10 items of the Fibonacci Sequence, `F`[0]=1, `F`[1]=1:

>> `LinearRecurrence[{1, 1}, {1, 1}, 10]`
$\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55\}$

Extract the 3rd to 5th elements:

```
>> LinearRecurrence[{1, 1}, {1, 1}, {3, 5}]
```
$\{2,3,5\}$

Now just the 6th element:

```
>> LinearRecurrence[{1, 1}, {1, 1}, {6}]
```
8

See also Fibonacci 30.4.1.

30.4.4. StirlingS1

Stirling numbers of first kind (WMA link)

`StirlingS1`[n, m]
: gives the Stirling number of the first kind.

Integer mathematical function, suitable for both symbolic and numerical manipulation. gives the number of permutations of n elements that contain exactly m cycles.

```
>> StirlingS1[50, 1]
```
$-608\,281\,864\,034\,267\,560\,872\,252\,163\,321\,295\,376\,887\,552\,831\,379\,210\,240\,000\,000\,000$

30.4.5. StirlingS2

Stirling numbers of second kind (WMA link)

`StirlingS2`[n, m]
: gives the Stirling number of the second kind. Returns the number of ways of partitioning a set of n elements into m non empty subsets.

```
>> Table[StirlingS2[10, m], {m, 10}]
```
$\{1,511,9330,34105,42525,22827,5880,750,45,1\}$

31. Integer and Number-Theoretical Functions

Contents

31.1. Algebraic Transformations

There are a number of built-in functions that perform:

- Structural Operations on Polynomials
- Finding the Structure of a Polynomial

- Structural Operations on Rational Expressions
- Polynomials over Algebraic Number Fields
- Simplification with or without Assumptions

31.1.1. Apart

WMA link

`Apart[`*expr*`]`
: writes *expr* as a sum of individual fractions.

`Apart[`*expr*, *var*`]`
: treats *var* as the main variable.

```
>> Apart[1 / (x^2 + 5x + 6)]
```

$$\frac{1}{2+x} - \frac{1}{3+x}$$

When several variables are involved, the results can be different depending on the main variable:

```
>> Apart[1 / (x^2 - y^2), x]
```

$$-\frac{1}{2y\left(x+y\right)} + \frac{1}{2y\left(x-y\right)}$$

```
>> Apart[1 / (x^2 - y^2), y]
```

$$\frac{1}{2x\left(x+y\right)} + \frac{1}{2x\left(x-y\right)}$$

`Apart` is `Listable`:

```
>> Apart[{1 / (x^2 + 5x + 6)}]
```

$$\left\{\frac{1}{2+x} - \frac{1}{3+x}\right\}$$

But it does not touch other expressions:

```
>> Sin[1 / (x ^ 2 - y ^ 2)] // Apart
```

$$\mathrm{Sin}\left[\frac{1}{x^2 - y^2}\right]$$

31.1.2. Cancel

WMA link

`Cancel`[*expr*]
: cancels out common factors in numerators and denominators.

```
>> Cancel[x / x ^ 2]
```
$$\frac{1}{x}$$

`Cancel` threads over sums:

```
>> Cancel[x / x ^ 2 + y / y ^ 2]
```
$$\frac{1}{x}+\frac{1}{y}$$

```
>> Cancel[f[x] / x + x * f[x] / x ^ 2]
```
$$\frac{2f[x]}{x}$$

31.1.3. Coefficient

WMA link

`Coefficient[expr, form]`
: returns the coefficient of *form* in the polynomial *expr*.

`Coefficient[expr, form, n]`
: return the coefficient of *form*^*n* in *expr*.

```
>> Coefficient[(x + y)^4, (x^2)* (y^2)]
```
6

```
>> Coefficient[a x^2 + b y^3 + c x + d y + 5, x]
```
c

```
>> Coefficient[(x + 3 y)^5, x]
```
$405y^4$

```
>> Coefficient[(x + 3 y)^5, x * y^4]
```
405

```
>> Coefficient[(x + 2)/(y - 3)+ (x + 3)/(y - 2), x]
```
$$\frac{1}{-3+y}+\frac{1}{-2+y}$$

```
>> Coefficient[x*Cos[x + 3] + 6*y, x]
```
$\mathrm{Cos}[3+x]$

```
>> Coefficient[(x + 1)^3, x, 2]
```
3

```
>> Coefficient[a x^2 + b y^3 + c x + d y + 5, y, 3]
```
b

Find the free term in a polynomial:

```
>> Coefficient[(x + 2)^3 + (x + 3)^2, x, 0]
```
17

```
>> Coefficient[(x + 2)^3 + (x + 3)^2, y, 0]
```
$(2 + x)^3 + (3 + x)^2$

```
>> Coefficient[a x^2 + b y^3 + c x + d y + 5, x, 0]
```
$5 + by^3 + dy$

31.1.4. CoefficientArrays

WMA link

`CoefficientArrays[`*polys*, *vars*`]`
: returns a list of arrays of coefficients of the variables *vars* in the polynomial *poly*.

```
>> CoefficientArrays[1 + x^3, x]
```
{1, {0}, {{0}}, {{{1}}}}

```
>> CoefficientArrays[1 + x y+ x^3, {x, y}]
```
{1, {0, 0}, {{0, 1}, {0, 0}}, {{{1, 0}, {0, 0}}, {{0, 0}, {0, 0}}}}

```
>> CoefficientArrays[{1 + x^2, x y}, {x, y}]
```
{{1, 0}, {{0, 0}, {0, 0}}, {{{1, 0}, {0, 0}}, {{0, 1}, {0, 0}}}}

```
>> CoefficientArrays[(x+y+Sin[z])^3, {x,y}]
```
$\left\{\mathrm{Sin}[z]^3, \left\{3\mathrm{Sin}[z]^2, 3\mathrm{Sin}[z]^2\right\}, \{\{3\mathrm{Sin}[z], 6\mathrm{Sin}[$
$z]\}, \{0, 3\mathrm{Sin}[z]\}\}, \{\{\{1, 3\}, \{0, 3\}\}, \{\{0, 0\}, \{0, 1\}\}\}\right\}$

```
>> CoefficientArrays[(x + y + Sin[z])^3, {x, z}]
(x + y + Sin[z]) ^ 3 is not a polynomial in {x, z}
```
$\mathrm{CoefficientArrays}\left[(x + y + \mathrm{Sin}[z])^3, \{x, z\}\right]$

31.1.5. CoefficientList

WMA link

`CoefficientList[poly, var]`
: returns a list of coefficients of powers of *var* in *poly*, starting with power 0.

`CoefficientList[poly, {var1, var2, ...}]`
: returns an array of coefficients of the *vari*.

```
>> CoefficientList[(x + 3)^5, x]
```
$\{243, 405, 270, 90, 15, 1\}$

```
>> CoefficientList[(x + y)^4, x]
```
$\{y^4, 4y^3, 6y^2, 4y, 1\}$

```
>> CoefficientList[a x^2 + b y^3 + c x + d y + 5, x]
```
$\{5 + by^3 + dy, c, a\}$

```
>> CoefficientList[(x + 2)/(y - 3)+ x/(y - 2), x]
```
$\left\{\frac{2}{-3+y}, \frac{1}{-3+y} + \frac{1}{-2+y}\right\}$

```
>> CoefficientList[(x + y)^3, z]
```
$\left\{(x+y)^3\right\}$

```
>> CoefficientList[a x^2 + b y^3 + c x + d y + 5, {x, y}]
```
$\{\{5, d, 0, b\}, \{c, 0, 0, 0\}, \{a, 0, 0, 0\}\}$

```
>> CoefficientList[(x - 2 y + 3 z)^3, {x, y, z}]
```
$\{\{\{0, 0, 0, 27\}, \{0, 0, -54, 0\}, \{0, 36, 0, 0\}, \{-8, 0, 0, 0\}\}, \{\{0, 0, 27, 0\}, \{0, -36, 0, 0\}, \{12, 0, 0, 0\}, \{0, 0, 0, 0\}\}, \{\{0,$

```
>> CoefficientList[Series[Log[1-x], {x, 0, 9}], x]
```
$\left\{0, -1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}, -\frac{1}{6}, -\frac{1}{7}, -\frac{1}{8}, -\frac{1}{9}\right\}$

```
>> CoefficientList[Series[2x, {x, 0, 9}], x]
```
$\{0, 2\}$

31.1.6. Collect

WMA link

`Collect[`$expr, x$`]`
: Expands $expr$ and collect together terms having the same power of x.

`Collect[`$expr, \{x_1, x_2, ...\}$`]`
: Expands $expr$ and collect together terms having the same powers of x_1, x_2, ...

`Collect[`$expr, \{x_1, x_2, ...\}, filter$`]`
: After collect the terms, applies $filter$ to each coefficient.

```
>> Collect[(x+y)^3, y]
```
$x^3 + 3x^2y + 3xy^2 + y^3$

```
>> Collect[2 Sin[x z] (x+2 y^2 + Sin[y] x), y]
```
$2x\text{Sin}[xz] + 2x\text{Sin}[xz]\,\text{Sin}[y] + 4y^2\text{Sin}[xz]$

```
>> Collect[3 x y+2 Sin[x z] (x+2 y^2 + x)+ (x+y)^3, y]
```

$4x\mathrm{Sin}\,[xz] + x^3 + y\left(3x + 3x^2\right) + y^2\,(3x + 4\mathrm{Sin}\,[xz]) + y^3$

```
>> Collect[3 x y+2 Sin[x z] (x+2 y^2 + x)+ (x+y)^3, {x,y}]
```

$4x\mathrm{Sin}\,[xz] + x^3 + 3xy + 3x^2y + 4y^2\mathrm{Sin}\,[xz] + 3xy^2 + y^3$

```
>> Collect[3 x y+2 Sin[x z] (x+2 y^2 + x)+ (x+y)^3, {x,y}, h]
```

$xh\,[4\mathrm{Sin}\,[xz]] + x^3h\,[1] + xyh\,[3] + x^2yh\,[3] + y^2h\,[4\mathrm{Sin}\,[xz]] + xy^2h\,[3] + y^3h\,[1]$

31.1.7. Denominator

WMA link

Denominator[*expr*]
gives the denominator in *expr*.

```
>> Denominator[a / b]
```

b

```
>> Denominator[2 / 3]
```

3

```
>> Denominator[a + b]
```

1

31.1.8. Expand

WMA link

Expand[*expr*]
expands out positive integer powers and products of sums in *expr*, as well as trigonometric identities.
Expand[*expr*, *target*]
just expands those parts involving *target*.

```
>> Expand[(x + y)^ 3]
```

$x^3 + 3x^2y + 3xy^2 + y^3$

```
>> Expand[(a + b)(a + c + d)]
```

$a^2 + ab + ac + ad + bc + bd$

```
>> Expand[(a + b)(a + c + d)(e + f)+ e a a]
```

$2a^2e + a^2f + abe + abf + ace + acf + ade + adf + bce + bcf + bde + bdf$

```
>> Expand[(a + b)^ 2 * (c + d)]
```

$a^2c + a^2d + 2abc + 2abd + b^2c + b^2d$

```
>> Expand[(x + y)^ 2 + x y]
```
$x^2 + 3xy + y^2$

```
>> Expand[((a + b)(c + d))^ 2 + b (1 + a)]
```
$a^2c^2 + 2a^2cd + a^2d^2 + b + ab + 2abc^2 + 4abcd + 2abd^2 + b^2c^2 + 2b^2cd + b^2d^2$

`Expand` expands items in lists and rules:

```
>> Expand[{4 (x + y), 2 (x + y)-> 4 (x + y)}]
```
$\{4x + 4y, 2x + 2y- > 4x + 4y\}$

`Expand` expands trigonometric identities

```
>> Expand[Sin[x + y], Trig -> True]
```
$\text{Cos}[x]\,\text{Sin}[y] + \text{Cos}[y]\,\text{Sin}[x]$

```
>> Expand[Tanh[x + y], Trig -> True]
```
$$\frac{\text{Cosh}[x]\,\text{Sinh}[y]}{\text{Cosh}[x]\,\text{Cosh}[y] + \text{Sinh}[x]\,\text{Sinh}[y]} + \frac{\text{Cosh}[y]\,\text{Sinh}[x]}{\text{Cosh}[x]\,\text{Cosh}[y] + \text{Sinh}[x]\,\text{Sinh}[y]}$$

`Expand` does not change any other expression.

```
>> Expand[Sin[x (1 + y)]]
```
$\text{Sin}[x(1 + y)]$

Using the second argument, the expression only expands those subexpressions containing *pat*:

```
>> Expand[(x+a)^2+(y+a)^2+(x+y)(x+a), y]
```
$a^2 + 2ay + x(a + x) + y(a + x) + y^2 + (a + x)^2$

`Expand` also works in Galois fields

```
>> Expand[(1 + a)^12, Modulus -> 3]
```
$1 + a^3 + a^9 + a^{12}$

```
>> Expand[(1 + a)^12, Modulus -> 4]
```
$1 + 2a^2 + 3a^4 + 3a^8 + 2a^{10} + a^{12}$

31.1.9. ExpandAll

WMA link

`ExpandAll`[*expr*]
: expands out negative integer powers and products of sums in *expr*.

`ExpandAll`[*expr*, *target*]
: just expands those parts involving *target*.

```
>> ExpandAll[(a + b)^ 2 / (c + d)^2]
```

$$\frac{a^2}{c^2 + 2cd + d^2} + \frac{2ab}{c^2 + 2cd + d^2} + \frac{b^2}{c^2 + 2cd + d^2}$$

`ExpandAll` descends into sub expressions

```
>> ExpandAll[(a + Sin[x (1 + y)])^2]
```

$$2a\mathrm{Sin}\left[x + xy\right] + a^2 + \mathrm{Sin}\left[x + xy\right]^2$$

```
>> ExpandAll[Sin[(x+y)^2]]
```

$$\mathrm{Sin}\left[x^2 + 2xy + y^2\right]$$

```
>> ExpandAll[Sin[(x+y)^2], Trig->True]
```

$$\mathrm{Cos}\left[x^2\right]\mathrm{Cos}\left[2xy\right]\mathrm{Sin}\left[y^2\right] + \mathrm{Cos}\left[x^2\right]\mathrm{Cos}\left[y^2\right]\mathrm{Sin}\left[2xy\right] + \mathrm{Cos}\left[2xy\right]\mathrm{Cos}\left[y^2\right]\mathrm{Sin}\left[x^2\right] - \mathrm{Sin}\left[x^2\right]\mathrm{Sin}\left[2xy\right]\mathrm{Sin}\left[y^2\right]$$

`ExpandAll` also expands heads

```
>> ExpandAll[((1 + x)(1 + y))[x]]
```

$$\left(1 + x + y + xy\right)\left[x\right]$$

`ExpandAll` can also work in finite fields

```
>> ExpandAll[(1 + a)^ 6 / (x + y)^3, Modulus -> 3]
```

$$\frac{1 + 2a^3 + a^6}{x^3 + y^3}$$

31.1.10. ExpandDenominator

WMA link

`ExpandDenominator[`*expr*`]`
    expands out negative integer powers and products of sums in *expr*.

```
>> ExpandDenominator[(a + b)^ 2 / ((c + d)^2 (e + f))]
```

$$\frac{(a + b)^2}{c^2e + c^2f + 2cde + 2cdf + d^2e + d^2f}$$

31.1.11. Exponent

WMA link

`Exponent[expr, form]`
: returns the maximum power with which *form* appears in the expanded form of *expr*.

`Exponent[expr, form, h]`
: applies *h* to the set of exponents with which *form* appears in *expr*.

```
>> Exponent[5 x^2 - 3 x + 7, x]
```
2

```
>> Exponent[(x^3 + 1)^2 + 1, x]
```
6

```
>> Exponent[x^(n + 1)+ Sqrt[x] + 1, x]
```
$\mathrm{Max}\left[\frac{1}{2}, 1+n\right]$

```
>> Exponent[x / y, y]
```
-1

```
>> Exponent[(x^2 + 1)^3 - 1, x, Min]
```
2

```
>> Exponent[0, x]
```
$-\infty$

```
>> Exponent[1, x]
```
0

31.1.12. Factor

WMA link

`Factor[`*expr*`]`
: factors the polynomial expression *expr*.

```
>> Factor[x ^ 2 + 2 x + 1]
```
$(1+x)^2$

```
>> Factor[1 / (x^2+2x+1)+ 1 / (x^4+2x^2+1)]
```
$$\frac{2+2x+3x^2+x^4}{(1+x)^2\left(1+x^2\right)^2}$$

Factor can also be used with equations:

```
>> Factor[x a == x b + x c]
```
$ax{==}x\,(b+c)$

And lists:

```
>> Factor[{x + x^2, 2 x + 2 y + 2}]
```
$\{x(1+x), 2(1+x+y)\}$

It also works with more complex expressions:

```
>> Factor[x ^ 3 + 3 x ^ 2 y + 3 x y ^ 2 + y ^ 3]
```
$(x+y)^3$

You can use Factor to find when a polynomial is zero:

```
>> x^2 - x == 0 // Factor
```
$x(-1+x)==0$

31.1.13. FactorTermsList

WMA link

`FactorTermsList[poly]`
: returns a list of 2 elements. The first element is the numerical factor in *poly*. The second one is the remaining of the polynomial with numerical factor removed.

`FactorTermsList[poly, {x1, x2, ...}]`
: returns a list of factors in *poly*. The first element is the numerical factor in *poly*. The next ones are factors that are independent of variables lists which are created by removing each variable *xi* from right to left. The last one is the remaining of polynomial after dividing *poly* to all previous factors.

```
>> FactorTermsList[2 x^2 - 2]
```
$\{2, -1+x^2\}$

```
>> FactorTermsList[x^2 - 2 x + 1]
```
$\{1, 1-2x+x^2\}$

```
>> f = 3 (-1 + 2 x)(-1 + y)(1 - a)
```
$3(-1+2x)(-1+y)(1-a)$

```
>> FactorTermsList[f]
```
$\{-3, -1+a-2ax-ay+2x+y-2xy+2axy\}$

```
>> FactorTermsList[f, x]
```
$\{-3, 1-a-y+ay, -1+2x\}$

31.1.14. FullSimplify

WMA link

`FullSimplify[`*expr*`]`
: simplifies *expr* using an extended set of simplification rules.

`FullSimplify[`*expr*, *assump*`]`
: simplifies *expr* assuming *assump* instead of *Assumptions*.

TODO: implement the extension. By now, this does the same than Simplify...

```
>> FullSimplify[2*Sin[x]^2 + 2*Cos[x]^2]
```
2

31.1.15. MinimalPolynomial

WMA link

`MinimalPolynomial[s, x]`
: gives the minimal polynomial in x for which the algebraic number s is a root.

```
>> MinimalPolynomial[7, x]
```
$-7 + x$

```
>> MinimalPolynomial[Sqrt[2] + Sqrt[3], x]
```
$1 - 10x^2 + x^4$

```
>> MinimalPolynomial[Sqrt[1 + Sqrt[3]], x]
```
$-2 - 2x^2 + x^4$

```
>> MinimalPolynomial[Sqrt[I + Sqrt[6]], x]
```
$49 - 10x^4 + x^8$

31.1.16. Numerator

WMA link

`Numerator[`*expr*`]`
: gives the numerator in *expr*.

```
>> Numerator[a / b]
```
a

```
>> Numerator[2 / 3]
```
2

```
>> Numerator[a + b]
```
$a + b$

31.1.17. PolynomialQ

Polynomial (SymPy, WMA)

`PolynomialQ[expr]`
: returns True if *expr* is a polynomial and returns False otherwise.

`PolynomialQ[expr, var]`
: returns True if *expr* is a polynomial in *var*, and returns False otherwise.

`PolynomialQ[expr, {var1, ...}]`
: tests whether *expr* is a polynomial in the *vari*.

`PolynomialQ` with no explicit variable mentioned:

```
>> PolynomialQ[x^2]
True
```

A number is a degenerate kind of polynomial:

```
>> PolynomialQ[2]
True
```

The following is not a polynomial because y is raised to the power -1:

```
>> PolynomialQ[x^2 + x/y]
False
```

`PolynomialQ` using an expression and a single variable:

```
>> PolynomialQ[x^3 - 2 x/y + 3xz, x]
True
```

In the above, there were no negative powers for x. In the below when we check with respect to y, we *do* find y is raised to a negative power:

```
>> PolynomialQ[x^3 - 2 x/y^2 + 3xz, y]
False
```

```
>> PolynomialQ[f[a] + f[a]^2, f[a]]
True
```

`PolynomialQ` using an expression and a list of variables:

```
>> PolynomialQ[x^2 + axy^2 - bSin[c], {x, y}]
True
```

```
>> PolynomialQ[x^2 + axy^2 - bSin[c], {a, b, c}]
False
```

31.1.18. PowerExpand

WMA link

`PowerExpand[`*expr*`]`
: expands out powers of the form $(x^y)^z$ and $(xy)^z$ in *expr*.

```
>> PowerExpand[(a ^ b)^ c]
```
a^{bc}

```
>> PowerExpand[(a * b)^ c]
```
$a^c b^c$

`PowerExpand` is not correct without certain assumptions:

```
>> PowerExpand[(x ^ 2)^ (1/2)]
```
x

31.1.19. Simplify

SymPy, WMA

`Simplify[`*expr*`]`
: simplifies *expr*.

`Simplify[`*expr*, *assump*`]`
: simplifies *expr* assuming *assump* instead of `$Assumptions`.

```
>> Simplify[2*Sin[x]^2 + 2*Cos[x]^2]
```
2

```
>> Simplify[x]
```
x

```
>> Simplify[f[x]]
```
$f[x]$

Simplify over conditional expressions uses `$Assumptions`, or *assump* to evaluate the condition:

```
>> $Assumptions={a <= 0};
```

```
>> Simplify[ConditionalExpression[1, a > 0]]
```
Undefined

The *assump* option override `$Assumption`:

```
>> Simplify[ConditionalExpression[1, a > 0] ConditionalExpression[1, b >
    0], { b > 0 }]
```

ConditionalExpression $[1, a > 0]$

On the other hand, `Assumptions` option does not override `$Assumptions`, but add to them:

```
>> Simplify[ConditionalExpression[1, a > 0] ConditionalExpression[1, b >
    0], Assumptions -> { b > 0 }]
```

ConditionalExpression $[1, a > 0]$

Passing both options overwrites `$Assumptions` with the union of *assump* the option

```
>> Simplify[ConditionalExpression[1, a > 0] ConditionalExpression[1, b >
    0], {a>0},Assumptions -> { b > 0 }]
```

1

```
>> $Assumptions={};
```

The option `ComplexityFunction` allows to control the way in which the evaluator decides if one expression is simpler than another. For example, by default, `Simplify` tries to avoid expressions involving numbers with many digits:

```
>> Simplify[20 Log[2]]
```

20Log [2]

This behaviour can be modified by setting `LeafCount` as the `ComplexityFunction`:

```
>> Simplify[20 Log[2], ComplexityFunction->LeafCount]
```

Log [1048576]

31.1.20. Together

WMA link

`Together[`*expr*`]`
: writes sums of fractions in *expr* together.

```
>> Together[a / c + b / c]
```

$$\frac{a+b}{c}$$

`Together` operates on lists:

```
>> Together[{x / (y+1)+ x / (y+1)^2}]
```

$$\left\{\frac{x\,(2+y)}{(1+y)^2}\right\}$$

But it does not touch other functions:

```
>> Together[f[a / c + b / c]]
```

$f\left[\frac{a}{c}+\frac{b}{c}\right]$

31.1.21. Variables

WMA link

`Variables[`*expr*`]`
: gives a list of the variables that appear in the polynomial *expr*.

```
>> Variables[a x^2 + b x + c]
```

$\{a,b,c,x\}$

```
>> Variables[{a + b x, c y^2 + x/2}]
```

$\{a,b,c,x,y\}$

```
>> Variables[x + Sin[y]]
```

$\{x,\text{Sin}\left[y\right]\}$

31.2. Calculus

Originally called infinitesimal calculus or "the calculus of infinitesimals", is the mathematical study of continuous change, in the same way that geometry is the study of shape and algebra is the study of generalizations of arithmetic operations.

31.2.1. Complexes

WMA link

`Complexes`
: the domain of complex numbers, as in x in Complexes.

31.2.2. D

Derivative (WMA)

`D`[f, x]
: gives the partial derivative of f with respect to x.

`D`[f, x, y, ...]
: differentiates successively with respect to x, y, etc.

`D`[f, {x, n}]
: gives the multiple derivative of order n.

`D`[f, {{x_1, x_2, ...}}]
: gives the vector derivative of f with respect to x_1, x_2, etc.

First-order derivative of a polynomial:

```
>> D[x^3 + x^2, x]
```
$2x + 3x^2$

Second-order derivative:

```
>> D[x^3 + x^2, {x, 2}]
```
$2 + 6x$

Trigonometric derivatives:

```
>> D[Sin[Cos[x]], x]
```
$-\text{Cos}\,[\text{Cos}\,[x]]\,\text{Sin}\,[x]$

```
>> D[Sin[x], {x, 2}]
```
$-\text{Sin}\,[x]$

```
>> D[Cos[t], {t, 2}]
```
$-\text{Cos}\,[t]$

Unknown variables are treated as constant:

```
>> D[y, x]
```
0

```
>> D[x, x]
```
1

```
>> D[x + y, x]
```
1

Derivatives of unknown functions are represented using `Derivative`:

```
>> D[f[x], x]
```
$f'\,[x]$

```
>> D[f[x, x], x]
```
$f^{(0,1)}\,[x, x] + f^{(1,0)}\,[x, x]$

```
>> D[f[x, x], x] // InputForm
```
$\text{Derivative}\,[0, 1]\,[f]\,[x, x] + \text{Derivative}\,[1, 0]\,[f]\,[x, x]$

Chain rule:

```
>> D[f[2x+1, 2y, x+y], x]
```

$2f^{(1,0,0)}\left[1+2x,2y,x+y\right]+f^{(0,0,1)}\left[1+2x,2y,x+y\right]$

```
>> D[f[x^2, x, 2y], {x,2}, y] // Expand
```

$8xf^{(1,1,1)}\left[x^2,x,2y\right]+8x^2f^{(2,0,1)}\left[x^2,x,2y\right]+2f^{(0,2,1)}\left[x^2,x,2y\right]+4f^{(1,0,1)}\left[x^2,x,2y\right]$

Compute the gradient vector of a function:

```
>> D[x ^ 3 * Cos[y], {{x, y}}]
```

$\left\{3x^2\text{Cos}\left[y\right],-x^3\text{Sin}\left[y\right]\right\}$

Hesse matrix:

```
>> D[Sin[x] * Cos[y], {{x,y}, 2}]
```

$\{\{-\text{Cos}\left[y\right]\text{Sin}\left[x\right],-\text{Cos}\left[x\right]\text{Sin}\left[y\right]\},\{-\text{Cos}\left[x\right]\text{Sin}\left[y\right],-\text{Cos}\left[y\right]\text{Sin}\left[x\right]\}\}$

31.2.3. Derivative (')

WMA link

f'[x,...]
    represents the derivative of f with respect to the first argument x.
f''[x,...]
    represents the 2nd derivative of f with respect to x.
`Derivative`[n][f]
    represents the nth derivative of the function f.
`Derivative`[n_1, n_2, ...][f]
    represents a multivariate derivative.

```
>> Derivative[1][Sin]
```

Cos [#1] &

```
>> Derivative[3][Sin]
```

−Cos [#1] &

```
>> Derivative[2][# ^ 3&]
```

6#1&

`Derivative` can be entered using `'`:

```
>> Sin'[x]
```

Cos [x]

```
>> (# ^ 4&)''
```

12#1^2&

```
>> f'[x] // InputForm
```
Derivative [1] $[f]$ $[x]$

```
>> Derivative[1][#2 Sin[#1]+Cos[#2]&]
```
Cos [#1] #2&

```
>> Derivative[1,2][#2^3 Sin[#1]+Cos[#2]&]
```
6Cos [#1] #2&

Deriving with respect to an unknown parameter yields 0:

```
>> Derivative[1,2,1][#2^3 Sin[#1]+Cos[#2]&]
```
0&

The 0th derivative of any expression is the expression itself:

```
>> Derivative[0,0,0][a+b+c]
```
$a + b + c$

You can calculate the derivative of custom functions:

```
>> f[x_] := x ^ 2
```

```
>> f'[x]
```
$2x$

Unknown derivatives:

```
>> Derivative[2, 1][h]
```
$h^{(2,1)}$

```
>> Derivative[2, 0, 1, 0][h[g]]
```
$h\left[g\right]^{(2,0,1,0)}$

31.2.4. DiscreteLimit

WMA link

`DiscreteLimit`[f, k->Infinity]
    gives the limit of the sequence f as k tends to infinity.

```
>> DiscreteLimit[n/(n + 1), n -> Infinity]
```
1

```
>> DiscreteLimit[f[n], n -> Infinity]
```
$f\,[\infty]$

31.2.5. FindMaximum

WMA link

FindMaximum[f, {x, x_0}]
searches for a numerical maximum of f, starting from x=x_0.

FindMaximum by default uses Newtonś method, so the function of interest should have a first derivative.

```
>> FindMaximum[-(x-3)^2+2., {x, 1}]
   Encountered a gradient that is effectively zero. The result returned
   may not be a maximum; it may be a minimum or a saddle point.
```

$\{2., \{x- > 3.\}\}$

```
>> FindMaximum[-10*^-30 *(x-3)^2+2., {x, 1}]
   Encountered a gradient that is effectively zero. The result returned
   may not be a maximum; it may be a minimum or a saddle point.
```

$\{2., \{x- > 3.\}\}$

```
>> FindMaximum[Sin[x], {x, 1}]
```

$\{1., \{x- > 1.5708\}\}$

```
>> phi[x_?NumberQ]:=NIntegrate[u, {u, 0., x}, Method->"Internal"];
```

```
>> Quiet[FindMaximum[-phi[x] + x, {x, 1.2}, Method->"Newton"]]
```

$\{0.5, \{x- > 1.00001\}\}$

```
>> Clear[phi];
```

For a not so well behaving function, the result can be less accurate:

```
>> FindMaximum[-Exp[-1/x^2]+1., {x,1.2}, MaxIterations->10]
   The maximum number of iterations was exceeded. The result might be
   inaccurate.
```

$$\text{FindMaximum}\left[-\text{Exp}\left[-\frac{1}{x^2}\right]+1., \{x, 1.2\}, \text{MaxIterations}- > 10\right]$$

31.2.6. FindMinimum

WMA link

FindMinimum[f, {x, x_0}]
searches for a numerical minimum of f, starting from x=x_0.

FindMinimum by default uses Newtonś method, so the function of interest should have a first derivative.

```
>> FindMinimum[(x-3)^2+2., {x, 1}]
   Encountered a gradient that is effectively zero. The result returned
   may not be a minimum; it may be a maximum or a saddle point.
```
$\{2., \{x- > 3.\}\}$

```
>> FindMinimum[10*^-30 *(x-3)^2+2., {x, 1}]
   Encountered a gradient that is effectively zero. The result returned
   may not be a minimum; it may be a maximum or a saddle point.
```
$\{2., \{x- > 3.\}\}$

```
>> FindMinimum[Sin[x], {x, 1}]
```
$\{-1., \{x- > -1.5708\}\}$

```
>> phi[x_?NumberQ]:=NIntegrate[u,{u,0,x}, Method->"Internal"];
```

```
>> Quiet[FindMinimum[phi[x]-x,{x, 1.2}, Method->"Newton"]]
```
$\{-0.5, \{x- > 1.00001\}\}$

```
>> Clear[phi];
```

For a not so well behaving function, the result can be less accurate:

```
>> FindMinimum[Exp[-1/x^2]+1., {x,1.2}, MaxIterations->10]
   The maximum number of iterations was exceeded. The result might be
   inaccurate.
```
$$\text{FindMinimum}\left[\text{Exp}\left[-\frac{1}{x^2}\right]+1., \{x, 1.2\}, \text{MaxIterations}- > 10\right]$$

31.2.7. FindRoot

WMA link

`FindRoot`[f, {x, x_0}]
    searches for a numerical root of f, starting from x=x_0.
`FindRoot`[lhs == rhs, {x, x_0}]
    tries to solve the equation lhs == rhs.

`FindRoot` by default uses Newtonś method, so the function of interest should have a first derivative.

```
>> FindRoot[Cos[x], {x, 1}]
```
$\{x- > 1.5708\}$

```
>> FindRoot[Sin[x] + Exp[x],{x, 0}]
```
$\{x- > -0.588533\}$

```
>> FindRoot[Sin[x] + Exp[x] == Pi,{x, 0}]
```
$\{x- > 0.866815\}$

`FindRoot` has attribute `HoldAll` and effectively uses `Block` to localize x. However, in the result x will eventually still be replaced by its value.

```
>> x = "I am the result!";

>> FindRoot[Tan[x] + Sin[x] == Pi, {x, 1}]
```
{I am the result!$->1.14911\}$

```
>> Clear[x]
```

`FindRoot` stops after 100 iterations:

```
>> FindRoot[x^2 + x + 1, {x, 1}]
The maximum number of iterations was exceeded. The result might be
inaccurate.
```
$\{x->-1.\}$

Find complex roots:

```
>> FindRoot[x ^ 2 + x + 1, {x, -I}]
```
$\{x->-0.5-0.866025I\}$

The function has to return numerical values:

```
>> FindRoot[f[x] == 0, {x, 0}]
The function value is not a number at x = 0..
```
$\text{FindRoot}\left[f[x]-0,\{x,0\}\right]$

The derivative must not be 0:

```
>> FindRoot[Sin[x] == x, {x, 0}]
Encountered a singular derivative at the point x = 0..
```
$\text{FindRoot}\left[\text{Sin}[x]-x,\{x,0\}\right]$

```
>> FindRoot[x^2 - 2, {x, 1,3}, Method->"Secant"]
```
$\{x->1.41421\}$

31.2.8. Integers

WMA link

`Integers`
the domain of integer numbers, as in x in Integers.

Limit a solution to integer numbers:

```
>> Solve[-4 - 4 x + x^4 + x^5 == 0, x, Integers]
```
$\{\{x->-1\}\}$

```
>> Solve[x^4 == 4, x, Integers]
```
$\{\}$

31.2.9. Integrate

Integral (SymPy, WMA)

`Integrate[`f, x`]`
: integrates f with respect to x. The result does not contain the additive integration constant.

`Integrate[`$f, \{x, a, b\}$`]`
: computes the definite integral of f with respect to x from a to b.

Integrate a polynomial:

```
>> Integrate[6 x ^ 2 + 3 x ^ 2 - 4 x + 10, x]
```

$$x\left(10 - 2x + 3x^2\right)$$

Integrate trigonometric functions:

```
>> Integrate[Sin[x] ^ 5, x]
```

$$\text{Cos}[x]\left(-1 - \frac{\text{Cos}[x]^4}{5} + \frac{2\text{Cos}[x]^2}{3}\right)$$

Definite integrals:

```
>> Integrate[x ^ 2 + x, {x, 1, 3}]
```

$$\frac{38}{3}$$

```
>> Integrate[Sin[x], {x, 0, Pi/2}]
```

1

Some other integrals:

```
>> Integrate[1 / (1 - 4 x + x^2), x]
```

$$\frac{\sqrt{3}\left(\text{Log}\left[-2 - \sqrt{3} + x\right] - \text{Log}\left[-2 + \sqrt{3} + x\right]\right)}{6}$$

```
>> Integrate[4 Sin[x] Cos[x], x]
```

$$2\text{Sin}[x]^2$$

```
>> Integrate[-Infinity, {x, 0, Infinity}]
```

$$-\infty$$

Integrating something ill-defined returns the expression untouched:

```
>> Integrate[1, {x, Infinity, 0}]
```

$$\int_{\infty}^{0} 1\,dx$$

Here how is an example of converting integral equation to TeX:

```
>> Integrate[f[x], {x, a, b}] // TeXForm
```

\int_a^b f\left[x\right] \, dx

Sometimes there is a loss of precision during integration. You can check the precision of your result with the following sequence of commands.

```
>> Integrate[Abs[Sin[phi]], {phi, 0, 2Pi}] // N
```

4.

```
>> % // Precision
```

MachinePrecision

```
>> Integrate[ArcSin[x / 3], x]
```

$x\text{ArcSin}\left[\frac{x}{3}\right] + \sqrt{9 - x^2}$

```
>> Integrate[f'[x], {x, a, b}]
```

$f[b] - f[a]$

and,

```
>> D[Integrate[f[u, x],{u, a[x], b[x]}], x]
```

$$\int_{a[x]}^{b[x]} f^{(0,1)}[u, x]\,du + f[b[x], x]\,b'[x] - f[a[x], x]\,a'[x]$$

```
>> N[Integrate[Sin[Exp[-x^2 /2 ]],{x,1,2}]]
```

0.330804

31.2.10. Limit

WMA link

`Limit`[$expr$, x->x_0]
  gives the limit of $expr$ as x approaches x_0.
`Limit`[$expr$, x->x_0, Direction->1]
  approaches x_0 from smaller values.
`Limit`[$expr$, x->x_0, Direction->-1]
  approaches x_0 from larger values.

```
>> Limit[x, x->2]
```

2

```
>> Limit[Sin[x] / x, x->0]
```

1

```
>> Limit[1/x, x->0, Direction->-1]
```

∞

```
>> Limit[1/x, x->0, Direction->1]
```
$-\infty$

31.2.11. NIntegrate

WMA link

`NIntegrate[`*expr*, *interval*`]`
: returns a numeric approximation to the definite integral of *expr* with limits *interval* and with a precision of *prec* digits.

`NIntegrate[`*expr*, $interval_1$, $interval_2$, ...`]`
: returns a numeric approximation to the multiple integral of *expr* with limits $interval_1$, $interval_2$ and with a precision of *prec* digits.

```
>> NIntegrate[Exp[-x],{x,0,Infinity},Tolerance->1*^-6, Method->"Internal
   "]
```
1.

```
>> NIntegrate[Exp[x],{x,-Infinity, 0},Tolerance->1*^-6, Method->"
   Internal"]
```
1.

```
>> NIntegrate[Exp[-x^2/2.],{x,-Infinity, Infinity},Tolerance->1*^-6,
   Method->"Internal"]
```
2.50664

31.2.12. O

WMA link

`O[$x$]^n`
: Represents a term of order x^n.
O[x]^n is generated to represent omitted higher order terms in power series.

```
>> Series[1/(1-x),{x,0,2}]
```
$1 + x + x^2 + O[x]^3$

When called alone, a 'SeriesData' expression is built:

```
>> O[x] // FullForm
```
SeriesData $[x, 0, \{\}, 1, 1, 1]$

31.2.13. Reals

WMA link

`Reals`
: is the domain real numbers, as in x in Reals.

Limit a solution to real numbers:

```
>> Solve[x^3 == 1, x, Reals]
```
$\{\{x- > 1\}\}$

31.2.14. Root

WMA link

`Root`$[f, i]$
: represents the i-th complex root of the polynomial f.

```
>> Root[#1 ^ 2 - 1&, 1]
```
-1

```
>> Root[#1 ^ 2 - 1&, 2]
```
1

Roots that can't be represented by radicals:

```
>> Root[#1 ^ 5 + 2 #1 + 1&, 2]
```
$\text{Root}\left[1 + \#1^5 + 2\#1\&, 2\right]$

31.2.15. RootSum

WMA link

`RootSum`$[f, form]$
: sums $form[x]$ for all roots of the polynomial $f[x]$.

Integrating a rational function of any order:

```
>> Integrate[1/(x^5 + 11 x + 1), {x, 1, 3}]
```

$\text{RootSum}\left[-1-212\,960\#1^3-9\,680\#1^2-165\right.$
$\#1+41232181\#1^5\&,\left(\text{Log}\left[3749971-3\,512\,322\,106\,304\right.\right.$
$\left.\#1^4+453522741\#1+16326568676\#1^2+79825502416\#1^3\right]-4\text{Log}\,[$
$\left.\left.5]\right)\#1\&\right]-\text{RootSum}\left[-1-212\,960\#1^3-9\,680\#1^2-\sim\right.$
$\sim 165\#1+41232181\#1^5\&,\left(\text{Log}\left[3748721-3\,512\,322\,106\,304\right.\right.$
$\left.\left.\#1^4+453522741\#1+16326568676\#1^2+79825502416\#1^3\right]-4\text{Log}\,[5]\right)\#1\&\left.\right]$

```
>> N[%, 50]
```

0.051278805184286949884270940103072421286139857550894

Simplification of `RootSum` expression

```
>> RootSum[#^5 - 11 # + 1 &, (#^2 - 1)/(#^3 - 2 # + c)&]
```

$$\frac{538-88c+396c^2+5c^3-5c^4}{97-529c-53c^2+88c^3+c^5}$$

```
>> RootSum[#^5 - 3 # - 7 &, Sin] //N//Chop
```

0.292188

Use `Normal` to expand `RootSum`:

```
>> RootSum[1+#+#^2+#^3+#^4 &, Log[x + #] &]
```

$\text{RootSum}\left[1+\#1^2+\#1^3+\#1^4+\#1\&,\text{Log}\,[x+\#1]\,\&\right]$

```
>> %//Normal
```

$$\text{Log}\left[-\frac{1}{4}-\frac{\sqrt{5}}{4}-I\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}+x\right]+\text{Log}\left[-\frac{1}{4}-\frac{\sqrt{5}}{4}+I\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}+x\right]+\text{Log}\left[-\frac{1}{4}-I\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}+\frac{\sqrt{5}}{4}+x\right]+\text{Log}\left[-\frac{1}{4}+I\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}+\frac{\sqrt{5}}{4}+x\right]$$

31.2.16. Series

WMA link

`Series`$[f,\{x,x_0,n\}]$
Represents the series expansion around \$x\$=\$x_0\$ up to order n.

For elementary expressions, `Series` returns the explicit power series as a `SeriesData` expression:

```
>> series = Series[Exp[x^2], {x,0,2}]
```

$1+x^2+O\,[x]^3$

The expression created is a `SeriesData` object:

```
>> series // FullForm
```
SeriesData $[x, 0, \{1, 0, 1\}, 0, 3, 1]$

Replacing x with does a value produces another `SeriesData` object:

```
>> series /. x->4
```
$1 + 4^2 + O[4]^3$

`Normal` transforms a `SeriesData` expression into a polynomial:

```
>> series // Normal
```
$1 + x^2$

```
>> (series // Normal)/. x-> 4
```
17

```
>> Clear[series];
```

We can also expand over multiple variables:

```
>> Series[Exp[x-y], {x, 0, 2}, {y, 0, 2}]
```
$$\left(1 - y + \frac{1}{2}y^2 + O[y]^3\right) + \left(1 - y + \frac{1}{2}y^2 + O[y]^3\right) x + \left(\frac{1}{2} + \left(-\frac{1}{2}\right) y + \frac{1}{4}y^2 + O[y]^3\right) x^2 + O[x]^3$$

See also 'SeriesCoefficient' 31.2.17 and 'SeriesData' 31.2.18.

31.2.17. SeriesCoefficient

WMA link

`SeriesCoefficient`[*series*, n]
    Find the nth coefficient in the given *series*.
`SeriesCoefficient`[f, {x, x_0, n}]
    Find the $(x$-$x_0)$^n in the expansion of f about the point x=x_0.

First we list 5 terms of a series:

```
>> Series[Exp[Sin[x]], {x, 0, 5}]
```
$$1 + x + \frac{1}{2}x^2 + \left(-\frac{1}{8}\right) x^4 + \left(-\frac{1}{15}\right) x^5 + O[x]^6$$

Now get the x^4 coefficient:

```
>> SeriesCoefficient[%, 4]
```

$-\frac{1}{8}$

Do the same thing, but without calling `Series` first:

```
>> SeriesCoefficient[Exp[Sin[x]], {x, 0, 4}]
```

$-\frac{1}{8}$

```
>> SeriesCoefficient[2x, {x, 0, 2}]
```

0

```
>> SeriesCoefficient[SeriesData[x, c, Table[i^2, {i, 10}], 7, 17, 3],
   14/3]
```

64

```
>> SeriesCoefficient[SeriesData[x, c, Table[i^2, {i, 10}], 7, 17, 3],
   6/3]
```

0

```
>> SeriesCoefficient[SeriesData[x, c, Table[i^2, {i, 10}], 7, 17, 3],
   17/3]
```

Indeterminate

See also 'Series' 31.2.16 and 'SeriesData' 31.2.18.

31.2.18. SeriesData

WMA link

`SeriesData`[x, x_0, {a_0, a_1, ...}, n_{min}, n_{max}, den]
: produces a power series in the variable x about point x_0. The ai are the coefficients of the power series. The powers of (x-x_0) that appear are n_{min}/den, $(n_{min}+1)/den$, ..., n_{max}/den.

`SeriesData` is the `Head` of expressions generated by `Series`:

```
>> series = Series[Cosh[x],{x,0,2}]
```

$1+\frac{1}{2}x^2+O[x]^3$

```
>> Head[series]
```

SeriesData

```
>> series // FullForm
```

SeriesData $[x,0,\{1,0,\text{Rational}[1,2]\},0,3,1]$

You can apply certain mathematical operations to `SeriesData` objects to get new `SeriesData` objects truncated to the appropriate order.

```
>> series + Series[Sinh[x],{x,0,3}]
```
$$1 + x + \frac{1}{2}x^2 + O[x]^3$$

```
>> Series[f[x],{x,0,2}] * g[w]
```
$$f[0]\,g[w] + g[w]\,f'[0]\,x + \frac{g[w]\,f''[0]}{2}x^2 + O[x]^3$$

The product of two series on the same neighborhood of the same variable are multiplied:

```
>> Series[Exp[-a x],{x,0,2}] * Series[Exp[-b x],{x,0,2}]
```
$$1 + (-a - b)\,x + \left(\frac{a^2}{2} + ab + \frac{b^2}{2}\right)x^2 + O[x]^3$$

```
>> D[Series[Exp[-a x],{x,0,2}],a]
```
$$-x + ax^2 + O[x]^3$$

See also 'Series' 31.2.16 and 'SeriesCoefficient' 31.2.17.

31.2.19. Solve

Equation solving (SymPy, WMA)

`Solve`[*equation*, *vars*]
: attempts to solve *equation* for the variables *vars*.

`Solve`[*equation*, *vars*, *domain*]
: restricts variables to *domain*, which can be `Complexes` or `Reals` or `Integers`.

```
>> Solve[x ^ 2 - 3 x == 4, x]
```
$$\{\{x- > -1\}, \{x- > 4\}\}$$

```
>> Solve[4 y - 8 == 0, y]
```
$$\{\{y- > 2\}\}$$

Apply the solution:

```
>> sol = Solve[2 x^2 - 10 x - 12 == 0, x]
```
$$\{\{x- > -1\}, \{x- > 6\}\}$$

```
>> x /. sol
```
$$\{-1, 6\}$$

Contradiction:

```
>> Solve[x + 1 == x, x]
```
$$\{\}$$

Tautology:

```
>> Solve[x ^ 2 == x ^ 2, x]
```
$\{\{\}\}$

Rational equations:

```
>> Solve[x / (x ^ 2 + 1)== 1, x]
```
$$\left\{\left\{x->\frac{1}{2}-\frac{I}{2}\sqrt{3}\right\},\left\{x->\frac{1}{2}+\frac{I}{2}\sqrt{3}\right\}\right\}$$

```
>> Solve[(x^2 + 3 x + 2)/(4 x - 2)== 0, x]
```
$\{\{x->-2\},\{x->-1\}\}$

Transcendental equations:

```
>> Solve[Cos[x] == 0, x]
```
$$\left\{\left\{x->\frac{\pi}{2}\right\},\left\{x->\frac{3\pi}{2}\right\}\right\}$$

Solve can only solve equations with respect to symbols or functions:

```
>> Solve[f[x + y] == 3, f[x + y]]
```
$\{\{f[x+y]->3\}\}$

```
>> Solve[a + b == 2, a + b]
a + b is not a valid variable.
```
Solve $[a+b==2,a+b]$

This happens when solving with respect to an assigned symbol:

```
>> x = 3;
```

```
>> Solve[x == 2, x]
3 is not a valid variable.
```
Solve [False, 3]

```
>> Clear[x]
```

```
>> Solve[a < b, a]
a < b is not a well-formed equation.
```
Solve $[a<b,a]$

Solve a system of equations:

```
>> eqs = {3 x ^ 2 - 3 y == 0, 3 y ^ 2 - 3 x == 0};
```

```
>> sol = Solve[eqs, {x, y}] // Simplify
```

$$\left\{\{x->0, y->0\}, \{x->1, y->1\}, \left\{x->-\frac{1}{2}+\frac{I}{2}\sqrt{3}, y->-\frac{1}{2}-\frac{I}{2}\sqrt{3}\right\}, \left\{x->-\frac{1}{2}-\frac{I}{2}\sqrt{3}, y->-\frac{1}{2}+\frac{I}{2}\sqrt{3}\right\}\right\}$$

```
>> eqs /. sol // Simplify
```

{{True, True}, {True, True}, {True, True}, {True, True}}

Solve when given an underdetermined system:

```
>> Solve[x^2 == 1 && z^2 == -1, {x, y, z}]
Equations may not give solutions for all "solve" variables.
```

$\{\{x->-1, z->-I\}, \{x->-1, z->I\}, \{x->1, z->-I\}, \{x->1, z->I\}\}$

Examples using specifying the Domain in solutions:

```
>> Solve[x^2 == -1, x, Reals]
```

{}

```
>> Solve[x^2 == 1, x, Reals]
```

$\{\{x->-1\}, \{x->1\}\}$

```
>> Solve[x^2 == -1, x, Complexes]
```

$\{\{x->-I\}, \{x->I\}\}$

```
>> Solve[4 - 4 * x^2 - x^4 + x^6 == 0, x, Integers]
```

$\{\{x->-1\}, \{x->1\}\}$

31.3. Differential Equations

31.3.1. C

WMA link

`C`[n]
: represents the n-th constant in a solution to a differential equation.

31.3.2. DSolve

WMA link

`DSolve[$eq$, $y$`[x], x]
: solves a differential equation for the function $y[x]$.

```
>> DSolve[y''[x] == 0, y[x], x]
```
$\{\{y[x]->xC[2]+C[1]\}\}$

```
>> DSolve[y''[x] == y[x], y[x], x]
```
$\{\{y[x]->C[1]E^{-x}+C[2]E^{x}\}\}$

```
>> DSolve[y''[x] == y[x], y, x]
```
$\{\{y->\text{Function}[\{x\},C[1]E^{-x}+C[2]E^{x}]\}\}$

DSolve can also solve basic PDE

```
>> DSolve[D[f[x, y], x] / f[x, y] + 3 D[f[x, y], y] / f[x, y] == 2, f, {
   x, y}]
```
$\left\{\left\{f->\text{Function}\left[\{x,y\},E^{\frac{x}{5}+\frac{3y}{5}}C[1]\left[3x-y\right]\right]\right\}\right\}$

```
>> DSolve[D[f[x, y], x] x + D[f[x, y], y] y == 2, f[x, y], {x, y}]
```
$\left\{\left\{f\left[x,y\right]->2\text{Log}[x]+C[1]\left[\frac{y}{x}\right]\right\}\right\}$

```
>> DSolve[D[y[x, t], t] + 2 D[y[x, t], x] == 0, y[x, t], {x, t}]
```
$\{\{y[x,t]->C[1][x-2t]\}\}$

31.4. Exponential Functions

Numerical values and derivatives can be computed; however, most special exact values and simplification rules are not implemented yet.

31.4.1. Exp

WMA link

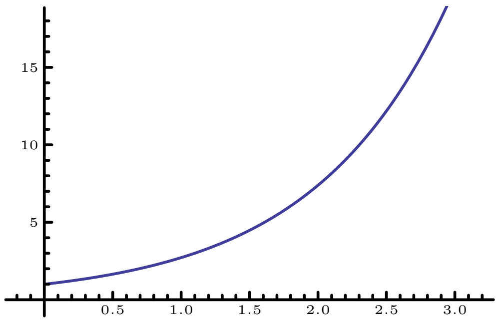
Exp[z]
    returns the exponential function of z.

```
>> Exp[1]
```
E

```
>> Exp[10.0]
```
22026.5

```
>> Exp[x] //FullForm
```
Power[E, x]

```
>> Plot[Exp[x], {x, 0, 3}]
```



31.4.2. Log

WMA link

> Log[z]
> returns the natural logarithm of z.

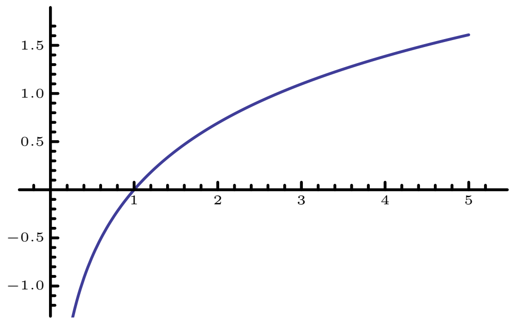
```
>> Log[{0, 1, E, E * E, E ^ 3, E ^ x}]
```

$\{-\infty, 0, 1, 2, 3, \text{Log}\left[E^x\right]\}$

```
>> Log[0.]
```

Indeterminate

```
>> Plot[Log[x], {x, 0, 5}]
```



31.4.3. Log10

WMA link

> Log10[z]
> returns the base-10 logarithm of z.

```
>> Log10[1000]
```

3

```
>> Log10[{2., 5.}]
```
$\{0.30103, 0.69897\}$

```
>> Log10[E ^ 3]
```
$$\frac{3}{\text{Log}\,[10]}$$

31.4.4. Log2

WMA link

`Log2[z]`
: returns the base-2 logarithm of z.

```
>> Log2[4 ^ 8]
```
16

```
>> Log2[5.6]
```
2.48543

```
>> Log2[E ^ 2]
```
$$\frac{2}{\text{Log}\,[2]}$$

31.4.5. LogisticSigmoid

WMA link

`LogisticSigmoid[z]`
: returns the logistic sigmoid of z.

```
>> LogisticSigmoid[0.5]
```
0.622459

```
>> LogisticSigmoid[0.5 + 2.3 I]
```
$1.06475 + 0.808177I$

```
>> LogisticSigmoid[{-0.2, 0.1, 0.3}]
```
$\{0.450166, 0.524979, 0.574443\}$

31.5. Hyperbolic Functions

Hyperbolic functions are analogues of the ordinary trigonometric functions, but defined using the hyperbola rather than the circle.

Numerical values and derivatives can be computed; however, most special exact values and simplification rules are not implemented yet.

31.5.1. ArcCosh

Inverse hyperbolic cosine (SymPy, mpmath, WMA)

`ArcCosh[`z`]`
: returns the inverse hyperbolic cosine of z.

```
>> ArcCosh[0]
```
$\frac{I}{2}\pi$

```
>> ArcCosh[0.]
```
$0. + 1.5708I$

```
>> ArcCosh[0.00000000000000000000000000000000000000]
```
$1.5707963267948966192313216916397514421I$

31.5.2. ArcCoth

Inverse hyperbolic cotangent (SymPy, mpmath, WMA)

`ArcCoth[`z`]`
: returns the inverse hyperbolic cotangent of z.

```
>> ArcCoth[0]
```
$\frac{I}{2}\pi$

```
>> ArcCoth[1]
```
∞

```
>> ArcCoth[0.0]
```
$0. + 1.5708I$

```
>> ArcCoth[0.5]
```
$0.549306 - 1.5708I$

31.5.3. ArcCsch

Inverse hyperbolic cosecant (SymPy, mpmath, WMA)

ArcCsch[*z*]
: returns the inverse hyperbolic cosecant of *z*.

```
>> ArcCsch[0]
```
ComplexInfinity

```
>> ArcCsch[1.0]
```
0.881374

31.5.4. ArcSech

WMA link

ArcSech[*z*]
: returns the inverse hyperbolic secant of *z*.

```
>> ArcSech[0]
```
∞

```
>> ArcSech[1]
```
0

```
>> ArcSech[0.5]
```
1.31696

31.5.5. ArcSinh

WMA link

ArcSinh[*z*]
: returns the inverse hyperbolic sine of *z*.

```
>> ArcSinh[0]
```
0

```
>> ArcSinh[0.]
```
0.

```
>> ArcSinh[1.0]
```
0.881374

31.5.6. ArcTanh

WMA link

`ArcTanh`[z]
: returns the inverse hyperbolic tangent of z.

```
>> ArcTanh[0]
```
0

```
>> ArcTanh[1]
```
∞

```
>> ArcTanh[0]
```
0

```
>> ArcTanh[.5 + 2 I]
```
$0.0964156 + 1.12656I$

```
>> ArcTanh[2 + I]
```
$\text{ArcTanh}[2 + I]$

31.5.7. ComplexExpand

(SymPy, WMA)

`ComplexExpand`[$expr$]
: expands $expr$ assuming that all variables are real.

`ComplexExpand`[$expr$,{x_1,x_2, ...}]
: expands $expr$ assuming that variables matching any of the xi are complex.

Note: we get equivalent, but different results from WMA:

```
>> ComplexExpand[3^(I x)]
```
$3^{-\text{Im}[x]}\text{Re}\left[3^{I\text{Re}[x]}\right] + I\text{Im}\left[3^{I\text{Re}[x]}\right]3^{-\text{Im}[x]}$

Assume that both x and y and are real:

```
>> ComplexExpand[Sin[x + I y]]
```
$\text{Cosh}\left[y\right]\text{Sin}[x] + I\text{Cos}[x]\text{Sinh}\left[y\right]$

Take x to be complex:

```
>> ComplexExpand[Sin[x], x]
```
$\text{Cosh}[\text{Im}[x]]\text{Sin}[\text{Re}[x]] + I\text{Cos}[\text{Re}[x]]\text{Sinh}[\text{Im}[x]]$

Polynomials:

```
>> ComplexExpand[Re[z^5 - 2 z^3 - z + 1], z]
```
$1 + \text{Re}[z]^5 - 2\text{Re}[z]^3 - \text{Re}[z] - 10\text{Im}[z]^2\text{Re}[z]^3 + 5\text{Im}[z]^4\text{Re}[z] + 6\text{Im}[z]^2\text{Re}[z]$

Trigonometric and hyperbolic functions

>> `ComplexExpand[Cos[x + I y] + Tanh[z], {z}]`

$$\mathrm{Cos}[x]\,\mathrm{Cosh}\left[y\right] - I\mathrm{Sin}[x]\,\mathrm{Sinh}\left[y\right] + \frac{\mathrm{Cosh}[\mathrm{Re}[z]]\,\mathrm{Sinh}[\mathrm{Re}[z]]}{\mathrm{Cos}[\mathrm{Im}[z]]^2 + \mathrm{Sinh}[\mathrm{Re}[z]]^2} + \frac{I\mathrm{Cos}[\mathrm{Im}[z]]\,\mathrm{Sin}[\mathrm{Im}[z]]}{\mathrm{Cos}[\mathrm{Im}[z]]^2 + \mathrm{Sinh}[\mathrm{Re}[z]]^2}$$

Exponential and logarithmic functions:

>> `ComplexExpand[Abs[2^z Log[2 z]], z]`

$$\mathrm{Abs}\left[I\mathrm{Arg}[\mathrm{Re}[z] + I\mathrm{Im}[z]] + \frac{\mathrm{Log}\left[4\mathrm{Im}[z]^2 + 4\mathrm{Re}[z]^2\right]}{2}\right]2^{\mathrm{Re}[z]}$$

Specify that variable z is taken to be complex:

>> `ComplexExpand[Re[2 z^3 - z + 1], z]`

$$1 - \mathrm{Re}[z] + 2\mathrm{Re}[z]^3 - 6\mathrm{Im}[z]^2\,\mathrm{Re}[z]$$

31.5.8. Cosh

WMA link

`Cosh[`z`]`
returns the hyperbolic cosine of z.

>> `Cosh[0]`

1

31.5.9. Coth

WMA link

`Coth[`z`]`
returns the hyperbolic cotangent of z.

>> `Coth[0]`

ComplexInfinity

31.5.10. Gudermannian

Gudermannian function (WMA, MathWorld)

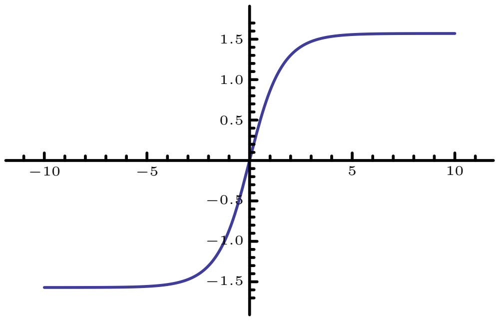
```
Gudermannian[z]
```
returns the Gudermannian function $gd(z)$.

```
>> Gudermannian[4.2]
```
1.54081

`Gudermannian[-$z$] == - Gudermannian[$z$]:`

```
>> Gudermannian[-4.2] == -Gudermannian[4.2]
```
True

```
>> Plot[Gudermannian[x], {x, -10, 10}]
```



31.5.11. InverseGudermannian

Inverse Gudermannian function (WMA, MathWorld)

```
InverseGudermannian[z]
```
returns the inverse Gudermannian function gd^-1(z).

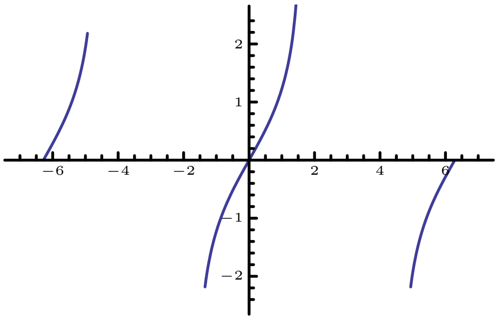
```
>> InverseGudermannian[.5]
```
0.522238

`InverseGudermannian[-$z$] == -InversGudermannian[$z$]:`

```
>> InverseGudermannian[-.5] == -InverseGudermannian[.5]
```
True

InverseGudermannian is 0 at multiples of 8 Pi: = 0

```
>> Plot[InverseGudermannian[x], {x, -2 Pi, 2 Pi}]
```



31.5.12. Sech

WMA link

`Sech[`*z*`]`
: returns the hyperbolic secant of *z*.

```
>> Sech[0]
```
1

31.5.13. Sinh

WMA link

`Sinh[`*z*`]`
: returns the hyperbolic sine of *z*.

```
>> Sinh[0]
```
0

31.5.14. Tanh

WMA link

`Tanh[`*z*`]`
: returns the hyperbolic tangent of *z*.

```
>> Tanh[0]
```
0

31.6. Integer Functions

31.6.1. BitLength

WMA link

`BitLength[`x`]`
: gives the number of bits needed to represent the integer x. x's sign is ignored.

>> `BitLength[1023]`

10

>> `BitLength[100]`

7

>> `BitLength[-5]`

3

>> `BitLength[0]`

0

31.6.2. Ceiling

WMA link

`Ceiling[`x`]`
: gives the smallest integer greater than or equal to x.

>> `Ceiling[1.2]`

2

>> `Ceiling[3/2]`

2

For complex x, take the ceiling of real an imaginary parts.

>> `Ceiling[1.3 + 0.7 I]`

$2 + I$

31.6.3. DigitCount

WMA link

`DigitCount`[n, b, d]
: returns the number of times digit d occurs in the base b representation of n.

`DigitCount`[n, b]
: returns a list indicating the number of times each digit occurs in the base b representation of n.

`DigitCount`[n, b]
: returns a list indicating the number of times each digit occurs in the decimal representation of n.

>> `DigitCount[1022]`

$\{1, 2, 0, 0, 0, 0, 0, 0, 0, 1\}$

>> `DigitCount[Floor[Pi * 10^100]]`

$\{8, 12, 12, 10, 8, 9, 8, 12, 14, 8\}$

>> `DigitCount[1022, 2]`

$\{9, 1\}$

>> `DigitCount[1022, 2, 1]`

9

31.6.4. Floor

WMA link

`Floor`[x]
: gives the greatest integer less than or equal to x.

`Floor`[x, a]
: gives the greatest multiple of a less than or equal to x.

>> `Floor[10.4]`

10

>> `Floor[10/3]`

3

>> `Floor[10]`

10

>> `Floor[21, 2]`

20

>> `Floor[2.6, 0.5]`

2.5

>> `Floor[-10.4]`

-11

For complex x, take the floor of real an imaginary parts.

```
>> Floor[1.5 + 2.7 I]
```
$1 + 2I$

For negative a, the smallest multiple of a greater than or equal to x is returned.

```
>> Floor[10.4, -1]
```
11

```
>> Floor[-10.4, -1]
```
-10

31.6.5. FromDigits

WMA link

`FromDigits[`*l*`]`
: returns the integer corresponding to the decimal representation given by l. l can be a list of digits or a string.

`FromDigits[`*l*, *b*`]`
: returns the integer corresponding to the base b representation given by l. l can be a list of digits or a string.

```
>> FromDigits["123"]
```
123

```
>> FromDigits[{1, 2, 3}]
```
123

```
>> FromDigits[{1, 0, 1}, 1000]
```
1000001

FromDigits can handle symbolic input:

```
>> FromDigits[{a, b, c}, 5]
```
$c + 5 (5a + b)$

Note that FromDigits does not automatically detect if you are providing a non-decimal representation:

```
>> FromDigits["a0"]
```
100

```
>> FromDigits["a0", 16]
```
160

FromDigits on empty lists or strings returns 0:

```
>> FromDigits[{}]
```
0

```
>> FromDigits[""]
```
0

31.6.6. IntegerDigits

WMA link

`IntegerDigits`[n]
: returns the decimal representation of integer x as list of digits. x's sign is ignored.

`IntegerDigits`[n, b]
: returns the base b representation of integer x as list of digits. x's sign is ignored.

`IntegerDigits`[n, b, $length$]
: returns a list of length $length$. If the number is too short, the list gets padded with 0 on the left. If the number is too long, the $length$ least significant digits are returned.

```
>> IntegerDigits[76543]
```
$\{7,6,5,4,3\}$

The same thing specifying base 10 explicitly:

```
>> IntegerDigits[76543, 10]
```
$\{7,6,5,4,3\}$

The sign is discarded:

```
>> IntegerDigits[-76543]
```
$\{7,6,5,4,3\}$

Just the last 3 digits:

```
>> IntegerDigits[76543, 10, 3]
```
$\{5,4,3\}$

A geeky way to relate Christmas with Halloween is to note that Dec(imal) 25 is Oct(al) 31

```
>> IntegerDigits[25, 8]
```
$\{3,1\}$

31.6.7. IntegerReverse

WMA link

`IntegerReverse`[n]
: returns the integer that has the reverse decimal representation of x without sign.

`IntegerReverse`[n, b]
: returns the integer that has the reverse base b representation of x without sign.

```
>> IntegerReverse[1234]
```
4321

```
>> IntegerReverse[1022, 2]
```
511

```
>> IntegerReverse[-123]
```
321

31.6.8. IntegerString

WMA link

`IntegerString`[n]
: returns the decimal representation of integer x as string. x's sign is ignored.

`IntegerString`[n, b]
: returns the base b representation of integer x as string. x's sign is ignored.

`IntegerString`[n, b, $length$]
: returns a string of length $length$. If the number is too short, the string gets padded with 0 on the left. If the number is too long, the $length$ least significant digits are returned.

For bases > 10, alphabetic characters a, b, ... are used to represent digits 11, 12, ... Note that base must be an integer in the range from 2 to 36.

```
>> IntegerString[12345]
```
12345

```
>> IntegerString[-500]
```
500

```
>> IntegerString[12345, 10, 8]
```
00012345

```
>> IntegerString[12345, 10, 3]
```
345

```
>> IntegerString[11, 2]
```
1011

```
>> IntegerString[123, 8]
```
173

```
>> IntegerString[32767, 16]
```
7fff

```
>> IntegerString[98765, 20]
```
c6i5

31.7. Linear algebra

31.7.1. DesignMatrix

WMA link

`DesignMatrix`[m, f, x]
    returns the design matrix for a linear model f in the variables x.

```
>> DesignMatrix[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, x, x]
```
$\{\{1,2\},\{1,3\},\{1,5\},\{1,7\}\}$

```
>> DesignMatrix[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, f[x], x]
```
$\{\{1,f[2]\},\{1,f[3]\},\{1,f[5]\},\{1,f[7]\}\}$

31.7.2. Det

Matrix Determinant (WMA link)

`Det`[m]
    computes the determinant of the matrix m.

```
>> Det[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]
```
-2

Symbolic determinant:

```
>> Det[{{a, b, c}, {d, e, f}, {g, h, i}}]
```
$aei - afh - bdi + bfg + cdh - ceg$

31.7.3. Eigensystem

Matrix Eigenvalues (WMA)

`Eigensystem`[m]
    returns the list `{Eigenvalues[$m$], Eigenvectors[$m$]}`.

>> `Eigensystem[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]`

$\{\{2, -1, 1\}, \{\{1, 1, 1\}, \{1, -2, 1\}, \{-1, 0, 1\}\}\}$

31.7.4. Eigenvalues

Matrix Eigenvalues (WMA link)

`Eigenvalues[`*m*`]`
    computes the eigenvalues of the matrix *m*.
By default, Sympy's routine is used. Sometimes this is slow and less good than the corresponding mpmath routine.
Use option Method->"mpmath" if you want to use mpmath's routine instead.

Numeric eigenvalues are sorted in order of decreasing absolute value:

>> `Eigenvalues[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]`

$\{2, -1, 1\}$

Symbolic eigenvalues:

>> `Eigenvalues[{{Cos[theta],Sin[theta],0},{-Sin[theta],Cos[theta],0},{0,0,1}}] // Sort`

$\{1, \mathrm{Cos}[\mathrm{theta}] + \sqrt{(-1 + \mathrm{Cos}[\mathrm{theta}])(1 + \mathrm{Cos}[\mathrm{theta}])}, \mathrm{Cos}[\mathrm{theta}] - \sqrt{(-1 + \mathrm{Cos}[\mathrm{theta}])(1 + \mathrm{Cos}[\mathrm{theta}])}\}$

>> `Eigenvalues[{{7, 1}, {-4, 3}}]`

$\{5, 5\}$

31.7.5. Eigenvectors

Matrix Eigenvalues (WMA link)

`Eigenvectors[`*m*`]`
    computes the eigenvectors of the matrix *m*.

>> `Eigenvectors[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]`

$\{\{1, 1, 1\}, \{1, -2, 1\}, \{-1, 0, 1\}\}$

>> `Eigenvectors[{{1, 0, 0}, {0, 1, 0}, {0, 0, 0}}]`

$\{\{0, 1, 0\}, \{1, 0, 0\}, \{0, 0, 1\}\}$

>> `Eigenvectors[{{2, 0, 0}, {0, -1, 0}, {0, 0, 0}}]`

$\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$

```
>> Eigenvectors[{{0.1, 0.2}, {0.8, 0.5}}]
```
$\{\{-0.355518, -1.15048\}, \{-0.62896, 0.777438\}\}$

31.7.6. FittedModel

WMA link

`FittedModel[...]`
Result of a linear fit

31.7.7. Inverse

WMA link

`Inverse[`m`]`
computes the inverse of the matrix m.

```
>> Inverse[{{1, 2, 0}, {2, 3, 0}, {3, 4, 1}}]
```
$\{\{-3,2,0\},\{2,-1,0\},\{1,-2,1\}\}$

```
>> Inverse[{{1, 0}, {0, 0}}]
The matrix {{1, 0}, {0, 0}} is singular.
```
Inverse $[\{\{1,0\},\{0,0\}\}]$

31.7.8. LeastSquares

WMA link

`LeastSquares[`m, b`]`
computes the least squares solution to $m\ x = b$, finding an x that solves for b optimally.

```
>> LeastSquares[{{1, 2}, {2, 3}, {5, 6}}, {1, 5, 3}]
```
$$\left\{-\frac{28}{13}, \frac{31}{13}\right\}$$

```
>> Simplify[LeastSquares[{{1, 2}, {2, 3}, {5, 6}}, {1, x, 3}]]
```
$$\left\{\frac{12}{13} - \frac{8x}{13}, -\frac{4}{13} + \frac{7x}{13}\right\}$$

```
>> LeastSquares[{{1, 1, 1}, {1, 1, 2}}, {1, 3}]
Solving for underdetermined system not implemented.
```
LeastSquares $[\{\{1,1,1\},\{1,1,2\}\},\{1,3\}]$

31.7.9. LinearModelFit

WMA link

`LinearModelFit`[m, f, x]
fits a linear model f in the variables x to the dataset m.

```
>> m = LinearModelFit[{{2, 1}, {3, 4}, {5, 3}, {7, 6}}, x, x];
```

```
>> m["BasisFunctions"]
```
$\{1, x\}$

```
>> m["BestFit"]
```
$0.186441 + 0.779661x$

```
>> m["BestFitParameters"]
```
$\{0.186441, 0.779661\}$

```
>> m["DesignMatrix"]
```
$\{\{1,2\},\{1,3\},\{1,5\},\{1,7\}\}$

```
>> m["Function"]
```
$0.186441 + 0.779661\#1\&$

```
>> m["Response"]
```
$\{1, 4, 3, 6\}$

```
>> m["FitResiduals"]
```
$\{-0.745763, 1.47458, -1.08475, 0.355932\}$

```
>> m = LinearModelFit[{{2, 2, 1}, {3, 2, 4}, {5, 6, 3}, {7, 9, 6}}, {Sin
[x], Cos[y]}, {x, y}];
```

```
>> m["BasisFunctions"]
```
$\{1, \mathrm{Sin}[x], \mathrm{Cos}[y]\}$

```
>> m["Function"]
```
$3.33077 - 5.65221\mathrm{Cos}[\#2] - 5.01042\mathrm{Sin}[\#1]\&$

```
>> m = LinearModelFit[{{{1, 4}, {1, 5}, {1, 7}}, {1, 2, 3}}];
```

```
>> m["BasisFunctions"]
```
$\{\#1, \#2\}$

```
>> m["FitResiduals"]
```
$\{-0.142857, 0.214286, -0.0714286\}$

31.7.10. LinearSolve

WMA link

`LinearSolve[`*matrix*, *right*`]`
: solves the linear equation system $matrix$. x = $right$ and returns one corresponding solution *x*.

```
>> LinearSolve[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}, {1, 2, 3}]
```
$\{0,1,2\}$

Test the solution:

```
>> {{1, 1, 0}, {1, 0, 1}, {0, 1, 1}} . {0, 1, 2}
```
$\{1,2,3\}$

If there are several solutions, one arbitrary solution is returned:

```
>> LinearSolve[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, {1, 1, 1}]
```
$\{-1,1,0\}$

Infeasible systems are reported:

```
>> LinearSolve[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, {1, -2, 3}]
Linear equation encountered that has no solution.
```
LinearSolve $[\{\{1,2,3\},\{4,5,6\},\{7,8,9\}\},\{1,-2,3\}]$

31.7.11. MatrixExp

WMA link

`MatrixExp[`*m*`]`
: computes the exponential of the matrix *m*.

```
>> MatrixExp[{{0, 2}, {0, 1}}]
```
$\{\{1,-2+2E\},\{0,E\}\}$

```
>> MatrixExp[{{1.5, 0.5}, {0.5, 2.0}}]
```
$\{\{5.16266,3.02952\},\{3.02952,8.19218\}\}$

31.7.12. MatrixPower

WMA link

`MatrixPower[`*m*, *n*`]`
: computes the *n*th power of a matrix *m*.

>> `MatrixPower[{{1, 2}, {1, 1}}, 10]`

$\{\{3363, 4756\}, \{2378, 3363\}\}$

>> `MatrixPower[{{1, 2}, {2, 5}}, -3]`

$\{\{169, -70\}, \{-70, 29\}\}$

31.7.13. MatrixRank

WMA link

`MatrixRank[`*matrix*`]`
returns the rank of *matrix*.

>> `MatrixRank[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]`

2

>> `MatrixRank[{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}]`

3

>> `MatrixRank[{{a, b}, {3 a, 3 b}}]`

1

31.7.14. NullSpace

Kernel (null space) (WMA link)

`NullSpace[`*matrix*`]`
returns a list of vectors that span the nullspace of *matrix*.

>> `NullSpace[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]`

$\{\{1, -2, 1\}\}$

>> `A = {{1, 1, 0}, {1, 0, 1}, {0, 1, 1}};`

>> `NullSpace[A]`

$\{\}$

>> `MatrixRank[A]`

3

31.7.15. PseudoInverse

WMA link

`PseudoInverse[`*m*`]`
: computes the Moore-Penrose pseudoinverse of the matrix *m*. If *m* is invertible, the pseudoinverse equals the inverse.

```
>> PseudoInverse[{{1, 2}, {2, 3}, {3, 4}}]
```

$$\left\{\left\{-\frac{11}{6},-\frac{1}{3},\frac{7}{6}\right\},\left\{\frac{4}{3},\frac{1}{3},-\frac{2}{3}\right\}\right\}$$

```
>> PseudoInverse[{{1, 2, 0}, {2, 3, 0}, {3, 4, 1}}]
```

$\{\{-3,2,0\},\{2,-1,0\},\{1,-2,1\}\}$

```
>> PseudoInverse[{{1.0, 2.5}, {2.5, 1.0}}]
```

$\{\{-0.190476,0.47619\},\{0.47619,-0.190476\}\}$

31.7.16. QRDecomposition

QR Decomposition (WMA link)

`QRDecomposition[`*m*`]`
: computes the QR decomposition of the matrix *m*.

```
>> QRDecomposition[{{1, 2}, {3, 4}, {5, 6}}]
```

$$\left\{\left\{\left\{\frac{\sqrt{35}}{35},\frac{3\sqrt{35}}{35},\frac{\sqrt{35}}{7}\right\},\left\{\frac{13\sqrt{210}}{210},\frac{2\sqrt{210}}{105},-\frac{\sqrt{210}}{42}\right\}\right\},\left\{\left\{\sqrt{35},\frac{44\sqrt{35}}{35}\right\},\left\{0,\frac{2\sqrt{210}}{35}\right\}\right\}\right\}$$

31.7.17. RowReduce

WMA link

`RowReduce[`*matrix*`]`
: returns the reduced row-echelon form of *matrix*.

```
>> RowReduce[{{1, 0, a}, {1, 1, b}}]
```

$\{\{1,0,a\},\{0,1,-a+b\}\}$

```
>> RowReduce[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

31.7.18. SingularValueDecomposition

Singular Value Decomposition (WMA link)

`SingularValueDecomposition[`*m*`]`
: calculates the singular value decomposition for the matrix *m*.

`SingularValueDecomposition` returns *u*, *s*, *w* such that *m*=*u s v*, *u*u=1, v*v*=1, and *s* is diagonal.

```
>> SingularValueDecomposition[{{1.5, 2.0}, {2.5, 3.0}}]
```
$\{\{\{0.538954, 0.842335\}, \{0.842335, -0.538954\}\}, \{\{4.63555, 0.\}, \{0., 0.107862\}\}, \{\{0.628678, 0.777666\}, \{-0.777666, 0.628678\}\}\}$

31.7.19. Tr

Matrix trace (WMA link)

`Tr[`*m*`]`
: computes the trace of the matrix *m*.

```
>> Tr[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
```
15

Symbolic trace:

```
>> Tr[{{a, b, c}, {d, e, f}, {g, h, i}}]
```
$a + e + i$

31.8. Mathematical Constants

Numeric, Arithmetic, or Symbolic constants like Pi, E, or Infinity.

31.8.1. Catalan

Catalan's constant (SymPy, WMA)

`Catalan`
: is Catalan's constant with numerical value 0.915966.

```
>> Catalan // N
```
0.915966

```
>> N[Catalan, 20]
```
0.91596559417721901505

31.8.2. ComplexInfinity

Complex Infinity is an infinite number in the complex plane whose complex argument is unknown or undefined. (SymPy, MathWorld, WMA)

`ComplexInfinity`
: represents an infinite complex quantity of undetermined direction.

ComplexInfinity can appear as the result of a computation such as dividing by zero:

```
>> 1 / 0
Infinite expression 1 / 0 encountered.
```
ComplexInfinity

But it can be used as an explicit value in an expression:

```
>> 1 / ComplexInfinity
```
0

```
>> ComplexInfinity * Infinity
```
ComplexInfinity

ComplexInfinity though is a special case of DirectedInfinity:

```
>> FullForm[ComplexInfinity]
```
DirectedInfinity []

See also 'DirectedInfinity' 37.8.

31.8.3. Degree

Degree (angle) (WMA)

`Degree`
: is the number of radians in one degree. It has a numerical value of π / 180.

```
>> Cos[60 Degree]
```
$\frac{1}{2}$

Degree has the value of Pi / 180

```
>> Degree == Pi / 180
```
True

```
>> N[\[Degree]] == N[Degree]
```
True

31.8.4. E

Euler's number (SymPy, WMA)

`E`
    is the constant with numerical value 2.71828.

```
>> N[E]
```
2.71828

```
>> N[E, 50]
```
2.7182818284590452353602874713526624977572470937000

31.8.5. EulerGamma

Euler's constant (SymPy, WMA)

`EulerGamma`
    is Euler's constant γ with numerical value 0.577216.

```
>> EulerGamma // N
```
0.577216

```
>> N[EulerGamma, 40]
```
0.5772156649015328606065120900824024310422

31.8.6. Glaisher

Glaisher–Kinkelin constant (mpmath, WMA)

`Glaisher`
    is Glaisher's constant, with numerical value 1.28243.

```
>> N[Glaisher]
```
1.28243

```
>> N[Glaisher, 50]
```
1.2824271291006226368753425688697917277676889273250

1.2824271291006219541941391071304678916931152343750

31.8.7. GoldenRatio

Golden ratio (mpmath, WMA)

`GoldenRatio`
  is the golden ratio, ϕ = (1+Sqrt[5])/2.

```
>> GoldenRatio // N
   1.61803
>> N[GoldenRatio, 40]
   1.618033988749894848204586834365638117720
```

31.8.8. Indeterminate

Indeterminate form (SymPy, WMA)

`Indeterminate`
  represents an indeterminate result.

```
>> 0^0
   Indeterminate expression 0 ^ 0 encountered.
   Indeterminate
>> Tan[Indeterminate]
   Indeterminate
```

31.8.9. Infinity

Infinity (SymPy, WMA)

`Infinity`
  a symbol that represents an infinite real quantity.

`Infinity` sometimes appears as the result of a calculation:

```
>> Precision[1]
   ∞
```

But `Infinity` it often used as a value in expressions:

```
>> 1 / Infinity
```
0

```
>> Infinity + 100
```
∞

`Infinity` often appears in sum and limit calculations:

```
>> Sum[1/x^2, {x, 1, Infinity}]
```
$\frac{\pi^2}{6}$

```
>> Limit[1/x, x->0]
```
$-\infty$

However, `Infinity` a shorthand for `DirectedInfinity[1]` :

```
>> FullForm[Infinity]
```
DirectedInfinity [1]

See also 'DirectedInfinity' 37.8.

31.8.10. Khinchin

Khinchin's constant (mpmath, WMA)

`Khinchin`
is Khinchin's constant, with numerical value 2.68545.

```
>> N[Khinchin]
```
2.68545

```
>> N[Khinchin, 50]
```
2.6854520010653064453097148354817956938203822939945

= 2.6854520010653075701156922150403261184692382812500

31.8.11. $MaxMachineNumber

Largest normalizable machine number (WMA)

`$MaxMachineNumber`
Represents the largest positive number that can be represented as a normalized machine number in the system.

The product of `$MaxMachineNumber` and `$MinMachineNumber` is a constant:

```
>> $MaxMachineNumber * $MinMachineNumber
```
4.

31.8.12. $MinMachineNumber

Smallest normalizable machine number (WMA)

`$MinMachineNumber`
: Represents the smallest positive number that can be represented as a normalized machine number in the system.

`MachinePrecision` minus the `Log` base 10 of this number is the `Accuracy` of 0':

```
>> MachinePrecision -Log[10., $MinMachineNumber]==Accuracy[0`]
```
True

31.8.13. Overflow

Numeric Overflow (WMA)

See also Integer Overflow.

`Overflow[]`
: represents a number too large to be represented by Mathics.

```
>> Exp[10.*^20]
Overflow occurred in computation.
```
Overflow []

```
>> Table[Exp[10.^k],{k, 3}]
Overflow occurred in computation.
```
{22026.5, 2.68812*^43, Overflow []}

```
>> 1 / Underflow[]
```
Overflow []

31.8.14. Pi

Pi, π (SymPy, WMA)

`Pi`
: is the constant π.

>> `Pi`
π

>> `N[Pi]`
3.14159

Pi to a numeric precision of 20 digits:

>> `N[Pi, 20]`
3.1415926535897932385

Note that the above is not the same thing as the number of digits *after* the decimal point. This may differ from similar concepts from other mathematical libraries, including those which Mathics uses!

Use numpy to compute Pi to 20 digits:

>> `N[Pi, 20, Method->"numpy"]`
3.1415926535897930000

"sympy" is the default method.

>> `Attributes[Pi]`
{Constant, Protected, ReadProtected}

31.8.15. Undefined

Undefined symbol/value (WMA)

`Undefined`
    a symbol that represents a quantity with no defined value.

>> `ConditionalExpression[a, False]`
Undefined

>> `Attributes[Undefined]`
{Protected}

31.8.16. Underflow

Arithmetic underflow (WMA)

`Overflow[]`
    represents a number too small to be represented by Mathics.

>> `1 / Overflow[]`
Underflow []

```
>> 5 * Underflow[]
```
5Underflow []

```
>> % // N
```
0.

Underflow[] is kept symbolic in operations against integer numbers, but taken as 0. in numeric evaluations:

```
>> 1 - Underflow[]
```
1 − Underflow []

```
>> % // N
```
1.

31.9. Number theoretic functions

31.9.1. ContinuedFraction

Continued fraction (SymPy, WMA)

`ContinuedFraction`[x, n]
: generate the first n terms in the continued fraction representation of x.

`ContinuedFraction`[x]
: the complete continued fraction representation for a rational or quadratic irrational number.

```
>> ContinuedFraction[Pi, 10]
```
$\{3,7,15,1,292,1,1,1,2,1\}$

```
>> ContinuedFraction[(1 + 2 Sqrt[3])/5]
```
$\{0,1,\{8,3,34,3\}\}$

```
>> ContinuedFraction[Sqrt[70]]
```
$\{8,\{2,1,2,1,2,16\}\}$

31.9.2. DivisorSigma

Divisor function (SymPy, WMA)

`DivisorSigma`[k, n]
: returns $\sigma_k(n)$

For reference, let us first get the integer divisors of 20:

```
>> Divisors[20]
```
$\{1, 2, 4, 5, 10, 20\}$

The DivisorSigma function counts this sum:

```
>> DivisorSigma[1, 20]
```
42

This is the same thing as:

```
>> DivisorSum[20, # &]
```
42

To get a sum of the second power of the factors of 20:

```
>> DivisorSigma[2, 20]
```
546

Doing this with `DivisorSum` instead:

```
>> DivisorSum[20, #^2 &]
```
546

See also 'DivisorSum' 31.9.3 and Divisors 31.9.4.

31.9.3. DivisorSum

WMA

`DivisorSum[`n, $form$`]`
: transform the divisors of n using $form$ and take their sum

```
>> DivisorSum[20, # &]
```
42

```
>> DivisorSum[20, #^2 &]
```
546

See also 'DivisorSigma' 31.9.2 and Divisors 31.9.4.

31.9.4. Divisors

WMA link

`Divisors[`n`]`
: returns a list of the integers that divide n.

```
>> Divisors[20]
```
$\{1, 2, 4, 5, 10, 20\}$

```
>> Divisors[704]
```
$\{1, 2, 4, 8, 11, 16, 22, 32, 44, 64, 88, 176, 352, 704\}$

```
>> Divisors[{87, 106, 202, 305}]
```
$\{\{1, 3, 29, 87\}, \{1, 2, 53, 106\}, \{1, 2, 101, 202\}, \{1, 5, 61, 305\}\}$

See also 'DivisorSigma' 31.9.2 and DivisorSum 31.9.3.

31.9.5. EulerPhi

Euler's totient function (SymPy, WMA) This function counts positive integers up to n that are relatively prime to n. It is typically used in cryptography and in many applications in elementary number theory.

`EulerPhi[`n`]`
    returns the Euler totient function .

Compute the Euler totient function:

```
>> EulerPhi[9]
```
6

`EulerPhi` of a negative integer is same as its positive counterpart:

```
>> EulerPhi[-11] == EulerPhi[11]
```
True

```
>> EulerPhi[0]
```
0

Large arguments are computed quickly:

```
>> EulerPhi[40!]
```
121343746763281707274905415180804423680000000000

`EulerPhi` threads over lists:

```
>> EulerPhi[Range[1, 17, 2]]
```
$\{1, 2, 4, 6, 6, 10, 12, 8, 16\}$

Above, we get consecutive even numbers when the input is prime.

Compare the results above with:

```
>> EulerPhi[Range[1, 17]]
```
$\{1, 1, 2, 2, 4, 2, 6, 4, 6, 4, 10, 4, 12, 6, 8, 8, 16\}$

31.9.6. FactorInteger

WMA link

`FactorInteger[`*n*`]`
: returns the factorization of *n* as a list of factors and exponents.

```
>> factors = FactorInteger[2010]
```
{{2,1},{3,1},{5,1},{67,1}}

To get back the original number:

```
>> Times @@ Power @@@ factors
```
2010

`FactorInteger` factors rationals using negative exponents:

```
>> FactorInteger[2010 / 2011]
```
{{2,1},{3,1},{5,1},{67,1},{2011,−1}}

31.9.7. FractionalPart

WMA link

`FractionalPart[`*n*`]`
: finds the fractional part of *n*.

```
>> FractionalPart[4.1]
```
0.1

```
>> FractionalPart[-5.25]
```
− 0.25

31.9.8. FromContinuedFraction

WMA link

`FromContinuedFraction[`*list*`]`
: reconstructs a number from the list of its continued fraction terms.

```
>> FromContinuedFraction[{3, 7, 15, 1, 292, 1, 1, 1, 2, 1}]
```
$$\frac{1146408}{364913}$$

```
>> FromContinuedFraction[Range[5]]
```
$\frac{225}{157}$

31.9.9. IntegerPart

WMA link

`IntegerPart[`n`]`
: finds the integer part of n.

```
>> IntegerPart[4.1]
```
4

```
>> IntegerPart[-5.25]
```
-5

31.9.10. IntegerPartitions

Integer partition (SymPy, WMA)

`IntegerPartitions[`n`]`
: lists all possible ways to partition integer n into smaller integers.

`IntegerPartitions[`n, k`]`
: lists all partitions into at most k integers.

`IntegerPartitions[`n, {k}`]`
: lists all partitions with exactly k integers.

`IntegerPartitions[`n, {k_{min}, k_{max}}`]`
: lists partitions between k_{min} and k_{max} integers.

`IntegerPartitions[`n, $kspec$, {s_1, s_2, ...}`]`
: lists partitions involving only the s_i.

All partitions of positive integers that add to 5:

```
>> IntegerPartitions[5]
```
$\{\{5\},\{4,1\},\{3,2\},\{3,1,1\},\{2,2,1\},\{2,1,1,1\},\{1,1,1,1,1\}\}$

Limit the above to just the first 3 elements:

```
>> IntegerPartitions[5, All, All, 3]
```
$\{\{5\},\{4,1\},\{3,2\}\}$

Partitions of 5 with at most 3 integers:

```
>> IntegerPartitions[5, 3]
```
$\{\{5\},\{4,1\},\{3,2\},\{3,1,1\},\{2,2,1\}\}$

Partitions of 5 with exactly 3 integers; this is a subset of "at most 3" above:

```
>> IntegerPartitions[5, {3}]
```
$\{\{3,1,1\},\{2,2,1\}\}$

Partitions of 5 that involve only integers 1, and 2:

```
>> IntegerPartitions[5, All, {1, 2}]
```
$\{\{2,2,1\},\{2,1,1,1\},\{1,1,1,1,1\}\}$

Partitions of 4 with exactly 2 elements and involve only integers -1, 0, 1, 4, and 5:

```
>> IntegerPartitions[4, {2}, {-1, 0, 1, 4, 5}]
```
$\{\{5,-1\},\{4,0\}\}$

31.9.11. JacobiSymbol

Jacobi symbol (WMA)

`JacobiSymbol[`a, n`]`
returns the Jacobi symbol (a/n).

```
>> Table[JacobiSymbol[n, m], {n, 0, 10}, {m, 1, n, 2}]
```
$\{\{\},\{1\},\{1\},\{1,0\},\{1,1\},\{1,-1,0\},\{1,0,1\},\{1,1,-1,0\},\{1,-1,-1,1\},\{1,0,1,1,0\},\{1,1,0,-1,1\}\}$

31.9.12. KroneckerSymbol

Kronecker symbol (WMA)

`KroneckerSymbol[`a, n`]`
returns the Kronecker symbol (a/n).

```
>> Table[KroneckerSymbol[n, m], {n, 5}, {m, 5}]
```
$\{\{1,1,1,1,1\},\{1,0,-1,0,-1\},\{1,-1,0,1,-1\},\{1,0,1,0,1\},\{1,-1,-1,1,0\}\}$

31.9.13. MantissaExponent

WMA link

`MantissaExponent`[n]
finds a list containing the mantissa and exponent of a given number n.
`MantissaExponent`[n, b]
finds the base b mantissa and exponent of n.

>> **`MantissaExponent[2.5*10^20]`**
$\{0.25, 21\}$

>> **`MantissaExponent[125.24]`**
$\{0.12524, 3\}$

>> **`MantissaExponent[125., 2]`**
$\{0.976563, 7\}$

>> **`MantissaExponent[10, b]`**
MantissaExponent $[10, b]$

31.9.14. MersennePrimeExponent

Mersenne Prime exponent (SymPy, WMA)

`MersennePrimeExponent`[n]
returns the exponent of the n-th Mersenne prime.

>> **`Table[MersennePrimeExponent[n], {n, 10}]`**
$\{2, 3, 5, 7, 13, 17, 19, 31, 61, 89\}$

31.9.15. MoebiusMu

Mobius function (SymPy, WMA)

`MoebiusMu`[n]
returns $\mu(n)$.

>> **`Array[MoebiusMu, 10]`**
$\{1, -1, -1, 0, -1, 1, -1, 0, 0, 1\}$

31.9.16. NextPrime

WMA link

`NextPrime[`n`]`
: gives the next prime after n.

`NextPrime[`n,k`]`
: gives the kth prime after n.

```
>> NextPrime[100]
```
101

The the first number does not have to be an integer:

```
>> NextPrime[100.5, 2]
```
103

However, when the second value, the step value is not an integer is given, we do nothing:

```
>> NextPrime[100, 2.5]
```
NextPrime [100, 2.5]

With a negative number, we find a prime number *before* the given number:

```
>> NextPrime[100, -1]
```
97

And with negative counts, it is possible to get *negative* prime numbers:

```
>> NextPrime[2, -1]
```
-2

31.9.17. PartitionsP

WMA link

`PartitionsP[`n`]`
: return the number $p(n)$ of unrestricted partitions of the integer n.

```
>> Table[PartitionsP[k], {k, -2, 12}]
```
$\{0, 0, 1, 1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77\}$

31.9.18. PowersRepresentations

WMA

`PowersRepresentations`[n, k, p]
: represent n as a sum of k non-negative integers raised to the power of p.

Get the ways licence plate number 1729 can be represented as the sum of two cubes:

```
>> PowersRepresentations[1729, 2, 3]
```
$\{\{1,12\},\{9,10\}\}$

See 1729 for the full backstory.

Demonstrate the validity of the Pythagorian triple: 3^2 + 4^2 == 5^2

```
>> PowersRepresentations[25, 2, 2]
```
$\{\{0,5\},\{3,4\}\}$

Since 0 is allowed in the sum, `PowersRepresentations[$n$, $k$+1, $p$]` is includes `PowersRepresentations [$n$, $k$, $p$]` with by inserting a zero element at the beginning:

```
>> PowersRepresentations[25, 3, 2]
```
$\{\{0,0,5\},\{0,3,4\}\}$

31.9.19. Prime

WMA link

`Prime`[n]
`Prime`[{n_0, n_1, ...}]
: returns the nth prime number where n is an positive Integer. If given a list of integers, the return value is a list with `Prime` applied to each.

Note that the first prime is 2, not 1:

```
>> Prime[1]
```
2

```
>> Prime[167]
```
991

When given a list of integers, a list is returned:

```
>> Prime[{5, 10, 15}]
```
$\{11,29,47\}$

1.2 isn't an integer

```
>> Prime[1.2]
```
Prime [1.2]

Since 0 is less than 1, like 1.2 it is invalid.

```
>> Prime[{0, 1, 1.2, 3}]
```
{Prime [0] , 2, Prime [1.2] , 5}

31.9.20. PrimePi

Prime numbers

`PrimePi[`*x*`]`
: gives the number of primes less than or equal to *x*.

PrimePi is the inverse of Prime:

```
>> PrimePi[2]
```
1

```
>> PrimePi[100]
```
25

```
>> PrimePi[-1]
```
0

```
>> PrimePi[3.5]
```
2

```
>> PrimePi[E]
```
1

31.9.21. PrimePowerQ

Prime numbers

`PrimePowerQ[`*n*`]`
: returns `True` if *n* is a power of a prime number.

```
>> PrimePowerQ[9]
```
True

```
>> PrimePowerQ[52142]
```
False

```
>> PrimePowerQ[-8]
```
True

```
>> PrimePowerQ[371293]
```
True

31.9.22. RandomPrime

Prime numbers

`RandomPrime[{`i_{min}, i_{max}`}]`
: gives a random prime between i_{min} and i_{max}.

`RandomPrime[`i_{max}`]`
: gives a random prime between 2 and i_{max}.

`RandomPrime[`*range*, n`]`
: gives a list of n random primes in *range*.

```
>> RandomPrime[{14, 17}]
```
17

```
>> RandomPrime[{14, 16}, 1]
There are no primes in the specified interval.
```
RandomPrime[{14, 16}, 1]

```
>> RandomPrime[{8,12}, 3]
```
$\{11, 11, 11\}$

```
>> RandomPrime[{10,30}, {2,5}]
```
$\{\{29, 29, 29, 29, 29\}, \{29, 29, 29, 29, 29\}\}$

31.9.23. SquaresR

Sum of squares function (WMA)

`SquaresR[`d, n`]`
: returns the number of ways to represent n as a sum of d squares.

```
>> Table[SquaresR[2, n], {n, 10}]
```
$\{4, 4, 0, 4, 8, 0, 0, 4, 4, 8\}$

```
>> Table[Sum[SquaresR[2, k], {k, 0, n^2}], {n, 5}]
```
$\{5, 13, 29, 49, 81\}$

```
>> Table[SquaresR[4, n], {n, 10}]
```
$\{8, 24, 32, 24, 48, 96, 64, 24, 104, 144\}$

```
>> Table[SquaresR[6, n], {n, 10}]
```
$\{12, 60, 160, 252, 312, 544, 960, 1020, 876, 1560\}$

```
>> Table[SquaresR[8, n], {n, 10}]
```
$\{16, 112, 448, 1136, 2016, 3136, 5504, 9328, 12112, 14112\}$

31.10. Random number generation

Random numbers are generated using the Mersenne Twister.

31.10.1. Random

Randomness (WMA link)

`Random[]`
: gives a pseudorandom real number in the range 0 to 1.

`Random[`*type*, *range*`]`
: gives a pseudorandom number of the type *type*, in the specified interval *range*. Possible types are `Integer`, `Real` or `Complex`.

Legacy function. Superseded by RandomReal 31.10.5, RandomInteger 31.10.4, and RandomComplex 31.10.3.

Four random numbers in the range 0..1:

```
>> Table[Random[], {4}]
```
$\{0.152343, 0.963858, 0.962405, 0.156925\}$

Eight random integers in the range 1..100:

```
>> Table[Random[Integer, {1, 100}], {8}]
```
$\{36, 100, 69, 47, 71, 69, 20, 36\}$

31.10.2. RandomChoice

WMA link

`RandomChoice[`*items*`]`
: randomly picks one item from *items*.

`RandomChoice[`*items*, *n*`]`
: randomly picks *n* items from *items*. Each pick in the *n* picks happens from the given set of *items*, so each item can be picked any number of times.

`RandomChoice[`*items*, {n_1, n_2, ...}`]`
: randomly picks items from *items* and arranges the picked items in the nested list structure described by {n_1, n_2, ...}.

`RandomChoice[`*weights* -> *items*, *n*`]`
: randomly picks *n* items from *items* and uses the corresponding numeric values in *weights* to determine how probable it is for each item in *items* to get picked (in the long run, items with higher weights will get picked more often than ones with lower weight).

`RandomChoice[`*weights* -> *items*`]`
: randomly picks one items from *items* using weights *weights*.

`RandomChoice[`*weights* -> *items*, {n_1, n_2, ...}`]`
: randomly picks a structured list of items from *items* using weights *weights*.

Note: `SeedRandom` is used below so we get repeatable "random" numbers that we can test.

```
>> SeedRandom[42]
```

```
>> RandomChoice[{a, b, c}]
```
$\{c\}$

```
>> SeedRandom[42] (* Set for repeatable randomness *)
```

```
>> RandomChoice[{a, b, c}, 20]
```
$\{c,a,c,c,a,a,c,b,c,c,c,c,a,c,b,a,b,b,b,b\}$

```
>> SeedRandom[42]
```

```
>> RandomChoice[{"a", {1, 2}, x, {}}, 10]
```
$\{x,\{\},\mathrm{a},x,x,\{\},\mathrm{a},\mathrm{a},x,\{1,2\}\}$

```
>> SeedRandom[42]
```

```
>> RandomChoice[{a, b, c}, {5, 2}]
```
$\{\{c,a\},\{c,c\},\{a,a\},\{c,b\},\{c,c\}\}$

```
>> SeedRandom[42]
```

```
>> RandomChoice[{1, 100, 5} -> {a, b, c}, 20]
```
$\{b,b,b,b,b,b,b,b,b,b,b,c,b,b,b,b,b,b,b,b\}$

31.10.3. RandomComplex

WMA link

RandomComplex[{z_{min}, z_{max}}]
: yields a pseudorandom complex number in the rectangle with complex corners z_{min} and z_{max}.

RandomComplex[z_{max}]
: yields a pseudorandom complex number in the rectangle with corners at the origin and at z_{max}.

RandomComplex[]
: yields a pseudorandom complex number with real and imaginary parts from 0 to 1.

RandomComplex[*range*, *n*]
: gives a list of *n* pseudorandom complex numbers.

RandomComplex[*range*, {n_1, n_2, ...}]
: gives a nested list of pseudorandom complex numbers.

```
>> RandomComplex[]
```
$0.600716 + 0.927319I$

```
>> RandomComplex[{1+I, 5+5I}]
```
$1.87984 + 1.23754I$

```
>> RandomComplex[1+I, 5]
```
$\{0.546229 + 0.0527457I, 0.335621 + 0.1937I, 0.905773 + 0.15793I, 0.777219 + 0.280257I, 0.775958 + 0.739345I\}$

```
>> RandomComplex[{1+I, 2+2I}, {2, 2}]
```
$\{\{1.05763 + 1.36355I, 1.33308 + 1.31125I\}, \{1.43273 + 1.61359I, 1.98435 + 1.49489I\}\}$

31.10.4. RandomInteger

WMA link

RandomInteger[{*min*, *max*}]
: yields a pseudorandom integer in the range from *min* to *max* inclusive.

RandomInteger[*max*]
: yields a pseudorandom integer in the range from 0 to *max* inclusive.

RandomInteger[]
: gives 0 or 1.

RandomInteger[*range*, *n*]
: gives a list of *n* pseudorandom integers.

RandomInteger[*range*, {n_1, n_2, ...}]
: gives a nested list of pseudorandom integers.

```
>> RandomInteger[{1, 5}]
```
4

```
>> RandomInteger[100, {2, 3}] // TableForm
```

45	62	51
54	17	40

Calling `RandomInteger` changes `$RandomState`:

```
>> previousState = $RandomState;
```

```
>> RandomInteger[]
```
0

```
>> $RandomState != previousState
```
True

31.10.5. RandomReal

WMA link

`RandomReal[{`*min*, *max*`}]`
: yields a pseudorandom real number in the range from *min* to *max*.

`RandomReal[`*max*`]`
: yields a pseudorandom real number in the range from 0 to *max*.

`RandomReal[]`
: yields a pseudorandom real number in the range from 0 to 1.

`RandomReal[`*range*, *n*`]`
: gives a list of *n* pseudorandom real numbers.

`RandomReal[`*range*, {n_1, n_2, ...}`]`
: gives an n_1 x n_2 array of pseudorandom real numbers.

```
>> RandomReal[]
```
0.789943

```
>> RandomReal[{1, 5}]
```
4.72288

31.10.6. RandomSample

WMA link

`RandomSample[`*items*`]`
: randomly picks one item from *items*.

`RandomSample[`*items*, *n*`]`
: randomly picks *n* items from *items*. Each pick in the *n* picks happens after the previous items picked have been removed from *items*, so each item can be picked at most once.

`RandomSample[`*items*, {n_1, n_2, ...}`]`
: randomly picks items from *items* and arranges the picked items in the nested list structure described by {n_1, n_2, ...}. Each item gets picked at most once.

`RandomSample[`*weights* -> *items*, *n*`]`
: randomly picks *n* items from *items* and uses the corresponding numeric values in *weights* to determine how probable it is for each item in *items* to get picked (in the long run, items with higher weights will get picked more often than ones with lower weight). Each item gets picked at most once.

`RandomSample[`*weights* -> *items*`]`
: randomly picks one items from *items* using weights *weights*. Each item gets picked at most once.

`RandomSample[`*weights* -> *items*, {n_1, n_2, ...}`]`
: randomly picks a structured list of items from *items* using weights *weights*. Each item gets picked at most once.

```
>> SeedRandom[42]
```

```
>> RandomSample[{a, b, c, d}]
```
$\{b, d, a, c\}$

```
>> SeedRandom[42]
```

```
>> RandomSample[{a, b, c, d, e, f, g, h}, 7]
```
$\{b, f, a, h, c, e, d\}$

```
>> SeedRandom[42]
```

```
>> RandomSample[{"a", {1, 2}, x, {}}, 3]
```
{{1,2} , {} , a}

```
>> SeedRandom[42]
```

```
>> RandomSample[Range[10]]
```
$\{9, 2, 6, 1, 8, 3, 10, 5, 4, 7\}$

```
>> SeedRandom[42]
```

```
>> RandomSample[Range[100], {2, 3}]
```
$\{\{84, 54, 71\}, \{46, 45, 40\}\}$

```
>> SeedRandom[42]
```

```
>> RandomSample[Range[100] -> Range[100], 5]
```
$\{62, 98, 86, 78, 40\}$

31.10.7. $RandomState

WMA link

`$RandomState`
: is a long number representing the internal state of the pseudo-random number generator.

```
>> Mod[$RandomState, 10^100]
```
8427713147997707315537988391814441438328804893015959100816005678284625982854741574294638579078698030

```
>> IntegerLength[$RandomState]
```
6442

So far, it is not possible to assign values to `$RandomState`.

```
>> $RandomState = 42
It is not possible to change the random state.
```
42

Not even to its own value:

```
>> $RandomState = $RandomState;
It is not possible to change the random state.
```

31.10.8. SeedRandom

WMA link

`SeedRandom[`*n*`]`
: resets the pseudorandom generator with seed n.

`SeedRandom[]`
: uses the current date and time as the seed.

`SeedRandom` can be used to get reproducible random numbers:

```
>> SeedRandom[42]
```

```
>> RandomInteger[100]
```
51

```
>> RandomInteger[100]
```
92

```
>> SeedRandom[42]
```

```
>> RandomInteger[100]
```
51

```
>> RandomInteger[100]
```
92

String seeds are supported as well:

```
>> SeedRandom["Mathics"]
```

```
>> RandomInteger[100]
```

27

Calling SeedRandom without arguments will seed the random number generator to a random state:

```
>> SeedRandom[]
```

```
>> RandomInteger[100]
```

48

31.11. Trigonometric Functions

Numerical values and derivatives can be computed; however, most special exact values and simplification rules are not implemented yet.

31.11.1. AnglePath

WMA link

AnglePath[{ϕ_1, ϕ_2, ...}]
: returns the points formed by a turtle starting at {0, 0} and angled at 0 degrees going through the turns given by angles ϕ_1, ϕ_2, ... and using distance 1 for each step.

AnglePath[{{r_1, ϕ_1}, {r_2, ϕ_2}, ...}]
: instead of using 1 as distance, use r_1, r_2, ... as distances for the respective steps.

AnglePath[ϕ_0, {ϕ_1, ϕ_2, ...}]
: starts with direction ϕ_0 instead of 0.

AnglePath[{x, y}, {ϕ_1, ϕ_2, ...}]
: starts at {x, y} instead of {0, 0}.

AnglePath[{{x, y}, ϕ_0}, {ϕ_1, ϕ_2, ...}]
: specifies initial position {x, y} and initial direction ϕ_0.

AnglePath[{{x, y}, {d_x, d_y}}, {ϕ_1, ϕ_2, ...}]
: specifies initial position {x, y} and a slope {d_x, d_y} that is understood to be the initial direction of the turtle.

```
>> AnglePath[{90 Degree, 90 Degree, 90 Degree, 90 Degree}]
```

$\{\{0,0\},\{0,1\},\{-1,1\},\{-1,0\},\{0,0\}\}$

```
>> AnglePath[{{1, 1}, 90 Degree}, {{1, 90 Degree}, {2, 90 Degree}, {1,
   90 Degree}, {2, 90 Degree}}]
```

$\{\{1,1\},\{0,1\},\{0,-1\},\{1,-1\},\{1,1\}\}$

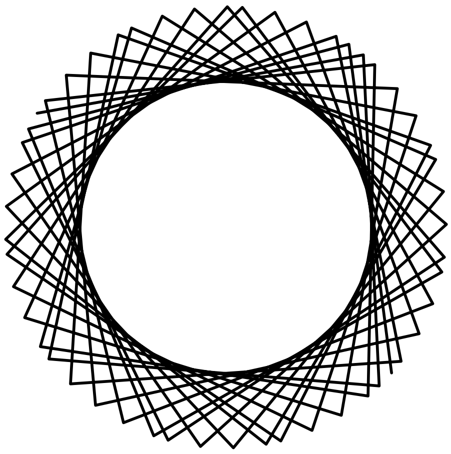
```
>> AnglePath[{a, b}]
```

$\{\{0,0\},\{\mathrm{Cos}[a],\mathrm{Sin}[a]\},\{\mathrm{Cos}[a]+\mathrm{Cos}[a+b],\mathrm{Sin}[a]+\mathrm{Sin}[a+b]\}\}$

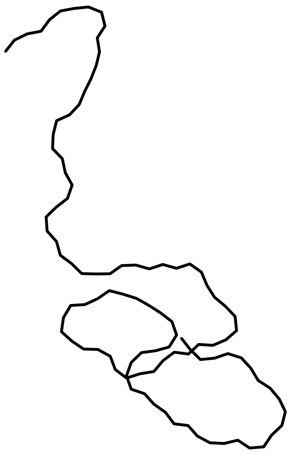
>> `Precision[Part[AnglePath[{N[1/3, 100], N[2/3, 100]}], 2, 1]]`

100.

>> `Graphics[Line[AnglePath[Table[1.7, {50}]]]]`



>> `Graphics[Line[AnglePath[RandomReal[{-1, 1}, {100}]]]]`



31.11.2. ArcCos

Inverse cosine, arccosine (SymPy, mpmath, WMA)

`ArcCos[`z`]`
: returns the inverse cosine of z.

>> `ArcCos[1]`

0

>> `ArcCos[0]`

$$\frac{\pi}{2}$$

>> `Integrate[ArcCos[x], {x, -1, 1}]`

π

31.11.3. ArcCot

Inverse cotangent, arccotangent (SymPy, mpmath, WMA)

`ArcCot[`z`]`
: returns the inverse cotangent of z.

```
>> ArcCot[0]
```
$\frac{\pi}{2}$

```
>> ArcCot[1]
```
$\frac{\pi}{4}$

31.11.4. ArcCsc

Inverse cosecant, arccosecant (SymPy, mpmath, WMA)

`ArcCsc[`z`]`
: returns the inverse cosecant of z.

```
>> ArcCsc[1]
```
$\frac{\pi}{2}$

```
>> ArcCsc[-1]
```
$-\frac{\pi}{2}$

31.11.5. ArcSec

Inverse secant, arcsecant (SymPy, mpmath, WMA)

`ArcSec[`z`]`
: returns the inverse secant of z.

```
>> ArcSec[1]
```
0

```
>> ArcSec[-1]
```
π

31.11.6. ArcSin

Inverse sine, arcsine (SymPy, mpmath, WMA)

`ArcSin[`z`]`
: returns the inverse sine of z.

```
>> ArcSin[0]
```
0

```
>> ArcSin[1]
```
$\frac{\pi}{2}$

31.11.7. ArcTan

Inverse tangent, arctangent (SymPy, mpmath, WMA)

`ArcTan[`z`]`
: returns the inverse tangent of z.

```
>> ArcTan[1]
```
$\frac{\pi}{4}$

```
>> ArcTan[1.0]
```
0.785398

```
>> ArcTan[-1.0]
```
-0.785398

```
>> ArcTan[1, 1]
```
$\frac{\pi}{4}$

31.11.8. Cos

Cosine (SymPy, mpmath, WMA)

`Cos[`z`]`
: returns the cosine of z.

```
>> Cos[3 Pi]
```
-1

31.11.9. Cot

Cotangent (SymPy, mpmath, WMA)

`Cot`[z]
: returns the cotangent of z.

>> `Cot[0]`
ComplexInfinity

>> `Cot[1.]`
0.642093

31.11.10. Csc

Cosecant (SymPy, mpmath, WMA)

`Csc`[z]
: returns the cosecant of z.

>> `Csc[0]`
ComplexInfinity

>> `Csc[1] (* Csc[1] in Mathematica *)`
$\frac{1}{\text{Sin}[1]}$

>> `Csc[1.]`
1.1884

31.11.11. Haversine

WMA link

`Haversine`[z]
: returns the haversine function of z.

>> `Haversine[1.5]`
0.464631

>> `Haversine[0.5 + 2I]`
$-1.15082 + 0.869405I$

31.11.12. InverseHaversine

WMA link

`InverseHaversine[`z`]`
: returns the inverse haversine function of z.

```
>> InverseHaversine[0.5]
```
1.5708

```
>> InverseHaversine[1 + 2.5 I]
```
$1.76459 + 2.33097I$

31.11.13. Sec

Secant (SymPy, mpmath, WMA)

`Sec[`z`]`
: returns the secant of z.

```
>> Sec[0]
```
1

```
>> Sec[1] (* Sec[1] in Mathematica *)
```
$$\frac{1}{\text{Cos}\,[1]}$$

```
>> Sec[1.]
```
1.85082

31.11.14. Sin

Sine (SymPy, mpmath, WMA)

`Sin[`z`]`
: returns the sine of z.

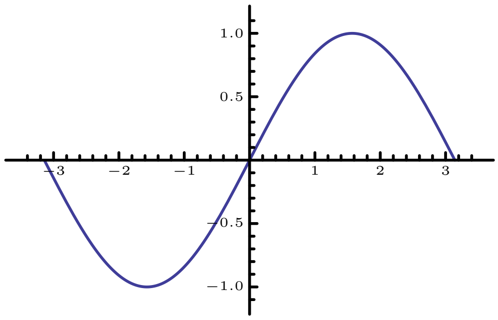
```
>> Sin[0]
```
0

```
>> Sin[0.5]
```
0.479426

```
>> Sin[3 Pi]
```
0

```
>> Sin[1.0 + I]
```
$1.29846 + 0.634964I$

```
>> Plot[Sin[x], {x, -Pi, Pi}]
```



31.11.15. Tan

Tangent (SymPy, mpmath, WMA)

`Tan[`*z*`]`
: returns the tangent of *z*.

```
>> Tan[0]
```
0

```
>> Tan[Pi / 2]
```
ComplexInfinity

32. Interactive Manipulation

33. Kernel Sessions

Contents

33.1. Exit

WMA link

`Exit[]`
: Terminates the Mathics session.

`Exit[`*n*`]`
: Terminates the mathics session with exit code *n*.

`Exit` is the same thing as `Quit`.

33.2. Out

WMA

`%$k$` or `Out[`*k*`]`
: gives the result of the *k*-th input line.

`%`
: gives the last result.

”%%’
: gives the result before the previous input line.

```
>> 42
   42
>> %
   42
>> 43;
>> %
   43
```

>> `44`
44

>> `%1`
42

>> `%%`
44

>> `Hold[Out[-1]]`
Hold [%]

>> `Hold[%4]`
Hold [%4]

>> `Out[0]`
Out [0]

33.3. Quit

WMA link

`Quit[]`
    Terminates the Mathics session.
`Quit[`*n*`]`
    Terminates the mathics session with exit code *n*.

`Quit` is the same thing as `Exit`.

34. Layout

This module contains symbols used to define the high level layout for expression formatting.

For instance, to represent a set of consecutive expressions in a row, we can use `Row`.

Contents

34.1. Center

WMA link

`Center`
 is used with the `ColumnAlignments` option to `Grid` or `TableForm` to specify a centered column.

34.2. Format

WMA link

`Format[`*expr*`]`
 holds values specifying how *expr* should be printed.

Assign values to `Format` to control how particular expressions should be formatted when printed to the user.

```
>> Format[f[x___]] := Infix[{x}, "~"]

>> f[1, 2, 3]
```
$1 \sim 2 \sim 3$

```
>> f[1]
1
```

Raw objects cannot be formatted:

```
>> Format[3] = "three";
Cannot assign to raw object 3.
```

Format types must be symbols:

```
>> Format[r, a + b] = "r";
Format type a + b is not a symbol.
```

Formats must be attached to the head of an expression:

```
>> f /: Format[g[f]] = "my f";
Tag f not found or too deep for an assigned rule.
```

34.3. Grid

WMA link

`Grid`[{{a_1, a_2, ...}, {b_1, b_2, ...}, ...}]
: formats several expressions inside a `GridBox`.

```
>> Grid[{{a, b}, {c, d}}]
```

a	*b*
c	*d*

For shallow lists, elements are shown as a column:

```
>> Grid[{a, b, c}]
```

a
b
c

If the sublists have different sizes, the grid has the number of columns of the largest one. Incomplete rows are completed with empty strings:

```
>> Grid[{{"first", "second", "third"},{a},{1, 2, 3}}]
```

first	second	third
a		
1	2	3

If the list is a mixture of lists and other expressions, the non-list expressions are shown as rows:

```
>> Grid[{"This is a long title", {"first", "second", "third"},{a},{1, 2,
   3}}]
```

This is a long title		
first	second	third
a		
1	2	3

34.4. Infix

WMA link

`Infix`[*expr*, *oper*, *prec*, *assoc*]
displays *expr* with the infix operator *oper*, with precedence *prec* and associativity *assoc*.

`Infix` can be used with `Format` to display certain forms with user-defined infix notation:

```
>> Format[g[x_, y_]] := Infix[{x, y}, "#", 350, Left]
```

```
>> g[a, g[b, c]]
```
$a\#(b\#c)$

```
>> g[g[a, b], c]
```
$a\#b\#c$

```
>> g[a + b, c]
```
$(a+b)\#c$

```
>> g[a * b, c]
```
$ab\#c$

```
>> g[a, b] + c
```
$c + a\#b$

```
>> g[a, b] * c
```
$c\,(a\#b)$

```
>> Infix[{a, b, c}, {"+", "-"}]
```
$a + b - c$

34.5. Left

WMA link

`Left`
is used with operator formatting constructs to specify a left-associative operator.

34.6. NonAssociative

on, logic, comparison, datentime, attributes and binary)

`NonAssociative`
is used with operator formatting constructs to specify a non-associative operator.

34.7. Postfix (//)

WMA link

`$x$ // $f$`
is equivalent to `$f$[$x$]` .

```
>> b // a
```
$a[b]$

```
>> c // b // a
```
$a[b[c]]$

The postfix operator // is parsed to an expression before evaluation:

```
>> Hold[x // a // b // c // d // e // f]
```
$\text{Hold}\left[f[e[d[c[b[a[x]]]]]]\right]$

34.8. Precedence

on, logic, comparison, datentime, attributes and binary)

`Precedence[`*op*`]`
returns the precedence of the built-in operator *op*.

```
>> Precedence[Plus]
```
310.

```
>> Precedence[Plus] < Precedence[Times]
```
True

Unknown symbols have precedence 670:

```
>> Precedence[f]
```
670.

Other expressions have precedence 1000:

```
>> Precedence[a + b]
```
1000.

34.9. PrecedenceForm

WMA link

`PrecedenceForm[`*expr*, *prec*`]`
: format *expr* parenthesized as it would be if it contained an operator of precedence *prec*.

34.10. Prefix

WMA link

`$f$ @ $x$`
: is equivalent to `$f$[$x$]`.

```
>> a @ b
```
$a[b]$

```
>> a @ b @ c
```
$a[b[c]]$

```
>> Format[p[x_]] := Prefix[{x}, "*"]
```

```
>> p[3]
```
$*3$

```
>> Format[q[x_]] := Prefix[{x}, "~", 350]
```

```
>> q[a+b]
```
$\sim (a+b)$

```
>> q[a*b]
```
$\sim ab$

```
>> q[a]+b
```
$b+\sim a$

The prefix operator `@` is parsed to an expression before evaluation:

```
>> Hold[a @ b @ c @ d @ e @ f @ x]
```
$\text{Hold}\left[a\left[b\left[c\left[d\left[e\left[f[x]\right]\right]\right]\right]\right]\right]$

34.11. Right

WMA link

`Right`
: is used with operator formatting constructs to specify a right-associative operator.

34.12. Row

WMA link

`Row`[{*expr*, ...}]
: formats several expressions inside a `RowBox`.

34.13. Style

WMA link

`Style`[*expr*, options]
: displays *expr* formatted using the specified option settings.

`Style`[*expr*, "style"]
: uses the option settings for the specified style in the current notebook.

`Style`[*expr*, *color*]
: displays using the specified color.

`Style`[*expr*, *Bold*]
: displays with fonts made bold.

`Style`[*expr*, *Italic*]
: displays with fonts made italic.

`Style`[*expr*, *Underlined*]
: displays with fonts underlined.

'Style[*expr*, *Larger*]
: displays with fonts made larger.

`Style`[*expr*, *Smaller*]
: displays with fonts made smaller.

`Style`[*expr*, *n*]
: displays with font size n.

`Style`[*expr*, *Tiny*]
`Style`[*expr*, *Small*], etc.
: display with fonts that are tiny, small, etc.

34.14. Subscript

WMA link

`Subscript`[a, i]
: displays as a_i.

```
>> Subscript[x,1,2,3] // TeXForm
```

x_{1,2,3}

34.15. Subsuperscript

WMA link

`Subsuperscript`[a, b, c]
: displays as a_b^c.

```
>> Subsuperscript[a, b, c] // TeXForm
```

a_b^c

34.16. Superscript

WMA link

`Superscript`[x, y]
: displays as x^y.

```
>> Superscript[x,3] // TeXForm
```

x^3

35. List Functions

Generalized Lists make up a core part of Mathics. In fact, to first approximation Evaluation works on a special kind of List called an M-Expression.

As a result, there about a hundred list functions.

Contents

35.1. Associations

An Association maps keys to values and is similar to a dictionary in Python; it is often sparse in that their key space is much larger than the number of actual keys found in the collection.

35.1.1. Association

WMA link

`Association`[key_1 -> val_1, key_2 -> val_2, ...]
`<|$key_1$ -> $val_1$, $key_2$ -> $val_2$, ...|>`
    represents an association between keys and values.

`Association` is the head of associations:

```
>> Head[<|a -> x, b -> y, c -> z|>]
```
Association

```
>> <|a -> x, b -> y|>
```
$<|a->x,b->y|>$

```
>> Association[{a -> x, b -> y}]
```
$<|a->x,b->y|>$

Associations can be nested:

```
>> <|a -> x, b -> y, <|a -> z, d -> t|>|>
```
$<|a->z,b->y,d->t|>$

35.1.2. AssociationQ

WMA link

`AssociationQ`[*expr*]
    return True if *expr* is a valid Association object, and False otherwise.

```
>> AssociationQ[<|a -> 1, b :> 2|>]
```
True

```
>> AssociationQ[<|a, b|>]
```
False

35.1.3. Key

WMA link

`Key`[*key*]
: represents a key used to access a value in an association.

`Key`[*key*][*assoc*]

35.1.4. Keys

WMA link

`Keys`[<| key_1 -> val_1, key_2 -> val_2, ...|>]
: return a list of the keys *keyi* in an association.

`Keys`[{key_1 -> val_1, key_2 -> val_2, ...}]
: return a list of the key_i in a list of rules.

```
>> Keys[<|a -> x, b -> y|>]
```
$\{a, b\}$

```
>> Keys[{a -> x, b -> y}]
```
$\{a, b\}$

Keys automatically threads over lists:

```
>> Keys[{<|a -> x, b -> y|>, {w -> z, {}}}]
```
$\{\{a, b\}, \{w, \{\}\}\}$

Keys are listed in the order of their appearance:

```
>> Keys[{c -> z, b -> y, a -> x}]
```
$\{c, b, a\}$

35.1.5. Lookup

WMA link

Lookup[*assoc*, *key*]
: looks up the value associated with *key* in the association *assoc*, or Missing[*KeyAbsent*].

35.1.6. Missing

WMA link

> `Missing[]`
> represents a data that is missing.

```
>> ElementData["Meitnerium","MeltingPoint"]
```
Missing [NotAvailable]

35.1.7. Values

WMA link

> `Values[<|`key_1 `->` val_1, key_2 `->` val_2, ...`|>]`
> return a list of the values val_i in an association.
> `Values[{`key_1 `->` val_1, key_2 `->` val_2, ...`}]`
> return a list of the val_i in a list of rules.

```
>> Values[<|a -> x, b -> y|>]
```
$\{x, y\}$

```
>> Values[{a -> x, b -> y}]
```
$\{x, y\}$

Values automatically threads over lists:

```
>> Values[{<|a -> x, b -> y|>, {c -> z, {}}}]
```
$\{\{x, y\}, \{z, \{\}\}\}$

Values are listed in the order of their appearance:

```
>> Values[{c -> z, b -> y, a -> x}]
```
$\{z, y, x\}$

35.2. Constructing Lists

Functions for constructing lists of various sizes and structure.

See also Constructing Vectors.

35.2.1. Array

WMA link

`Array`[f, n]
: returns the n-element list `{$f$[1], ..., $f$[$n$]}`.

`Array`[f, n, a]
: returns the n-element list `{$f$[$a$], ..., $f$[$a$ + $n$]}`.

`Array`[f, {n, m}, {a, b}]
: returns an n-by-m matrix created by applying f to indices ranging from `($a$, $b$)` to `($a$ + $n$, $b$ + $m$)`.

`Array`[f, $dims$, $origins$, h]
: returns an expression with the specified dimensions and index origins, with head h (instead of `List`).

```
>> Array[f, 4]
```
$\{f[1], f[2], f[3], f[4]\}$

```
>> Array[f, {2, 3}]
```
$\{\{f[1,1], f[1,2], f[1,3]\}, \{f[2,1], f[2,2], f[2,3]\}\}$

```
>> Array[f, {2, 3}, 3]
```
$\{\{f[3,3], f[3,4], f[3,5]\}, \{f[4,3], f[4,4], f[4,5]\}\}$

```
>> Array[f, {2, 3}, {4, 6}]
```
$\{\{f[4,6], f[4,7], f[4,8]\}, \{f[5,6], f[5,7], f[5,8]\}\}$

```
>> Array[f, {2, 3}, 1, Plus]
```
$f[1,1] + f[1,2] + f[1,3] + f[2,1] + f[2,2] + f[2,3]$

35.2.2. ConstantArray

WMA link

`ConstantArray`[$expr$, n]
: returns a list of n copies of $expr$.

```
>> ConstantArray[a, 3]
```
$\{a, a, a\}$

```
>> ConstantArray[a, {2, 3}]
```
$\{\{a, a, a\}, \{a, a, a\}\}$

35.2.3. List

WMA link

`List[`$e_1, e_2, ..., ei$`]`
`{$e_1$, $e_2$, ..., $ei$}`
    represents a list containing the elements $e_1...ei$.

`List` is the head of lists:

```
>> Head[{1, 2, 3}]
```
List

Lists can be nested:

```
>> {{a, b, {c, d}}}
```
$\{\{a, b, \{c, d\}\}\}$

35.2.4. Normal

WMA link

`Normal[expr_]`
    Brings special expressions to a normal expression from different special forms.

```
>> Normal[Pi]
```
π

```
>> Series[Exp[x], {x, 0, 5}]
```
$$1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + O[x]^6$$

```
>> Normal[%]
```
$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}$$

35.2.5. Permutations

WMA link

`Permutations[`*list*`]`
    gives all possible orderings of the items in *list*.
`Permutations[`*list*, *n*`]`
    gives permutations up to length *n*.
`Permutations[`*list*, {*n*}`]`
    gives permutations of length *n*.

```
>> Permutations[{y, 1, x}]
```
$\{\{y, 1, x\}, \{y, x, 1\}, \{1, y, x\}, \{1, x, y\}, \{x, y, 1\}, \{x, 1, y\}\}$

Elements are differentiated by their position in *list*, not their value.

```
>> Permutations[{a, b, b}]
```
$\{\{a,b,b\},\{a,b,b\},\{b,a,b\},\{b,b,a\},\{b,a,b\},\{b,b,a\}\}$

```
>> Permutations[{1, 2, 3}, 2]
```
$\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,1\},\{2,3\},\{3,1\},\{3,2\}\}$

```
>> Permutations[{1, 2, 3}, {2}]
```
$\{\{1,2\},\{1,3\},\{2,1\},\{2,3\},\{3,1\},\{3,2\}\}$

35.2.6. Range

WMA link

`Range`[n]
: returns a list of integers from 1 to n.

`Range`[a, b]
: returns a list of (Integer, Rational, Real) numbers from a to b.

`Range`[a, b, di]
: returns a list of numbers from a to b using step di. More specifically, `Range` starts from a and successively adds increments of di until the result is greater (if $di > 0$) or less (if $di < 0$) than b.

```
>> Range[5]
```
$\{1,2,3,4,5\}$

```
>> Range[-3, 2]
```
$\{-3,-2,-1,0,1,2\}$

```
>> Range[5, 1, -2]
```
$\{5,3,1\}$

```
>> Range[1.0, 2.3]
```
$\{1.,2.\}$

```
>> Range[0, 2, 1/3]
```
$$\left\{0,\frac{1}{3},\frac{2}{3},1,\frac{4}{3},\frac{5}{3},2\right\}$$

```
>> Range[1.0, 2.3, .5]
```
$\{1.,1.5,2.\}$

35.2.7. Reap

WMA link

Reap[*expr*]
: gives the result of evaluating *expr*, together with all values sown during this evaluation. Values sown with different tags are given in different lists.

Reap[*expr*, *pattern*]
: only yields values sown with a tag matching *pattern*. `Reap[$expr$]` is equivalent to `Reap[$expr$, _]`.

Reap[*expr*, {$pattern_1$, $pattern_2$, ...}]
: uses multiple patterns.

Reap[*expr*, *pattern*, *f*]
: applies *f* on each tag and the corresponding values sown in the form `$f$[$tag$, {$e_1$, $e_2$, ...}]`.

```
>> Reap[Sow[3]; Sow[1]]
```
$\{1, \{\{3, 1\}\}\}$

```
>> Reap[Sow[2, {x, x, x}]; Sow[3, x]; Sow[4, y]; Sow[4, 1], {_Symbol,
   _Integer, x}, f]
```
$\{4, \{\{f[x, \{2, 2, 2, 3\}], f[y, \{4\}]\}, \{f[1, \{4\}]\}, \{f[x, \{2, 2, 2, 3\}]\}\}\}$

Find the unique elements of a list, keeping their order:

```
>> Reap[Sow[Null, {a, a, b, d, c, a}], _, # &][[2]]
```
$\{a, b, d, c\}$

Sown values are reaped by the innermost matching `Reap`:

```
>> Reap[Reap[Sow[a, x]; Sow[b, 1], _Symbol, Print["Inner: ", #1]&];, _,
   f]
Inner: x
```
$\{\text{Null}, \{f[1, \{b\}]\}\}$

When no value is sown, an empty list is returned:

```
>> Reap[x]
```
$\{x, \{\}\}$

35.2.8. Sow

WMA link

Sow[*e*]
: sends the value *e* to the innermost `Reap`.

Sow[*e*, *tag*]
: sows *e* using *tag*. `Sow[$e$]` is equivalent to `Sow[$e$, Null]`.

Sow[*e*, {tag_1, tag_2, ...}]
: uses multiple tags.

35.2.9. Table

WMA link

`Table`[*expr*, *n*]
    generates a list of *n* copies of *expr*.
`Table`[*expr*, {*i*, *n*}]
    generates a list of the values of expr when *i* runs from 1 to *n*.
`Table`[*expr*, {*i*, *start*, *stop*, *step*}]
    evaluates *expr* with *i* ranging from *start* to *stop*, incrementing by *step*.
`Table`[*expr*, {*i*, {e_1, e_2, ..., *ei*}}]
    evaluates *expr* with *i* taking on the values e_1, e_2, ..., *ei*.

```
>> Table[x, 3]
```
$\{x, x, x\}$

```
>> n = 0; Table[n = n + 1, {5}]
```
$\{1, 2, 3, 4, 5\}$

```
>> Table[i, {i, 4}]
```
$\{1, 2, 3, 4\}$

```
>> Table[i, {i, 2, 5}]
```
$\{2, 3, 4, 5\}$

```
>> Table[i, {i, 2, 6, 2}]
```
$\{2, 4, 6\}$

```
>> Table[i, {i, Pi, 2 Pi, Pi / 2}]
```
$$\left\{\pi, \frac{3\pi}{2}, 2\pi\right\}$$

```
>> Table[x^2, {x, {a, b, c}}]
```
$$\left\{a^2, b^2, c^2\right\}$$

`Table` supports multi-dimensional tables:

```
>> Table[{i, j}, {i, {a, b}}, {j, 1, 2}]
```
$\{\{\{a, 1\}, \{a, 2\}\}, \{\{b, 1\}, \{b, 2\}\}\}$

Symbolic bounds:

```
>> Table[x, {x, a, a + 5 n, n}]
```
$\{a, 5 + a, 10 + a, 15 + a, 20 + a, 25 + a\}$

The lower bound is always included even for large step sizes:

```
>> Table[i, {i, 1, 9, Infinity}]
```
$\{1\}$

35.2.10. Tuples

WMA link

`Tuples`[*list*, *n*]
: returns a list of all *n*-tuples of elements in *list*.

`Tuples`[{$list_1$, $list_2$, ...}]
: returns a list of tuples with elements from the given lists.

```
>> Tuples[{a, b, c}, 2]
```
$\{\{a,a\},\{a,b\},\{a,c\},\{b,a\},\{b,b\},\{b,c\},\{c,a\},\{c,b\},\{c,c\}\}$

```
>> Tuples[{}, 2]
```
$\{\}$

```
>> Tuples[{a, b, c}, 0]
```
$\{\{\}\}$

```
>> Tuples[{{a, b}, {1, 2, 3}}]
```
$\{\{a,1\},\{a,2\},\{a,3\},\{b,1\},\{b,2\},\{b,3\}\}$

The head of *list* need not be `List`:

```
>> Tuples[f[a, b, c], 2]
```
$\{f[a,a],f[a,b],f[a,c],f[b,a],f[b,b],f[b,c],f[c,a],f[c,b],f[c,c]\}$

However, when specifying multiple expressions, `List` is always used:

```
>> Tuples[{f[a, b], g[c, d]}]
```
$\{\{a,c\},\{a,d\},\{b,c\},\{b,d\}\}$

35.3. Elements of Lists

Functions for accessing elements of lists using either indices, positions, or patterns of criteria.

35.3.1. Append

WMA link

`Append`[*expr*, *elem*]
: returns *expr* with *elem* appended.

```
>> Append[{1, 2, 3}, 4]
```
$\{1,2,3,4\}$

`Append` works on expressions with heads other than `List`:

```
>> Append[f[a, b], c]
```

$f[a, b, c]$

Unlike `Join`, `Append` does not flatten lists in *item*:

```
>> Append[{a, b}, {c, d}]
```

$\{a, b, \{c, d\}\}$

35.3.2. AppendTo

WMA link

`AppendTo`[*s*, *elem*]
: append *elem* to value of *s* and sets *s* to the result.

```
>> s = {};
```

```
>> AppendTo[s, 1]
```

$\{1\}$

```
>> s
```

$\{1\}$

`Append` works on expressions with heads other than `List`:

```
>> y = f[];
```

```
>> AppendTo[y, x]
```

$f[x]$

```
>> y
```

$f[x]$

35.3.3. Cases

WMA link

`Cases`[*list*, *pattern*]
: returns the elements of *list* that match *pattern*.

`Cases`[*list*, *pattern*, *ls*]
: returns the elements matching at levelspec *ls*.

`Cases`[*list*, *pattern*, Heads->*bool*]
: Match including the head of the expression in the search.

```
>> Cases[{a, 1, 2.5, "string"}, _Integer|_Real]
```
$\{1, 2.5\}$

```
>> Cases[_Complex][{1, 2I, 3, 4-I, 5}]
```
$\{2I, 4 - I\}$

Find symbols among the elements of an expression:

```
>> Cases[{b, 6, \[Pi]}, _Symbol]
```
$\{b, \pi\}$

Also include the head of the expression in the previous search:

```
>> Cases[{b, 6, \[Pi]}, _Symbol, Heads -> True]
```
$\{\text{List}, b, \pi\}$

See also 'MatchQ' 55.2.2.

35.3.4. Count

WMA link

`Count`[*list*, *pattern*]
: returns the number of times *pattern* appears in *list*.

`Count`[*list*, *pattern*, *ls*]
: counts the elements matching at levelspec *ls*.

```
>> Count[{3, 7, 10, 7, 5, 3, 7, 10}, 3]
```
2

```
>> Count[{{a, a}, {a, a, a}, a}, a, {2}]
```
5

35.3.5. Delete

WMA link

`Delete`[*expr*, *i*]
: deletes the element at position *i* in *expr*. The position is counted from the end if *i* is negative.

`Delete`[*expr*, {*m*, *n*, ...}]
: deletes the element at position {*m*, *n*, ...}.

`Delete`[*expr*, {{m_1, n_1, ...}, {m_2, n_2, ...}, ...}]
: deletes the elements at several positions.

Delete the element at position 3:

```
>> Delete[{a, b, c, d}, 3]
```
$\{a, b, d\}$

Delete at position 2 from the end:

```
>> Delete[{a, b, c, d}, -2]
```
$\{a, b, d\}$

Delete at positions 1 and 3:

```
>> Delete[{a, b, c, d}, {{1}, {3}}]
```
$\{b, d\}$

Delete in a 2D array:

```
>> Delete[{{a, b}, {c, d}}, {2, 1}]
```
$\{\{a, b\}, \{d\}\}$

Deleting the head of a whole expression gives a Sequence object:

```
>> Delete[{a, b, c}, 0]
```
Sequence $[a, b, c]$

Delete in an expression with any head:

```
>> Delete[f[a, b, c, d], 3]
```
$f[a, b, d]$

Delete a head to splice in its arguments:

```
>> Delete[f[a, b, u + v, c], {3, 0}]
```
$f[a, b, u, v, c]$

```
>> Delete[{a, b, c}, 0]
```
Sequence $[a, b, c]$

Delete without the position:

```
>> Delete[{a, b, c, d}]
Delete called with 1 argument; 2 arguments are expected.
```
Delete $[\{a, b, c, d\}]$

Delete with many arguments:

```
>> Delete[{a, b, c, d}, 1, 2]
Delete called with 3 arguments; 2 arguments are expected.
```
Delete $[\{a, b, c, d\}, 1, 2]$

Delete the element out of range:

```
>> Delete[{a, b, c, d}, 5]
   Part {5} of {a, b, c, d} does not exist.
```
Delete $[\{a,b,c,d\},5]$

Delete the position not integer:

```
>> Delete[{a, b, c, d}, {1, n}]
   Position specification n in {a, b, c, d} is not a machine-sized
   integer or a list of machine-sized integers.
```
Delete $[\{a,b,c,d\},\{1,n\}]$

35.3.6. DeleteCases

WMA link

`DeleteCases`[*list*, *pattern*]
: returns the elements of *list* that do not match *pattern*.

`DeleteCases`[*list*, *pattern*, *levelspec*]
: removes all parts of *list* on levels specified by *levelspec* that match pattern (not fully implemented).

`DeleteCases`[*list*, *pattern*, *levelspec*, *n*]
: removes the first *n* parts of *list* that match *pattern*.

```
>> DeleteCases[{a, 1, 2.5, "string"}, _Integer|_Real]
```
$\{a, \text{string}\}$

```
>> DeleteCases[{a, b, 1, c, 2, 3}, _Symbol]
```
$\{1,2,3\}$

35.3.7. Drop

WMA link

`Drop`[*list*, *n*]
: returns *list* with the first *n* elements removed.

`Drop`[*list*, -*n*]
: returns *list* with its last *n* elements removed.

`Drop`[*list*, {*m*, *n*}]
: returns *list* with elements *m* though *n* removed.

Drop up until the third item from the beginning of a list:

```
>> Drop[{a, b, c, d}, 3]
```
$\{d\}$

Drop until the second item from the end of that list:

```
>> Drop[{a, b, c, d}, -2]
```
$\{a, b\}$

Drop from the second item to the second-to-the-end item:

```
>> Drop[{a, b, c, d, e}, {2, -2}]
```
$\{a, e\}$

Drop a submatrix:

```
>> A = Table[i*10 + j, {i, 4}, {j, 4}]
```
$\{\{11, 12, 13, 14\}, \{21, 22, 23, 24\}, \{31, 32, 33, 34\}, \{41, 42, 43, 44\}\}$

```
>> Drop[A, {2, 3}, {2, 3}]
```
$\{\{11, 14\}, \{41, 44\}\}$

Dropping the 0th element does nothing, and returns the list unmodified:

```
>> Drop[{a, b, c, d}, 0]
```
$\{a, b, c, d\}$

Even if the list is empty:

```
>> Drop[{}, 0]
```
$\{\}$

See also 'Take' 35.3.25.

35.3.8. Extract

WMA link

`Extract[`*expr*, *list*`]`
extracts parts of *expr* specified by *list*.
`Extract[`*expr*, {$list_1$, $list_2$, ...}`]`
extracts a list of parts.

`Extract[$expr$, $i$, $j$, ...]` is equivalent to `Part[$expr$, {$i$, $j$, ...}]`.

```
>> Extract[a + b + c, {2}]
```
b

```
>> Extract[{{a, b}, {c, d}}, {{1}, {2, 2}}]
   {{a,b},d}
```

35.3.9. First

WMA link

`First[`*expr*`]`
: returns the first element in *expr*.

`First[`*expr*, *def*`]`
: returns the first element in *expr* if it exists or *def* otherwise.

`First[$expr$]` is equivalent to `$expr$[[1]]`.

```
>> First[{a, b, c}]
   a
```

The first argument need not be a list:

```
>> First[a + b + c]
   a
```

However, the first argument must be Nonatomic when there is a single argument:

```
>> First[x]
   Nonatomic expression expected at position 1 in First[x].
   First[x]
```

Or if it is not, but a second default argument is provided, that is evaluated and returned:

```
>> First[10, 1+2]
   3
```

```
>> First[{}]
   {} has zero length and no first element.
   First[{}]
```

As before, the first argument is empty, but a default argument is given, evaluate and return the second argument:

```
>> First[{}, 1+2]
   3
```

35.3.10. FirstCase

WMA link

FirstCase[{e_1, e_2, ...}, *pattern*]
: gives the first *ei* to match *pattern*, or *Missing*["*NotFound*"] if none matching pattern is found.

FirstCase[{e_1,e_2, ...}, *pattern* -> *rhs*]
: gives the value of *rhs* corresponding to the first *ei* to match pattern.

FirstCase[*expr*, *pattern*, *default*]
: gives *default* if no element matching *pattern* is found.

FirstCase[*expr*, *pattern*, *default*, *levelspec*]
: finds only objects that appear on levels specified by *levelspec*.

FirstCase[*pattern*]
: represents an operator form of FirstCase that can be applied to an expression.

35.3.11. FirstPosition

WMA link

`FirstPosition`[*expr*, *pattern*]
: gives the position of the first element in *expr* that matches *pattern*, or Missing["NotFound"] if no such element is found.

`FirstPosition`[*expr*, *pattern*, *default*]
: gives default if no element matching *pattern* is found.

`FirstPosition`[*expr*, *pattern*, *default*, *levelspec*]
: finds only objects that appear on levels specified by *levelspec*.

```
>> FirstPosition[{a, b, a, a, b, c, b}, b]
```
{2}

```
>> FirstPosition[{{a, a, b}, {b, a, a}, {a, b, a}}, b]
```
{1,3}

```
>> FirstPosition[{x, y, z}, b]
```
Missing [NotFound]

Find the first position at which x^2 to appears:

```
>> FirstPosition[{1 + x^2, 5, x^4, a + (1 + x^2)^2}, x^2]
```
{1,2}

35.3.12. Insert

WMA link

`Insert`[*list*, *elem*, *n*]
: inserts *elem* at position *n* in *list*. When *n* is negative, the position is counted from the end.

```
>> Insert[{a,b,c,d,e}, x, 3]
```
$\{a, b, x, c, d, e\}$

```
>> Insert[{a,b,c,d,e}, x, -2]
```
$\{a, b, c, d, x, e\}$

35.3.13. Last

WMA link

`Last[`*expr*`]`
: returns the last element in *expr*.

`Last[`*expr*, *def*`]`
: returns the last element in *expr* if it exists or *def* otherwise.

`Last[$expr$]` is equivalent to `$expr$[[-1]]`.

```
>> Last[{a, b, c}]
```
c

The first argument need not be a list:

```
>> Last[a + b + c]
```
c

However, the first argument must be Nonatomic when there is a single argument:

```
>> Last[10]
Nonatomic expression expected at position 1 in Last[10].
```
Last [10]

Or if it is not, but a second default argument is provided, that is evaluated and returned:

```
>> Last[10, 1+2]
```
3

```
>> Last[{}]
{} has zero length and no last element.
```
Last [{}]

As before, the first argument is empty, but since default argument is given, evaluate and return the second argument:

```
>> Last[{}, 1+2]
```
3

35.3.14. Length

WMA link

`Length`[*expr*]
: returns the number of elements in *expr*.

Length of a list:

```
>> Length[{1, 2, 3}]
```
3

`Length` operates on the `FullForm` of expressions:

```
>> Length[Exp[x]]
```
2

```
>> FullForm[Exp[x]]
```
Power $[E, x]$

The length of atoms is 0:

```
>> Length[a]
```
0

Note that rational and complex numbers are atoms, although their `FullForm` might suggest the opposite:

```
>> Length[1/3]
```
0

```
>> FullForm[1/3]
```
Rational $[1, 3]$

35.3.15. Most

WMA link

`Most`[*expr*]
: returns *expr* with the last element removed.

`Most[$expr$]` is equivalent to `$expr$[[;;-2]]`.

```
>> Most[{a, b, c}]
```
$\{a, b\}$

```
>> Most[a + b + c]
```
$a + b$

```
>> Most[x]
Nonatomic expression expected at position 1 in Most[x].
```
Most$[x]$

35.3.16. Part

WMA link

`Part`[*expr*, *i*]
 returns part *i* of *expr*.

Extract an element from a list:

```
>> A = {a, b, c, d};
```

```
>> A[[3]]
```
c

Negative indices count from the end:

```
>> {a, b, c}[[-2]]
```
b

`Part` can be applied on any expression, not necessarily lists.

```
>> (a + b + c)[[2]]
```
b

`$expr$[[0]]` gives the head of *expr*:

```
>> (a + b + c)[[0]]
```
Plus

Parts of nested lists:

```
>> M = {{a, b}, {c, d}};
```

```
>> M[[1, 2]]
```
b

You can use `Span` to specify a range of parts:

```
>> {1, 2, 3, 4}[[2;;4]]
```
$\{2,3,4\}$

```
>> {1, 2, 3, 4}[[2;;-1]]
```
$\{2,3,4\}$

A list of parts extracts elements at certain indices:

```
>> {a, b, c, d}[[{1, 3, 3}]]
```
$\{a, c, c\}$

Get a certain column of a matrix:

```
>> B = {{a, b, c}, {d, e, f}, {g, h, i}};
```

```
>> B[[;;, 2]]
```
$\{b, e, h\}$

Extract a submatrix of 1st and 3rd row and the two last columns:

```
>> B = {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}};
```

```
>> B[[{1, 3}, -2;;-1]]
```
$\{\{2,3\},\{8,9\}\}$

The 3d column of a matrix:

```
>> {{a, b, c}, {d, e, f}, {g, h, i}}[[All, 3]]
```
$\{c, f, i\}$

Further examples:

```
>> (a+b+c+d)[[-1;;-2]]
```
0

```
>> x[[2]]
Part specification is longer than depth of object.
```
$x[[2]]$

Assignments to parts are possible:

```
>> B[[;;, 2]] = {10, 11, 12}
```
$\{10, 11, 12\}$

```
>> B
```
$\{\{1, 10, 3\},\{4, 11, 6\},\{7, 12, 9\}\}$

```
>> B[[;;, 3]] = 13
```
13

```
>> B
```
$\{\{1, 10, 13\},\{4, 11, 13\},\{7, 12, 13\}\}$

```
>> B[[1;;-2]] = t;
```

```
>> B
```
$\{t, t, \{7, 12, 13\}\}$

```
>> F = Table[i*j*k, {i, 1, 3}, {j, 1, 3}, {k, 1, 3}];
```

```
>> F[[;; All, 2 ;; 3, 2]] = t;
```

```
>> F
```
$\{\{\{1,2,3\},\{2,t,6\},\{3,t,9\}\},\{\{2,4,6\},\{4,t,12\},\{6,t,18\}\},\{\{3,6,9\},\{6,t,18\},\{9,t,27\}\}\}$

```
>> F[[;; All, 1 ;; 2, 3 ;; 3]] = k;
```

```
>> F
```
$\{\{\{1,2,k\},\{2,t,k\},\{3,t,9\}\},\{\{2,4,k\},\{4,t,k\},\{6,t,18\}\},\{\{3,6,k\},\{6,t,k\},\{9,t,27\}\}\}$

Of course, part specifications have precedence over most arithmetic operations:

```
>> A[[1]] + B[[2]] + C[[3]] // Hold // FullForm
```
Hold [Plus [Part [A, 1] , Part [B, 2] , Part [C, 3]]]

35.3.17. Pick

WMA link

`Pick`[*list*, *sel*]
  returns those items in *list* that are True in *sel*.
`Pick`[*list*, *sel*, *patt*]
  returns those items in *list* that match *patt* in *sel*.

```
>> Pick[{a, b, c}, {False, True, False}]
```
$\{b\}$

```
>> Pick[f[g[1, 2], h[3, 4]], {{True, False}, {False, True}}]
```
$f\left[g[1], h[4]\right]$

```
>> Pick[{a, b, c, d, e}, {1, 2, 3.5, 4, 5.5}, _Integer]
```
$\{a,b,d\}$

35.3.18. Position

WMA link

`Position`[*expr*, *patt*]
  returns the list of positions for which *expr* matches *patt*.
`Position`[*expr*, *patt*, *ls*]
  returns the positions on levels specified by levelspec *ls*.

```
>> Position[{1, 2, 2, 1, 2, 3, 2}, 2]
```
$\{\{2\},\{3\},\{5\},\{7\}\}$

Find positions upto 3 levels deep:

```
>> Position[{1 + Sin[x], x, (Tan[x] - y)^2}, x, 3]
```
$\{\{1,2,1\},\{2\}\}$

Find all powers of x:

```
>> Position[{1 + x^2, x y ^ 2, 4 y, x ^ z}, x^_]
```
$\{\{1,2\},\{4\}\}$

Use Position as an operator:

```
>> Position[_Integer][{1.5, 2, 2.5}]
```
$\{\{2\}\}$

35.3.19. Prepend

WMA link

`Prepend[`*expr*, *item*`]`
: returns *expr* with *item* prepended to its elements.

`Prepend[`*expr*`]`
: `Prepend[$elem$][$expr$]` is equivalent to `Prepend[$expr$,$elem$]`.

`Prepend` is similar to `Append`, but adds *item* to the beginning of *expr*:

```
>> Prepend[{2, 3, 4}, 1]
```
$\{1,2,3,4\}$

`Prepend` works on expressions with heads other than `List`:

```
>> Prepend[f[b, c], a]
```
$f[a,b,c]$

Unlike `Join`, `Prepend` does not flatten lists in *item*:

```
>> Prepend[{c, d}, {a, b}]
```
$\{\{a,b\},c,d\}$

35.3.20. PrependTo

WMA link

`PrependTo[`*s*, *item*`]`
: prepends *item* to value of *s* and sets *s* to the result.

Assign s to a list

```
>> s = {1, 2, 4, 9}
```
$\{1,2,4,9\}$

Add a new value at the beginning of the list:

```
>> PrependTo[s, 0]
```
$\{0,1,2,4,9\}$

The value assigned to s has changed:

```
>> s
```
$\{0,1,2,4,9\}$

`PrependTo` works with a head other than `List`:

```
>> y = f[a, b, c];
```

```
>> PrependTo[y, x]
```
$f[x,a,b,c]$

```
>> y
```
$f[x,a,b,c]$

35.3.21. ReplacePart

WMA link

`ReplacePart[`*expr*, *i* -> *new*`]`
: replaces part *i* in *expr* with *new*.

`ReplacePart[`*expr*, {{*i*, *j*} -> e_1, {*k*, *l*} -> e_2}`]`
: replaces parts *i* and *j* with e_1, and parts *k* and *l* with e_2.

```
>> ReplacePart[{a, b, c}, 1 -> t]
```
$\{t,b,c\}$

```
>> ReplacePart[{{a, b}, {c, d}}, {2, 1} -> t]
```
$\{\{a,b\},\{t,d\}\}$

```
>> ReplacePart[{{a, b}, {c, d}}, {{2, 1} -> t, {1, 1} -> t}]
```
$\{\{t,b\},\{t,d\}\}$

```
>> ReplacePart[{a, b, c}, {{1}, {2}} -> t]
```
$\{t, t, c\}$

Delayed rules are evaluated once for each replacement:

```
>> n = 1;
```

```
>> ReplacePart[{a, b, c, d}, {{1}, {3}} :> n++]
```
$\{1, b, 2, d\}$

Non-existing parts are simply ignored:

```
>> ReplacePart[{a, b, c}, 4 -> t]
```
$\{a, b, c\}$

You can replace heads by replacing part 0:

```
>> ReplacePart[{a, b, c}, 0 -> Times]
```
abc

(This is equivalent to `Apply`.)

Negative part numbers count from the end:

```
>> ReplacePart[{a, b, c}, -1 -> t]
```
$\{a, b, t\}$

35.3.22. Rest

WMA link

`Rest[`*expr*`]`
: returns *expr* with the first element removed.

`Rest[$expr$]` is equivalent to `$expr$[[2;;]]`.

```
>> Rest[{a, b, c}]
```
$\{b, c\}$

```
>> Rest[a + b + c]
```
$b + c$

```
>> Rest[x]
Nonatomic expression expected at position 1 in Rest[x].
```
Rest $[x]$

```
>> Rest[{}]
Cannot take Rest of expression {} with length zero.
```
Rest [{}]

35.3.23. Select

WMA link

`Select`[{e_1, e_2, ...}, *crit*]
: returns a list of the elements *ei* for which *crit*[*ei*] is `True`.

`Select`[{e_1, e_2, ...}, *crit*, n]
: returns a list of the first *n* elements *ei* for which *crit*[*ei*] is `True`.

Get a list of even numbers up to 10:

```
>> Select[Range[10], EvenQ]
```
$\{2, 4, 6, 8, 10\}$

Find numbers that are greater than zero in a list:

```
>> Select[{-3, 0, 10, 3, a}, #>0&]
```
$\{10, 3\}$

Find the first number that is list greater than zero in a list:

```
>> Select[{-3, 0, 10, 3, a}, #>0&, 1]
```
$\{10\}$

`Select` works on an expression with any head:

```
>> Select[f[a, 2, 3], NumberQ]
```
$f[2, 3]$

35.3.24. Span (;;)

WMA link

`Span`
: is the head of span ranges like `1;;3`.

```
>> ;; // FullForm
```
Span [1, All]

```
>> 1;;4;;2 // FullForm
```
Span [1, 4, 2]

```
>> 2;;-2 // FullForm
```
Span [2, − 2]

```
>> ;;3 // FullForm
```
Span [1, 3]

35.3.25. Take

WMA link

`Take`[*expr*, *n*]
: returns *expr* with all but the first *n* elements removed.

`Take`[*list*, -*n*]
: returns last *n* elements of *list*.

`Take`[*list*, {*m*, *n*}]
: returns elements *m* through *n* of *list*.

Get the first three elements:

```
>> Take[{a, b, c, d}, 3]
```
$\{a, b, c\}$

Get the last two elements:

```
>> Take[{a, b, c, d}, -2]
```
$\{c, d\}$

Get the elements from the second element through the next to last element:

```
>> Take[{a, b, c, d, e}, {2, -2}]
```
$\{b, c, d\}$

Take a submatrix:

```
>> A = {{a, b, c}, {d, e, f}};
```

```
>> Take[A, 2, 2]
```
$\{\{a, b\}, \{d, e\}\}$

Take a single column:

```
>> Take[A, All, {2}]
```
$\{\{b\}, \{e\}\}$

Taking the 0th element does nothing, and returns an empty list:

```
>> Take[{a, b, c, d}, 0]
```

{}

See also 'Drop' 35.3.7.

35.3.26. UpTo

WMA link

> UpTo[n]
> is a symbolic specification that represents up to n objects or positions. If n objects or positions are available, all are used. If fewer are available, only those available are used.

35.4. Math & Counting Operations on Lists

35.4.1. TakeLargestBy

WMA link

> TakeLargestBy[$list$, f, n]
> returns the a sorted list of the n largest items in $list$ using f to retrieve the items' keys to compare them.

For details on how to use the ExcludedForms option, see TakeLargest[].

```
>> TakeLargestBy[{{1, -1}, {10, 100}, {23, 7, 8}, {5, 1}}, Total, 2]
```

$\{\{10, 100\}, \{23, 7, 8\}\}$

```
>> TakeLargestBy[{"abc", "ab", "x"}, StringLength, 1]
```

{abc}

35.4.2. TakeSmallestBy

WMA link

> TakeSmallestBy[$list$, f, n]
> returns the a sorted list of the n smallest items in $list$ using f to retrieve the items' keys to compare them.

For details on how to use the ExcludedForms option, see TakeLargest[].

```
>> TakeSmallestBy[{{1, -1}, {10, 100}, {23, 7, 8}, {5, 1}}, Total, 2]
```
$\{\{1,-1\},\{5,1\}\}$

```
>> TakeSmallestBy[{"abc", "ab", "x"}, StringLength, 1]
```
$\{x\}$

35.5. Predicates on Lists

35.5.1. ContainsOnly

WMA link

`ContainsOnly`[$list_1$, $list_2$]
    yields True if $list_1$ contains only elements that appear in $list_2$.

```
>> ContainsOnly[{b, a, a}, {a, b, c}]
```
True

The first list contains elements not present in the second list:

```
>> ContainsOnly[{b, a, d}, {a, b, c}]
```
False

```
>> ContainsOnly[{}, {a, b, c}]
```
True

Use Equal as the comparison function to have numerical tolerance:

```
>> ContainsOnly[{a, 1.0}, {1, a, b}, {SameTest -> Equal}]
```
True

35.6. Rearranging and Restructuring Lists

These functions reorder and rearrange lists.

35.6.1. Catenate

WMA link

`Catenate`[{l_1, l_2, ...}]
    concatenates the lists l_1, l_2, ...

```
>> Catenate[{{1, 2, 3}, {4, 5}}]
```
$\{1,2,3,4,5\}$

35.6.2. Complement

WMA link

`Complement[`*all*, e_1, e_2, ...]
: returns an expression containing the elements in the set *all* that are not in any of e_1, e_2, etc.

`Complement[`*all*, e_1, e_2, ..., SameTest->*test*]
: applies *test* to the elements in *all* and each of the *ei* to determine equality.

The sets *all*, e_1, etc can have any head, which must all match.

The returned expression has the same head as the input expressions. The expression will be sorted and each element will only occur once.

```
>> Complement[{a, b, c}, {a, c}]
```
$\{b\}$

```
>> Complement[{a, b, c}, {a, c}, {b}]
```
$\{\}$

```
>> Complement[f[z, y, x, w], f[x], f[x, z]]
```
$f[w,y]$

```
>> Complement[{c, b, a}]
```
$\{a,b,c\}$

35.6.3. DeleteDuplicates

WMA link

`DeleteDuplicates[`*list*]
: deletes duplicates from *list*.

`DeleteDuplicates[`*list*, *test*]
: deletes elements from *list* based on whether the function *test* yields `True` on pairs of elements.

`DeleteDuplicates` does not change the order of the remaining elements.

```
>> DeleteDuplicates[{1, 7, 8, 4, 3, 4, 1, 9, 9, 2, 1}]
```
$\{1,7,8,4,3,9,2\}$

```
>> DeleteDuplicates[{3,2,1,2,3,4}, Less]
```
$\{3,2,1\}$

35.6.4. Flatten

WMA link

`Flatten`[*expr*]
: flattens out nested lists in *expr*.

`Flatten`[*expr*, *n*]
: stops flattening at level *n*.

`Flatten`[*expr*, *n*, *h*]
: flattens expressions with head *h* instead of `List`.

```
>> Flatten[{{a, b}, {c, {d}, e}, {f, {g, h}}}]
```
$\{a, b, c, d, e, f, g, h\}$

```
>> Flatten[{{a, b}, {c, {e}, e}, {f, {g, h}}}, 1]
```
$\{a, b, c, \{e\}, e, f, \{g, h\}\}$

```
>> Flatten[f[a, f[b, f[c, d]], e], Infinity, f]
```
$f[a, b, c, d, e]$

```
>> Flatten[{{a, b}, {c, d}}, {{2}, {1}}]
```
$\{\{a, c\}, \{b, d\}\}$

```
>> Flatten[{{a, b}, {c, d}}, {{1, 2}}]
```
$\{a, b, c, d\}$

Flatten also works in irregularly shaped arrays

```
>> Flatten[{{1, 2, 3}, {4}, {6, 7}, {8, 9, 10}}, {{2}, {1}}]
```
$\{\{1, 4, 6, 8\}, \{2, 7, 9\}, \{3, 10\}\}$

35.6.5. Gather

WMA link

`Gather`[*list*, *test*]
: gathers elements of *list* into sub lists of items that are the same according to *test*.

`Gather`[*list*]
: gathers elements of *list* into sub lists of items that are the same.

The order of the items inside the sub lists is the same as in the original list.

```
>> Gather[{1, 7, 3, 7, 2, 3, 9}]
```
$\{\{1\}, \{7, 7\}, \{3, 3\}, \{2\}, \{9\}\}$

```
>> Gather[{1/3, 2/6, 1/9}]
```
$\left\{\left\{\frac{1}{3}, \frac{1}{3}\right\}, \left\{\frac{1}{9}\right\}\right\}$

35.6.6. GatherBy

WMA link

`GatherBy`[*list*, *f*]
: gathers elements of *list* into sub lists of items whose image under *f* identical.

`GatherBy`[*list*, {*f*, *g*, ...}]
: gathers elements of *list* into sub lists of items whose image under *f* identical. Then, gathers these sub lists again into sub sub lists, that are identical under *g*.

```
>> GatherBy[{{1, 3}, {2, 2}, {1, 1}}, Total]
```
$\{\{\{1,3\},\{2,2\}\},\{\{1,1\}\}\}$

```
>> GatherBy[{"xy", "abc", "ab"}, StringLength]
```
{{xy, ab}, {abc}}

```
>> GatherBy[{{2, 0}, {1, 5}, {1, 0}}, Last]
```
$\{\{\{2,0\},\{1,0\}\},\{\{1,5\}\}\}$

```
>> GatherBy[{{1, 2}, {2, 1}, {3, 5}, {5, 1}, {2, 2, 2}}, {Total, Length
}]
```
$\{\{\{\{1,2\},\{2,1\}\}\},\{\{\{3,5\}\}\},\{\{\{5,1\}\},\{\{2,2,2\}\}\}\}$

35.6.7. Intersection

WMA link

`Intersection`[*a*, *b*, ...]
: gives the intersection of the sets. The resulting list will be sorted and each element will only occur once.

```
>> Intersection[{1000, 100, 10, 1}, {1, 5, 10, 15}]
```
$\{1,10\}$

```
>> Intersection[{{a, b}, {x, y}}, {{x, x}, {x, y}, {x, z}}]
```
$\{\{x,y\}\}$

```
>> Intersection[{c, b, a}]
```
$\{a,b,c\}$

```
>> Intersection[{1, 2, 3}, {2, 3, 4}, SameTest->Less]
```
$\{3\}$

35.6.8. Join

WMA link

`Join`[l_1, l_2]
    concatenates the lists l_1 and l_2.

`Join` concatenates lists:

```
>> Join[{a, b}, {c, d, e}]
```
$\{a, b, c, d, e\}$

```
>> Join[{{a, b}, {c, d}}, {{1, 2}, {3, 4}}]
```
$\{\{a, b\}, \{c, d\}, \{1, 2\}, \{3, 4\}\}$

The concatenated expressions may have any head:

```
>> Join[a + b, c + d, e + f]
```
$a + b + c + d + e + f$

However, it must be the same for all expressions:

```
>> Join[a + b, c * d]
Heads Plus and Times are expected to be the same.
```
$\text{Join}[a + b, cd]$

35.6.9. PadLeft

WMA link

`PadLeft`[*list*, n]
    pads *list* to length n by adding 0 on the left.
`PadLeft`[*list*, n, x]
    pads *list* to length n by adding x on the left.
`PadLeft`[*list*, $\{n_1, n_2, ...\}$, x]
    pads *list* to lengths n_1, n_2 at levels 1, 2, ... respectively by adding x on the left.
`PadLeft`[*list*, n, x, m]
    pads *list* to length n by adding x on the left and adding a margin of m on the right.
`PadLeft`[*list*, n, x, $\{m_1, m_2, ...\}$]
    pads *list* to length n by adding x on the left and adding margins of m_1, m_2, ... on levels 1, 2, ... on the right.
`PadLeft`[*list*]
    turns the ragged list *list* into a regular list by adding 0 on the left.

```
>> PadLeft[{1, 2, 3}, 5]
```
$\{0, 0, 1, 2, 3\}$

```
>> PadLeft[x[a, b, c], 5]
```
$x[0, 0, a, b, c]$

```
>> PadLeft[{1, 2, 3}, 2]
```
$\{2, 3\}$

>> `PadLeft[{{}, {1, 2}, {1, 2, 3}}]`

$\{\{0,0,0\},\{0,1,2\},\{1,2,3\}\}$

>> `PadLeft[{1, 2, 3}, 10, {a, b, c}, 2]`

$\{b,c,a,b,c,1,2,3,a,b\}$

>> `PadLeft[{{1, 2, 3}}, {5, 2}, x, 1]`

$\{\{x,x\},\{x,x\},\{x,x\},\{3,x\},\{x,x\}\}$

35.6.10. PadRight

WMA link

`PadRight`[*list*, *n*]
: pads *list* to length *n* by adding 0 on the right.

`PadRight`[*list*, *n*, *x*]
: pads *list* to length *n* by adding *x* on the right.

`PadRight`[*list*, {n_1, n_2, ...}, *x*]
: pads *list* to lengths n_1, n_2 at levels 1, 2, ... respectively by adding *x* on the right.

`PadRight`[*list*, *n*, *x*, *m*]
: pads *list* to length *n* by adding *x* on the left and adding a margin of *m* on the left.

`PadRight`[*list*, *n*, *x*, {m_1, m_2, ...}]
: pads *list* to length *n* by adding *x* on the right and adding margins of m_1, m_2, ... on levels 1, 2, ... on the left.

`PadRight`[*list*]
: turns the ragged list *list* into a regular list by adding 0 on the right.

>> `PadRight[{1, 2, 3}, 5]`

$\{1,2,3,0,0\}$

>> `PadRight[x[a, b, c], 5]`

$x[a,b,c,0,0]$

>> `PadRight[{1, 2, 3}, 2]`

$\{1,2\}$

>> `PadRight[{{}, {1, 2}, {1, 2, 3}}]`

$\{\{0,0,0\},\{1,2,0\},\{1,2,3\}\}$

>> `PadRight[{1, 2, 3}, 10, {a, b, c}, 2]`

$\{b,c,1,2,3,a,b,c,a,b\}$

>> `PadRight[{{1, 2, 3}}, {5, 2}, x, 1]`

$\{\{x,x\},\{x,1\},\{x,x\},\{x,x\},\{x,x\}\}$

35.6.11. Partition

WMA link

`Partition`[*list*, *n*]
: partitions *list* into sublists of length *n*.

`Partition`[*list*, *n*, *d*]
: partitions *list* into sublists of length *n* which overlap *d* indices.

>> **`Partition[{a, b, c, d, e, f}, 2]`**

$\{\{a,b\},\{c,d\},\{e,f\}\}$

>> **`Partition[{a, b, c, d, e, f}, 3, 1]`**

$\{\{a,b,c\},\{b,c,d\},\{c,d,e\},\{d,e,f\}\}$

35.6.12. Reverse

WMA link

`Reverse`[*expr*]
: reverses the order of *expr*'s items (on the top level)

`Reverse`[*expr*, *n*]
: reverses the order of items in *expr* on level *n*

`Reverse`[*expr*, {n_1, n_2, ...}]
: reverses the order of items in *expr* on levels n_1, n_2, ...

>> **`Reverse[{1, 2, 3}]`**

$\{3,2,1\}$

>> **`Reverse[x[a, b, c]]`**

$x[c,b,a]$

>> **`Reverse[{{1, 2}, {3, 4}}, 1]`**

$\{\{3,4\},\{1,2\}\}$

>> **`Reverse[{{1, 2}, {3, 4}}, 2]`**

$\{\{2,1\},\{4,3\}\}$

>> **`Reverse[{{1, 2}, {3, 4}}, {1, 2}]`**

$\{\{4,3\},\{2,1\}\}$

35.6.13. Riffle

WMA link

`Riffle`[*list*, *x*]
: inserts a copy of *x* between each element of *list*.

`Riffle`[{a_1, a_2, ...}, {b_1, b_2, ...}]
: interelements the elements of both lists, returning `{$a_1$, $b_1$, $a_2$, $b_2$, ...}`.

```
>> Riffle[{a, b, c}, x]
```
$\{a,x,b,x,c\}$

```
>> Riffle[{a, b, c}, {x, y, z}]
```
$\{a,x,b,y,c,z\}$

```
>> Riffle[{a, b, c, d, e, f}, {x, y, z}]
```
$\{a,x,b,y,c,z,d,x,e,y,f\}$

35.6.14. RotateLeft

WMA link

`RotateLeft`[*expr*]
: rotates the items of *expr'* by one item to the left.

`RotateLeft`[*expr*, *n*]
: rotates the items of *expr'* by *n* items to the left.

`RotateLeft`[*expr*, {n_1, n_2, ...}]
: rotates the items of *expr'* by n_1 items to the left at the first level, by n_2 items to the left at the second level, and so on.

```
>> RotateLeft[{1, 2, 3}]
```
$\{2,3,1\}$

```
>> RotateLeft[Range[10], 3]
```
$\{4,5,6,7,8,9,10,1,2,3\}$

```
>> RotateLeft[x[a, b, c], 2]
```
$x[c,a,b]$

```
>> RotateLeft[{{a, b, c}, {d, e, f}, {g, h, i}}, {1, 2}]
```
$\{\{f,d,e\},\{i,g,h\},\{c,a,b\}\}$

35.6.15. RotateRight

WMA link

`RotateRight`[*expr*]
: rotates the items of *expr'* by one item to the right.

`RotateRight`[*expr*, *n*]
: rotates the items of *expr'* by *n* items to the right.

`RotateRight`[*expr*, {n_1, n_2, ...}]
: rotates the items of *expr'* by n_1 items to the right at the first level, by n_2 items to the right at the second level, and so on.

```
>> RotateRight[{1, 2, 3}]
```
$\{3,1,2\}$

```
>> RotateRight[Range[10], 3]
```
$\{8,9,10,1,2,3,4,5,6,7\}$

```
>> RotateRight[x[a, b, c], 2]
```
$x[b,c,a]$

```
>> RotateRight[{{a, b, c}, {d, e, f}, {g, h, i}}, {1, 2}]
```
$\{\{h,i,g\},\{b,c,a\},\{e,f,d\}\}$

35.6.16. Split

WMA link

`Split[`*list*`]`
: splits *list* into collections of consecutive identical elements.

`Split[`*list*, *test*`]`
: splits *list* based on whether the function *test* yields `True` on consecutive elements.

```
>> Split[{x, x, x, y, x, y, y, z}]
```
$\{\{x,x,x\},\{y\},\{x\},\{y,y\},\{z\}\}$

Split into increasing or decreasing runs of elements

```
>> Split[{1, 5, 6, 3, 6, 1, 6, 3, 4, 5, 4}, Less]
```
$\{\{1,5,6\},\{3,6\},\{1,6\},\{3,4,5\},\{4\}\}$

```
>> Split[{1, 5, 6, 3, 6, 1, 6, 3, 4, 5, 4}, Greater]
```
$\{\{1\},\{5\},\{6,3\},\{6,1\},\{6,3\},\{4\},\{5,4\}\}$

Split based on first element

```
>> Split[{x -> a, x -> y, 2 -> a, z -> c, z -> a}, First[#1] === First
   [#2] &]
```
$\{\{x->a,x->y\},\{2->a\},\{z->c,z->a\}\}$

35.6.17. SplitBy

WMA link

`SplitBy[`*list*, f`]`
: splits *list* into collections of consecutive elements that give the same result when f is applied.

```
>> SplitBy[Range[1, 3, 1/3], Round]
```
$$\left\{\left\{1,\frac{4}{3}\right\},\left\{\frac{5}{3},2,\frac{7}{3}\right\},\left\{\frac{8}{3},3\right\}\right\}$$

```
>> SplitBy[{1, 2, 1, 1.2}, {Round, Identity}]
```
$\{\{\{1\}\},\{\{2\}\},\{\{1\},\{1.2\}\}\}$

35.6.18. Tally

WMA link

`Tally[`*list*`]`
: counts and returns the number of occurrences of objects and returns the result as a list of pairs {object, count}.

`Tally[`*list, test*`]`
: counts the number of occurrences of objects and uses *test* to determine if two objects should be counted in the same bin.

```
>> Tally[{a, b, c, b, a}]
```
$\{\{a,2\},\{b,2\},\{c,1\}\}$

Tally always returns items in the order as they first appear in *list*:

```
>> Tally[{b, b, a, a, a, d, d, d, d, c}]
```
$\{\{b,2\},\{a,3\},\{d,4\},\{c,1\}\}$

35.6.19. Union

WMA link

`Union[`*a, b, ...*`]`
: gives the union of the given set or sets. The resulting list will be sorted and each element will only occur once.

A union of two lists:

```
>> Union[{a, b, c}, {c, d, e}]
```
$\{a,b,c,d,e\}$

A union of two associations:

```
>> Union[{a -> b}, {c -> d}]
```
$\{a->b,c->d\}$

A union of one item is the item. Note that the list is sorted:

```
>> Union[{c, b, a}]
```

$\{a, b, c\}$

As usual, `Union` removes duplicate values:

```
>> Union[{5, 1, 3, 7, 1, 8, 3}]
```

$\{1, 3, 5, 7, 8\}$

`Union` using a custom test which compares using the last coordinate of each element list:

```
>> Union[{{a, 1}, {b, 2}}, {{c, 1}, {d, 3}}, SameTest->(SameQ[Last[#1],
   Last[#2]]&)]
```

$\{\{b, 2\}, \{c, 1\}, \{d, 3\}\}$

```
>> Union[{1, 2, 3}, {2, 3, 4}, SameTest->Less]
```

$\{1, 2, 2, 3, 4\}$

36. Low-level Format definitions

Contents

36.1. $BoxForms

WMA link

`$BoxForms`
: contains the list of box formats.

>> `$BoxForms`
{StandardForm, TraditionalForm}

36.2. MakeBoxes

WMA link

`MakeBoxes[`*expr*`]`
: is a low-level formatting primitive that converts *expr* to box form, without evaluating it.

`( ... )`
: directly inputs box objects.

String representation of boxes

>> `\(x \^ 2\)`
SuperscriptBox [x, 2]

>> `\(x \_ 2\)`
SubscriptBox [x, 2]

>> `\( a \+ b \% c\)`
UnderoverscriptBox [a, b, c]

```
>> \( a \& b \% c\)
```
UnderoverscriptBox [a, c, b]

```
>> \(x \& y \)
```
OverscriptBox [x, y]

```
>> \(x \+ y \)
```
UnderscriptBox [x, y]

36.3. ToBoxes

WMA link

`ToBoxes[`*expr*`]`
: evaluates *expr* and converts the result to box form.

Unlike `MakeBoxes`, `ToBoxes` evaluates its argument:

```
>> ToBoxes[a + a]
```
RowBox [{2, , a}]

```
>> ToBoxes[a + b]
```
RowBox [{a, +, b}]

```
>> ToBoxes[a ^ b] // FullForm
```
SuperscriptBox [“a”, “b”]

37. Mathematical Functions

Basic arithmetic functions, including complex number arithmetic.

Contents

37.1. $Assumptions

WMA link

`$Assumptions`
: is the default setting for the `Assumptions` option used in such functions as `Simplify`, `Refine`, and `Integrate`.

37.2. Arg

Argument (complex analysis) (WMA link)

`Arg`[*z*, `Method ->` "*option*"]
: returns the argument of a complex value *z*.

- `Arg`[*z*] is left unevaluated if *z* is not a numeric quantity.
- `Arg`[*z*] gives the phase angle of *z* in radians.
- The result from `Arg`[*z*] is always between -Pi and +Pi.
- `Arg`[*z*] has a branch cut discontinuity in the complex *z* plane running from -Infinity to 0.

- `Arg`[0] is 0.

>> `Arg[-3]`
π

Same as above, but using SymPy's method:

>> `Arg[-3, Method->"sympy"]`
π

>> `Arg[1-I]`
$-\frac{\pi}{4}$

`Arg` evaluates the direction of `DirectedInfinity` quantities by the `Arg` of its arguments:

>> `Arg[DirectedInfinity[1+I]]`
$\frac{\pi}{4}$

>> `Arg[DirectedInfinity[]]`
1

Arg for 0 is assumed to be 0:

>> `Arg[0]`
0

37.3. Assuming

WMA link

`Assuming`[*cond*, *expr*]
Evaluates *expr* assuming the conditions *cond*.

>> `$Assumptions = { x > 0 }`
$\{x > 0\}$

>> `Assuming[y>0, ConditionalExpression[y x^2, y>0]//Simplify]`
$x^2 y$

>> `Assuming[Not[y>0], ConditionalExpression[y x^2, y>0]//Simplify]`
Undefined

>> `ConditionalExpression[y x ^ 2, y > 0]//Simplify`
ConditionalExpression $\left[x^2 y, y > 0\right]$

37.4. Boole

WMA link

`Boole[expr]`
: returns 1 if expr is True and 0 if expr is False.

```
>> Boole[2 == 2]
```
1

```
>> Boole[7 < 5]
```
0

```
>> Boole[a == 7]
```
Boole [a==7]

37.5. Complex

WMA link

`Complex`
: is the head of complex numbers.

`Complex[`a, b`]`
: constructs the complex number `$a$ + I $b$`.

```
>> Head[2 + 3*I]
```
Complex

```
>> Complex[1, 2/3]
```
$1 + \frac{2I}{3}$

```
>> Abs[Complex[3, 4]]
```
5

37.6. ConditionalExpression

WMA link

`ConditionalExpression[`$expr$, $cond$`]`
: returns $expr$ if $cond$ evaluates to $True$, $Undefined$ if $cond$ evaluates to $False$.

```
>> ConditionalExpression[x^2, True]
```
x^2

```
>> ConditionalExpression[x^2, False]
```
Undefined

```
>> f = ConditionalExpression[x^2, x>0]
```
$\text{ConditionalExpression}\left[x^2, x > 0\right]$

```
>> f /. x -> 2
```
4

```
>> f /. x -> -2
```
Undefined

`ConditionalExpression` uses assumptions to evaluate the condition:

```
>> $Assumptions = x > 0;
```

```
>> ConditionalExpression[x ^ 2, x>0]//Simplify
```
x^2

```
>> $Assumptions = True;
```

» ConditionalExpression[ConditionalExpression[s,x>a], x<b] # = ConditionalExpression[s, And[x>a, x<b]]

37.7. Conjugate

Complex Conjugate WMA link

`Conjugate[`*z*`]`
   returns the complex conjugate of the complex number *z*.

```
>> Conjugate[3 + 4 I]
```
$3 - 4I$

```
>> Conjugate[3]
```
3

```
>> Conjugate[a + b * I]
```
$\text{Conjugate}[a] - I\text{Conjugate}[b]$

```
>> Conjugate[{{1, 2 + I 4, a + I b}, {I}}]
```
$\{\{1, 2 - 4I, \text{Conjugate}[a] - I\text{Conjugate}[b]\}, \{-I\}\}$

```
>> Conjugate[1.5 + 2.5 I]
```
$1.5 - 2.5I$

37.8. DirectedInfinity

WMA link

`DirectedInfinity[`z`]`
: represents an infinite multiple of the complex number z.

`DirectedInfinity[]`
: is the same as `ComplexInfinity`.

```
>> DirectedInfinity[1]
```
∞

```
>> DirectedInfinity[]
```
ComplexInfinity

```
>> DirectedInfinity[1 + I]
```
$$\left(\frac{1}{2}+\frac{I}{2}\right)\sqrt{2}\infty$$

```
>> 1 / DirectedInfinity[1 + I]
```
0

```
>> DirectedInfinity[1] + DirectedInfinity[-1]
Indeterminate expression -Infinity + Infinity encountered.
```
Indeterminate

```
>> DirectedInfinity[0]
```
ComplexInfinity

37.9. Element

Element of WMA link

`Element[`*expr*, *domain*`]`
: returns *True* if *expr* is an element of *domain*

`Element[`$expr_1$ | $expr_2$ | ..., *domain*`]`
: returns *True* if all the $expr_i$ belongs to *domain*, and *False* if one of the items doesn't.

Check if 3 and a are both integers. If a is not defined, then `Element` reduces the condition:

```
>> Element[3 | a, Integers]
```
Element $[a, \text{Integers}]$

Notice that standard domain names (`Primes`, `Integers`, `Rationals`, `Algebraics`, `Reals`, `Complexes`, and `Booleans`) are in plural form. If a singular form is used, a warning is shown:

```
>> Element[a, Real]
The second argument Real of Element should be one of: Primes,
Integers, Rationals, Algebraics, Reals, Complexes, or Booleans.
```

Element [a, Real]

37.10. I

Imaginary unit (WMA)

`I`
    represents the imaginary number `Sqrt[-1]`.

```
>> I^2
```

-1

```
>> (3+I)*(3-I)
```

10

37.11. Im

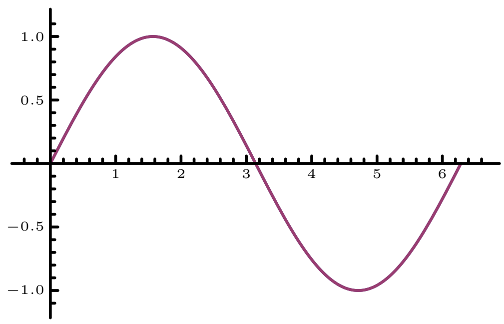
WMA link

`Im[`z`]`
    returns the imaginary component of the complex number z.

```
>> Im[3+4I]
```

4

```
>> Plot[{Sin[a], Im[E^(I a)]}, {a, 0, 2 Pi}]
```



37.12. Integer

WMA link

`Integer`
  is the head of integers.

```
>> Head[5]
```
Integer

37.13. Product (∏)

Direct product (SymPy, WMA)

`Product`[f, {i, i_{min}, i_{max}}]
  evaluates the discrete product of f with i ranging from i_{min} to i_{max}.

`Product`[f, {i, i_{max}}]
  same as `Product`[f, {i, 1, i_{max}}].

`Product`[f, {i, i_{min}, i_{max}, di}]
  i ranges from i_{min} to i_{max} in steps of di.

`Product`[f, {i, i_{min}, i_{max}}, {j, j_{min}, j_{max}}, ...]
  evaluates f as a multiple product, with {i, ...}, {j, ...}, ... being in outermost-to-innermost order.

`Product[k, {k, i, n}]` is defined in terms of Factorial 51.5.2:

```
>> Product[k, {k, i, n}]
```
$$\frac{n!}{(-1+i)!}$$

When i is 1, we get the factorial function:

```
>> Product[k, {k, 1, n}]
```
$n!$

Or more succinctly:

```
>> Product[k, {k, n}]
```
$n!$

Symbolic products involving the factorial are evaluated:

```
>> Product[k, {k, 3, n}]
```
$$\frac{n!}{2}$$

Examples of numeric evaluation using more complex functions:

```
>> Product[x^k, {k, 2, 20, 2}]
```
x^{110}

```
>> Product[2 ^ i, {i, 1, n}]
```
$2^{\frac{n}{2}+\frac{n^2}{2}}$

```
>> Product[f[i], {i, 1, 7}]
```
$f[1] f[2] f[3] f[4] f[5] f[6] f[7]$

Evaluate the n-th Primorial:

```
>> Primorial[0] = 1;
```

```
>> Primorial[n_Integer] := Product[Prime[k], {k, 1, n}];
```

```
>> Primorial[12]
```
7420738134810

37.14. Rational

WMA link

`Rational`
: is the head of rational numbers.

`Rational`[a, b]
: constructs the rational number a / b.

```
>> Head[1/2]
```
Rational

```
>> Rational[1, 2]
```
$\frac{1}{2}$

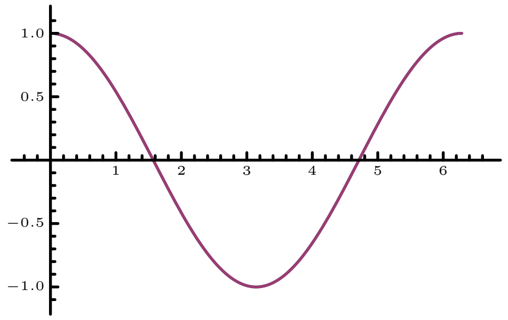
37.15. Re

WMA link

`Re`[z]
: returns the real component of the complex number z.

```
>> Re[3+4I]
```
3

>> `Plot[{Cos[a], Re[E^(I a)]}, {a, 0, 2 Pi}]`



37.16. Real

WMA link

`Real`
is the head of real (inexact) numbers.

>> `x = 3. ^ -20;`

>> `InputForm[x]`
2.8679719907924413*^ − 10

>> `Head[x]`
Real

37.17. RealValuedNumberQ

WMA link

`RealValuedNumberQ[`*expr*`]`
returns `True` if *expr* is an explicit number with no imaginary component.

>> `RealValuedNumberQ[10]`
True

>> `RealValuedNumberQ[4.0]`
True

>> `RealValuedNumberQ[1+I]`
False

>> `RealValuedNumberQ[0 * I]`
True

```
>> RealValuedNumberQ[0.0 * I]
```
False

"Underflow[]" and "Overflow[]" are considered Real valued numbers:

```
>> {RealValuedNumberQ[Underflow[]], RealValuedNumberQ[Overflow[]]}
```
$\{\text{True}, \text{True}\}$

37.18. Sum (∑)

Summation (SymPy, WMA)

`Sum[`$f, \{i, i_{min}, i_{max}\}$`]`
: evaluates the discrete sum of f with i ranging from i_{min} to i_{max}.

`Sum[`$f, \{i, i_{max}\}$`]`
: same as `Sum[`$f, \{i, 1, i_{max}\}$`]` .

`Sum[`$f, \{i, i_{min}, i_{max}, di\}$`]`
: i ranges from i_{min} to i_{max} in steps of di.

`Sum[`$f, \{i, i_{min}, i_{max}, \{j, j_{min}, j_{max}, ...$`]`
: evaluates f as a multiple sum, with $\{i, ...\}, \{j, ...\}, ...$ being in outermost-to-innermost order.

A sum that Gauss in elementary school was asked to do to kill time:

```
>> Sum[k, {k, 1, 10}]
```
55

The symbolic form he used:

```
>> Sum[k, {k, 1, n}]
```
$$\frac{n(1+n)}{2}$$

A Geometric series with a finite limit:

```
>> Sum[1 / 2 ^ i, {i, 1, k}]
```
$1 - 2^{-k}$

A Geometric series using Infinity:

```
>> Sum[1 / 2 ^ i, {i, 1, Infinity}]
```
1

Leibniz formula used in computing Pi:

```
>> Sum[1 / ((-1)^k (2k + 1)), {k, 0, Infinity}]
```
$$\frac{\pi}{4}$$

A table of double sums to compute squares:

```
>> Table[ Sum[i * j, {i, 0, n}, {j, 0, n}], {n, 0, 4} ]
```

$\{0,1,9,36,100\}$

Computing Harmonic using a sum

```
>> Sum[1 / k ^ 2, {k, 1, n}]
```

HarmonicNumber $[n,2]$

Other symbolic sums:

```
>> Sum[k, {k, n, 2 n}]
```

$\frac{3n(1+n)}{2}$

A sum with Complex-number iteration values

```
>> Sum[k, {k, I, I + 1}]
```

$1+2I$

```
>> Sum[k, {k, Range[5]}]
```

15

```
>> Sum[f[i], {i, 1, 7}]
```

$f[1]+f[2]+f[3]+f[4]+f[5]+f[6]+f[7]$

Verify algebraic identities:

```
>> Sum[x ^ 2, {x, 1, y}] - y * (y + 1)* (2 * y + 1)/ 6
```

0

Non-integer bounds:

```
>> Sum[i, {i, 1, 2.5}]
```

3

```
>> Sum[i, {i, 1.1, 2.5}]
```

3.2

```
>> Sum[k, {k, I, I + 1.5}]
```

$1+2I$

38. Mathematical Optimization

Mathematical optimization is the selection of a best element, with regard to some criterion, from some set of available alternatives.

Optimization problems of sorts arise in all quantitative disciplines from computer science and engineering to operations research and economics, and the development of solution methods has been of interest in mathematics for centuries.

We intend to provide local and global optimization techniques, both numeric and symbolic.

Contents

38.1. Maximize

WMA link

`Maximize`[f, x]
: compute the maximum of f respect x that change between a and b.

```
>> Maximize[-2 x^2 - 3 x + 5, x]
```

$$\left\{\left\{\frac{49}{8},\left\{x->-\frac{3}{4}\right\}\right\}\right\}$$

38.2. Minimize

WMA link

`Minimize`[f, x]
: compute the minimum of f respect x that change between a and b.

```
>> Minimize[2 x^2 - 3 x + 5, x]
```

$$\left\{\left\{\frac{31}{8},\left\{x->\frac{3}{4}\right\}\right\}\right\}$$

39. Matrices and Linear Algebra

Construction and manipulation of Matrices.

Contents

39.1. Constructing Matrices

Methods for constructing Matrices.

39.1.1. BoxMatrix

WMA link

`BoxMatrix`[*s*]
Gives a box shaped kernel of size 2 s + 1.

```
>> BoxMatrix[3]
```
{{1,1,1,1,1,1,1},{1,1,1,1,1,1,1},{1,1,1,1,1,1,1},{1,1,1,1,1,1,1},{1,1,1,1,1,1,1},{1,1,1,1,1,1,1},{1,1,1

39.1.2. DiagonalMatrix

WMA link

`DiagonalMatrix`[*list*]
gives a matrix with the values in *list* on its diagonal and zeroes elsewhere.

```
>> DiagonalMatrix[{1, 2, 3}]
```
{{1,0,0},{0,2,0},{0,0,3}}

```
>> MatrixForm[%]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

39.1.3. DiamondMatrix

WMA link

`DiamondMatrix[s]`
Gives a diamond shaped kernel of size 2 s + 1.

```
>> DiamondMatrix[3]
```

{{0,0,0,1,0,0,0},{0,0,1,1,1,0,0},{0,1,1,1,1,1,0},{1,1,1,1,1,1,1},{0,1,1,1,1,1,0},{0,0,1,1,1,0,0},{0,0,0

39.1.4. DiskMatrix

WMA link

`DiskMatrix[s]`
Gives a disk shaped kernel of size 2 s + 1.

```
>> DiskMatrix[3]
```

{{0,0,1,1,1,0,0},{0,1,1,1,1,1,0},{1,1,1,1,1,1,1},{1,1,1,1,1,1,1},{1,1,1,1,1,1,1},{0,1,1,1,1,1,0},{0,0,1

39.1.5. IdentityMatrix

WMA link

`IdentityMatrix[n]`
gives the identity matrix with n rows and columns.

```
>> IdentityMatrix[3]
```

{{1,0,0},{0,1,0},{0,0,1}}

39.2. Parts of Matrices

Methods for manipulating Matrices.

39.2.1. Diagonal

WMA link

`Diagonal[`m`]`
: gives a list with the values in the diagonal of the matrix m.

`Diagonal[`m, k`]`
: gives a list with the values in the k diagonal of the matrix m.

```
>> Diagonal[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
```
$\{1,5,9\}$

Get the superdiagonal:

```
>> Diagonal[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, 1]
```
$\{2,6\}$

Get the subdiagonal:

```
>> Diagonal[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}, -1]
```
$\{4,8\}$

Get the diagonal of a nonsquare matrix:

```
>> Diagonal[{{1, 2, 3}, {4, 5, 6}}]
```
$\{1,5\}$

40. Message-related functions.

Contents

40.1. $Aborted

WMA link

`$Aborted`
  is returned by a calculation that has been aborted.

40.2. $Failed

WMA link

`$Failed`
  is returned by some functions in the event of an error.

40.3. Check

WMA link

`Check`[*expr*, *failexpr*]
  evaluates *expr*, and returns the result, unless messages were generated, in which case it evaluates and *failexpr* will be returned.
`Check`[*expr*, *failexpr*, {s1::t1,s2::t2,...}]
  checks only for the specified messages.

Return err when a message is generated:

```
>> Check[1/0, err]
   Infinite expression 1 / 0 encountered.
```
err

Check only for specific messages:

```
>> Check[Sin[0^0], err, Sin::argx]
   Indeterminate expression 0 ^ 0 encountered.
```
Indeterminate

```
>> Check[1/0, err, Power::infy]
   Infinite expression 1 / 0 encountered.
```
err

40.4. Failure

WMA link

Failure[*tag*, *assoc*]
: represents a failure of a type indicated by *tag*, with details given by the association *assoc*.

40.5. General

WMA link

`General`
: is a symbol to which all general-purpose messages are assigned.

```
>> General::argr
```
'1' called with 1 argument; '2' arguments are expected.

```
>> Message[Rule::argr, Rule, 2]
   Rule called with 1 argument; 2 arguments are expected.
```

40.6. Message

WMA link

`Message[`*symbol*::*msg*, *expr*$_1$, *expr*$_2$, ...]
: displays the specified message, replacing placeholders in the message text with the corresponding expressions.

```
>> a::b = "Hello world!"
```
Hello world!

```
>> Message[a::b]
Hello world!
```

```
>> a::c := "Hello `1`, Mr 00`2`!"
```

```
>> Message[a::c, "you", 3 + 4]
Hello you, Mr 007!
```

40.7. MessageName (::)

WMA link

`MessageName[`*symbol*, *tag*]
`$symbol$::$tag$`
: identifies a message.

`MessageName` is the head of message IDs of the form `symbol::tag`.

```
>> FullForm[a::b]
```
MessageName [*a*, "b"]

The second parameter `tag` is interpreted as a string.

```
>> FullForm[a::"b"]
```
MessageName [*a*, "b"]

40.8. Off

WMA link

`Off[`*symbol*::*tag*]
: turns a message off so it is no longer printed.

```
>> Off[Power::infy]
```

```
>> 1 / 0
```
ComplexInfinity

```
>> Off[Power::indet, Syntax::com]
```

```
>> {0 ^ 0,}
```
{Indeterminate, Null}

40.9. On

WMA link

`On`[*symbol*::*tag*]
    turns a message on for printing.

```
>> Off[Power::infy]
```

```
>> 1 / 0
```
ComplexInfinity

```
>> On[Power::infy]
```

```
>> 1 / 0
Infinite expression 1 / 0 encountered.
```
ComplexInfinity

40.10. Quiet

WMA link

`Quiet`[$expr$, {s_1::t_1, ...}]
    evaluates $expr$, without messages `{$s_1$::$t_1$, ...}` being displayed.
`Quiet`[$expr$, All]
    evaluates $expr$, without any messages being displayed.
`Quiet`[$expr$, None]
    evaluates $expr$, without all messages being displayed.
`Quiet`[$expr$, off, on]
    evaluates $expr$, with messages off being suppressed, but messages on being displayed.

Evaluate without generating messages:

```
>> Quiet[1/0]
```
ComplexInfinity

Same as above:

```
>> Quiet[1/0, All]
```
ComplexInfinity

```
>> a::b = "Hello";
```

```
>> Quiet[x+x, {a::b}]
```
$2x$

```
>> Quiet[Message[a::b]; x+x, {a::b}]
```
$2x$

```
>> Message[a::b]; y=Quiet[Message[a::b]; x+x, {a::b}]; Message[a::b]; y
Hello
Hello
```
$2x$

```
>> Quiet[x + x, {a::b}, {a::b}]
In Quiet[x + x, {a::b}, {a::b}] the message name(s) {a::b} appear in
both the list of messages to switch off and the list of messages to
switch on.
```
$\text{Quiet}\left[x + x, \{a\text{::b}\}, \{a\text{::b}\}\right]$

40.11. Syntax

WMA link

`Syntax`
    is a symbol to which all syntax messages are assigned.

```
>> 1 +
Incomplete expression; more input is needed (line 1 of ).
```

```
>> Sin[1)
"Sin[1" cannot be followed by ")" (line 1 of ")").
```

```
>> ^ 2
Expression cannot begin with "^ 2" (line 1 of ).
```

```
>> 1.5``
"1.5`" cannot be followed by "`" (line 1 of "`").
```

41. Numerical Functions

Support for approximate real numbers and exact real numbers represented in algebraic or symbolic form.

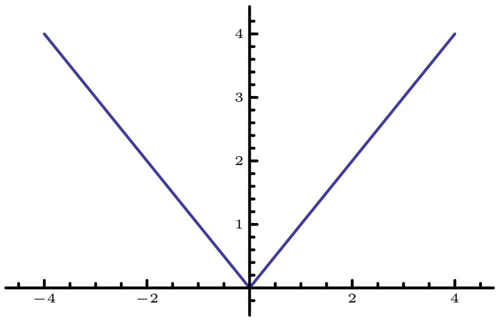
Contents

41.1. Abs

Absolute value (SymPy, WMA)

`Abs[`*x*`]`
    returns the absolute value of *x*.

```
>> Abs[-3]
```
3

```
>> Plot[Abs[x], {x, -4, 4}]
```


`Abs` returns the magnitude of complex numbers:

```
>> Abs[3 + I]
```
$\sqrt{10}$

```
>> Abs[3.0 + I]
```
3.16228

All of the below evaluate to Infinity:

```
>> Abs[Infinity] == Abs[I Infinity] == Abs[ComplexInfinity]
```
True

41.2. Chop

WMA link

`Chop[`*expr*`]`
: replaces floating point numbers close to 0 by 0.

`Chop[`*expr*, *delta*`]`
: uses a tolerance of *delta*. The default tolerance is `10^-10`.

```
>> Chop[10.0 ^ -16]
```
0

```
>> Chop[10.0 ^ -9]
```
$1.*^{\wedge} - 9$

```
>> Chop[10 ^ -11 I]
```
$$\frac{I}{100000000000}$$

```
>> Chop[0. + 10 ^ -11 I]
```
0

41.3. N

WMA link

`N[`*expr*, *prec*`]`
: evaluates *expr* numerically with a precision of *prec* digits.

```
>> N[Pi, 50]
```
3.1415926535897932384626433832795028841971693993751

```
>> N[1/7]
```
0.142857

```
>> N[1/7, 5]
```
0.14286

You can manually assign numerical values to symbols.

When you do not specify a precision, `MachinePrecision` is taken.

```
>> N[a] = 10.9
```
10.9

```
>> a
```
a

`N` automatically threads over expressions, except when a symbol has attributes `NHoldAll`, `NHoldFirst`, or `NHoldRest`.

```
>> N[a + b]
```
$10.9 + b$

```
>> N[a, 20]
```
a

```
>> N[a, 20] = 11;
```

```
>> N[a + b, 20]
```
$11.000000000000000000 + b$

```
>> N[f[a, b]]
```
$f[10.9, b]$

```
>> SetAttributes[f, NHoldAll]
```

```
>> N[f[a, b]]
```
$f[a, b]$

The precision can be a pattern:

```
>> N[c, p_?(#>10&)] := p
```

```
>> N[c, 3]
```
c

```
>> N[c, 11]
```
11.000000000

You can also use `UpSet` or `TagSet` to specify values for `N`:

```
>> N[d] ^= 5;
```

However, the value will not be stored in `UpValues`, but in `NValues` (as for `Set`):

```
>> UpValues[d]
```
{}

```
>> NValues[d]
```
{HoldPattern [*N* [*d*, MachinePrecision]] :>5}

```
>> e /: N[e] = 6;
```

```
>> N[e]
```
6.

Values for `N[$expr$]` must be associated with the head of *expr*:

```
>> f /: N[e[f]] = 7;
   Tag f not found or too deep for an assigned rule.
```

You can use `Condition`:

```
>> N[g[x_, y_], p_] := x + y * Pi /; x + y > 3
```

```
>> SetAttributes[g, NHoldRest]
```

```
>> N[g[1, 1]]
```

$g[1., 1]$

```
>> N[g[2, 2]] // InputForm
```

8.283185307179586

The precision of the result is no higher than the precision of the input

```
>> N[Exp[0.1], 100]
```

1.10517

```
>> % // Precision
```

MachinePrecision

```
>> N[Exp[1/10], 100]
```

1.105170918075647624811707826490246668224547194737518718792863289440967966747654302989143318970748654

```
>> % // Precision
```

100.

```
>> N[Exp[1.0`20], 100]
```

2.7182818284590452354

```
>> % // Precision
```

20.

N can also accept an option "Method". This establishes what is the prefrered underlying method to compute numerical values:

```
>> N[F[Pi], 30, Method->"numpy"]
```

$F[3.14159265358979300000000000000]$

```
>> N[F[Pi], 30, Method->"sympy"]
```

$F[3.14159265358979323846264338328]$

41.4. Piecewise

SymPy, WMA

```
Piecewise[{{expr1, cond1}, ...}]
```
represents a piecewise function.

```
Piecewise[{{expr1, cond1}, ...}, expr]
```
represents a piecewise function with default `expr`.

Heaviside function

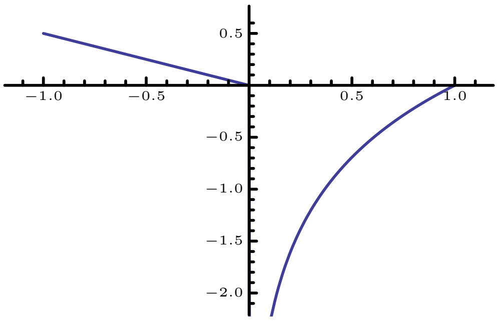
```
>> Piecewise[{{0, x <= 0}}, 1]
```
Piecewise $\left[\{\{0, x{<=}0\}\}, 1\right]$

```
>> Integrate[Piecewise[{{1, x <= 0}, {-1, x > 0}}], x]
```
Piecewise $\left[\{\{x, x{<=}0\}\}, -x\right]$

```
>> Integrate[Piecewise[{{1, x <= 0}, {-1, x > 0}}], {x, -1, 2}]
```
-1

Piecewise defaults to 0 if no other case is matching.

```
>> Piecewise[{{1, False}}]
```
0

```
>> Plot[Piecewise[{{Log[x], x > 0}, {x*-0.5, x < 0}}], {x, -1, 1}]
```



```
>> Piecewise[{{0 ^ 0, False}}, -1]
```
-1

41.5. Rationalize

WMA link

`Rationalize[`x`]`
converts a real number x to a nearby rational number with small denominator.

`Rationalize[`x, d_x`]`
finds the rational number lies within d_x of x.

```
>> Rationalize[2.2]
```
$\frac{11}{5}$

For negative x, `-Rationalize[-$x$] == Rationalize[$x$]` which gives symmetric results:

```
>> Rationalize[-11.5, 1]
```
-11

Not all numbers can be well approximated.

```
>> Rationalize[N[Pi]]
```
3.14159

Find the exact rational representation of `N[Pi]`

```
>> Rationalize[N[Pi], 0]
```
$$\frac{245850922}{78256779}$$

41.6. RealAbs

Abs (Real) (WMA link)

`RealAbs`[x]
: returns the absolute value of a real number x.

`RealAbs` is also known as modulus. It is evaluated if x can be compared with 0.

```
>> RealAbs[-3.]
```
3.

`RealAbs[$z$]` is left unevaluated for complex z:

```
>> RealAbs[2. + 3. I]
```
$\text{RealAbs}[2. + 3.I]$

```
>> D[RealAbs[x ^ 2], x]
```
$$\frac{2x^3}{\text{RealAbs}\left[x^2\right]}$$

41.7. RealSign

Sign function (WMA link)

`RealSign`[x]
: returns -1, 0 or 1 depending on whether x is negative, zero or positive.

`RealSign` is also known as *sgn* or *signum* function.

```
>> RealSign[-3.]
```
-1

`RealSign[$z$]` is left unevaluated for complex z:

```
>> RealSign[2. + 3. I]
```
RealSign [2. + 3.I]

```
>> D[RealSign[x^2],x]
```
$2x\text{Piecewise}\left[\left\{\left\{0, x^2!=0\right\}\right\}, \text{Indeterminate}\right]$

```
>> Integrate[RealSign[u],{u,0,x}]
```
RealAbs [x]

41.8. Round

WMA link

`Round[`*expr*`]`
 rounds *expr* to the nearest integer.

`Round[`*expr*, *k*`]`
 rounds *expr* to the closest multiple of *k*.

```
>> Round[10.6]
```
11

```
>> Round[0.06, 0.1]
```
0.1

```
>> Round[0.04, 0.1]
```
0.

Constants can be rounded too

```
>> Round[Pi, .5]
```
3.

```
>> Round[Pi^2]
```
10

Round to exact value

```
>> Round[2.6, 1/3]
```
$\frac{8}{3}$

```
>> Round[10, Pi]
```
3π

Round complex numbers

```
>> Round[6/(2 + 3 I)]
```
$1 - I$

```
>> Round[1 + 2 I, 2 I]
```
$2I$

Round Negative numbers too

```
>> Round[-1.4]
```
-1

Expressions other than numbers remain unevaluated:

```
>> Round[x]
```
Round $[x]$

```
>> Round[1.5, k]
```
Round $[1.5, k]$

41.9. Sign

Sign (WMA link)

`Sign[`x`]`
: return -1, 0, or 1 depending on whether x is negative, zero, or positive.

```
>> Sign[19]
```
1

```
>> Sign[-6]
```
-1

```
>> Sign[0]
```
0

```
>> Sign[{-5, -10, 15, 20, 0}]
```
$\{-1, -1, 1, 1, 0\}$

For a complex number, `Sign` returns the phase of the number:

```
>> Sign[3 - 4*I]
```
$$\frac{3}{5} - \frac{4I}{5}$$

41.10. UnitStep

Heaviside step function (WMA link)

`UnitStep[`x`]`
: return 0 if $x < 0$, and 1 if $x >= 0$.

`UnitStep[`x_1, x_2, ...`]`
: return the multidimensional unit step function which is 1 only if none of the xi are negative.

Evaluation numerically:

```
>> UnitStep[0.7]
   1
```

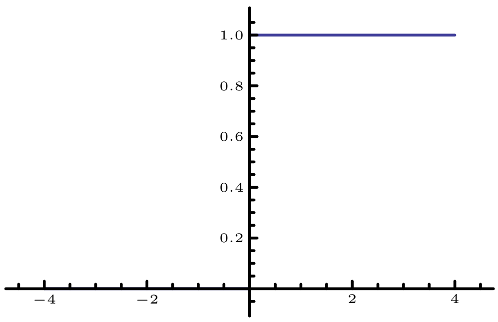
We can use `UnitStep` on irrational numbers and infinities:

```
>> Map[UnitStep, {Pi, Infinity, -Infinity}]
   {1,1,0}
```

```
>> Table[UnitStep[x], {x, -3, 3}]
   {0,0,0,1,1,1,1}
```

Plot in one dimension:

```
>> Plot[UnitStep[x], {x, -4, 4}]
```



42. Operations on Vectors

In mathematics and physics, a vector is a term that refers colloquially to some quantities that cannot be expressed by a single number. It is also a row or column of a matrix.

In computer science, it is an array data structure consisting of collection of elements identified by at least on array index or key.

In *Mathics3* vectors as are Lists. One never needs to distinguish between row and column vectors. As with other objects vectors can mix number and symbolic elements.

Vectors can be long, dense, or sparse.

Here are the grouping we have for Vector Operations:

Contents

42.1. Constructing Vectors

Functions for constructing lists of various sizes and structure.

See also Constructing Lists.

42.1.1. AngleVector

WMA link

`AngleVector[`ϕ`]`
: returns the point at angle ϕ on the unit circle.

`AngleVector[`$\{r, \phi\}$`]`
: returns the point at angle ϕ on a circle of radius r.

`AngleVector[`$\{x, y\}, \phi$`]`
: returns the point at angle ϕ on a circle of radius 1 centered at $\{x, y\}$.

`AngleVector[`$\{x, y\}, \{r, \phi\}$`]`
: returns point at angle ϕ on a circle of radius r centered at $\{x, y\}$.

```
>> AngleVector[90 Degree]
```
$\{0,1\}$

```
>> AngleVector[{1, 10}, a]
```
$\{1+\mathrm{Cos}[a], 10+\mathrm{Sin}[a]\}$

42.2. Mathematical Operations

42.2.1. Cross

Cross product (SymPy, WMA)

`Cross[`*a*, *b*`]`
computes the vector cross product of *a* and *b*.

Three-dimensional cross product:

```
>> Cross[{x1, y1, z1}, {x2, y2, z2}]
```
$\{\mathrm{y1z2}-\mathrm{y2z1}, -\mathrm{x1z2}+\mathrm{x2z1}, \mathrm{x1y2}-\mathrm{x2y1}\}$

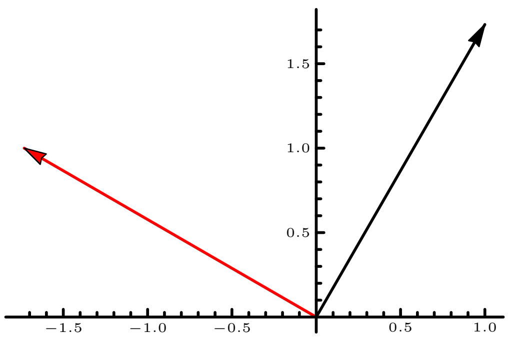
`Cross` is antisymmetric, so:

```
>> Cross[{x, y}]
```
$\{-y, x\}$

Graph two-Dimensional cross product:

```
>> v1 = {1, Sqrt[3]}; v2 = Cross[v1]
```
$\left\{-\sqrt{3}, 1\right\}$

Visualize this:

```
>> Graphics[{Arrow[{{0, 0}, v1}], Red, Arrow[{{0, 0}, v2}]}, Axes ->
True]
```


```
>> Cross[{1, 2}, {3, 4, 5}]
The arguments are expected to be vectors of equal length, and the
number of arguments is expected to be 1 less than their length.
```

Cross [{1,2} , {3,4,5}]

42.2.2. Curl

Curl (SymPy, WMA)

`Curl`[{f_1, f_2}, {x_1, x_2}]
    returns the curl df_2/dx_1 - df_1/dx_2

`Curl`[{f_1, f_2, f_3} {x_1, x_2, x_3}]
    returns the curl (df_3/dx_2 - df_2/dx_3, dx_3/df_3 - df_3/dx_1, df_2/df_1 - df_1/dx_2)

Two-dimensional `Curl`:

```
>> Curl[{y, -x}, {x, y}]
```

-2

```
>> v[x_, y_] := {Cos[x] Sin[y], Cos[y] Sin[x]}
```

```
>> Curl[v[x, y], {x, y}]
```

0

Three-dimensional `Curl`:

```
>> Curl[{y, -x, 2 z}, {x, y, z}]
```

$\{0,0,-2\}$

42.2.3. Norm

Matrix norms induced by vector p-norms (SymPy, WMA)

`Norm`[m, p]
    computes the p-norm of matrix m.

`Norm`[m]
    computes the 2-norm of matrix m.

The `Norm` of of a vector is its Euclidean distance:

```
>> Norm[{x, y, z}]
```

$\sqrt{\text{Abs}[x]^2 + \text{Abs}[y]^2 + \text{Abs}[z]^2}$

By default, 2-norm is used for vectors, but you can be explicit:

```
>> Norm[{3, 4}, 2]
```
5

The 1-norm is the sum of the values:

```
>> Norm[{10, 100, 200}, 1]
```
310

```
>> Norm[{x, y, z}, Infinity]
```
$\text{Max}\left[\left\{\text{Abs}[x], \text{Abs}[y], \text{Abs}[z]\right\}\right]$

```
>> Norm[{-100, 2, 3, 4}, Infinity]
```
100

For complex numbers, `Norm[$z$]` is `Abs[$z$]`:

```
>> Norm[1 + I]
```
$\sqrt{2}$

So the norm is always real, even when the input is complex.

`Norm`[m,"Frobenius"] gives the Frobenius norm of m:

```
>> Norm[Array[Subscript[a, ##] &, {2, 2}], "Frobenius"]
```
$\sqrt{\text{Abs}\left[a_{1,1}\right]^2 + \text{Abs}\left[a_{1,2}\right]^2 + \text{Abs}\left[a_{2,1}\right]^2 + \text{Abs}\left[a_{2,2}\right]^2}$

42.3. Vector Space Operations

42.3.1. KroneckerProduct

Kronecker product (SymPy, WMA)

`KroneckerProduct`[m_1, m_2, ...]
    returns the Kronecker product of the arrays m_i

Show symbolically how the Kronecker product works on two two-dimensional arrays:

```
>> a = {{a11, a12}, {a21, a22}}; b = {{b11, b12}, {b21, b22}};
```

```
>> KroneckerProduct[a, b]
```
{{a11b11, a11b12, a12b11, a12b12}, {a11b21, a11b22, a12b21, a12b22}, {a21b11, a21b12, a22b11, a22b12}, {a21b21, a

Now do the same with discrete values:

```
>> a = {{0, 1}, {-1, 0}}; b = {{1, 2}, {3, 4}};
```

```
>> KroneckerProduct[a, b] // MatrixForm
```
$$\begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \\ -1 & -2 & 0 & 0 \\ -3 & -4 & 0 & 0 \end{pmatrix}$$

42.3.2. Normalize

WMA link

`Normalize[`v`]`
: calculates the normalized vector v.

`Normalize[`z`]`
: calculates the normalized complex number z.

```
>> Normalize[{1, 1, 1, 1}]
```
$$\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$$

```
>> Normalize[1 + I]
```
$$\left(\frac{1}{2} + \frac{I}{2}\right)\sqrt{2}$$

42.3.3. Projection

WMA link

`Projection[`u, v`]`
: gives the projection of the vector u onto v

For vectors u and v, the projection is taken to be (v . u / v . v) v

For complex vectors u and v, the projection is taken to be (v^* . u / v^* . v) v where v^* is `Conjugate[v]`.

Projection of two three-dimensional Integer vectors:

```
>> Projection[{5, 6, 7}, {1, 0, 0}]
```
$\{5, 0, 0\}$

Projection of two two-dimensional Integer vectors:

```
>> Projection[{2, 3}, {1, 2}]
```
$$\left\{\frac{8}{5}, \frac{16}{5}\right\}$$

Projection of a machine-precision vector onto another:

```
>> Projection[{1.3, 2.1, 3.1}, {-0.3, 4.2, 5.3}]
```
$\{-0.162767, 2.27874, 2.87556\}$

Projection of two complex vectors:

```
>> Projection[{3 + I, 2, 2 - I}, {2, 4, 5 I}]
```
$\left\{\frac{2}{5}-\frac{16I}{45}, \frac{4}{5}-\frac{32I}{45}, \frac{8}{9}+I\right\}$

Project a symbol vector onto a numeric vector:

```
>> Projection[{a, b, c}, {1, 1, 1}]
```
$\left\{\frac{a+b+c}{3}, \frac{a+b+c}{3}, \frac{a+b+c}{3}\right\}$

The projection of vector u onto vector v is in the direction of v:

```
>> {u, v} = RandomReal[1, {2, 6}];
```

```
>> Abs[VectorAngle[Projection[u, v], v]] < 0. + 10^-7
```
True

42.3.4. UnitVector

Unit vector (WMA)

`UnitVector`[n, k]
: returns the n-dimensional unit vector with a 1 in position k.

`UnitVector`[k]
: is equivalent to `UnitVector[2, $k$]`.

```
>> UnitVector[2]
```
$\{0, 1\}$

```
>> UnitVector[4, 3]
```
$\{0, 0, 1, 0\}$

42.3.5. VectorAngle

WMA link

`VectorAngle`[u, v]
: gives the angles between vectors u and v

>> `VectorAngle[{1, 0}, {0, 1}]`

$\frac{\pi}{2}$

>> `VectorAngle[{1, 2}, {3, 1}]`

$\frac{\pi}{4}$

>> `VectorAngle[{1, 1, 0}, {1, 0, 1}]`

$\frac{\pi}{3}$

43. Operators without Built-in Meanings

Not all operators recognized by the Mathics3 are associated with functions that have built-in meanings. You can use these operators as a way to build up your own notation within Mathics3.

Contents

43.1. Infix Operators that require Additional Mathics3 Modules

Some Infix operators require loading Mathics3 Modules before the operators is used in a special way.

Right now, this happens for directed and undirected edges of a network graph. Before issuing `LoadModule["pymathics.g` the operators here have no meaning and can be user defined like other operators that have no pre-set meaning.

43.1.1. DirectedEdge (→)

WML link

`DirectedEdge`[x, y, ...]
    displays $x \rightarrow y \rightarrow$ ...
Directed edges are typically used in network graphs. In Mathics3, network graphs are supported through a Mathics3 module.
Issue `LoadModule["pymathics.graph"]` after `pip` installing Python package `pymathics-graph`.

```
>> DirectedEdge[x, y, z]
```
$x \rightarrow y \rightarrow z$

```
>> a \[DirectedEdge] b
```
$a \rightarrow b$

43.1.2. UndirectedEdge (↔)

WML link

`UndirectedEdge`[x, y, ...]
    displays $x \leftrightarrow y$ ...

Undirected edges are typically used in network graphs. In Mathics3, network graphs are supported through a Mathics3 module.
Issue `LoadModule["pymathics.graph"]` after `pip` installing Python package `pymathics-graph`.

```
>> UndirectedEdge[x, y, z]
```
$x \leftrightarrow y \leftrightarrow z$

```
>> a <-> b
```
$a \leftrightarrow b$

43.2. Infix Operators without Built-in Meanings

43.2.1. Backslash

WML link

`Backslash`[x, y, ...]
    displays x y ...

```
>> Backslash[x, y, z]
```
x
y
z

```
>> a \[Backslash] b
```
a
b

43.2.2. Because (∵)

WML link

`Because`[x, y, ...]
    displays $x \because y \because$ ...

```
>> Because[x, y, z]
```
$x \because y \because z$

```
>> a \[Because] b
```
$a \because b$

43.2.3. Cap (∩)

WML link

`Cap`[x, y, ...]
: displays $x \cap y \cap ...$

```
>> Cap[x, y, z]
```
$x \cap y \cap z$

```
>> a \[Cap] b
```
$a \cap b$

43.2.4. CenterDot (·)

WML link

`CenterDot`[x, y, ...]
: displays $x \cdot y \cdot ...$

```
>> CenterDot[x, y, z]
```
$x \cdot y \cdot z$

```
>> a \[CenterDot] b
```
$a \cdot b$

43.2.5. CircleDot (⊙)

WML link

`CircleDot`[x, y, ...]
: displays $x \odot y \odot ...$

```
>> CircleDot[x, y, z]
```
$x \odot y \odot z$

```
>> a \[CircleDot] b
```
$a \odot b$

43.2.6. CircleMinus (⊖)

WML link

CircleMinus[$x, y, ...$]
displays $x \ominus y \ominus ...$

```
>> CircleMinus[x, y, z]
```
$x \ominus y \ominus z$

```
>> a \[CircleMinus] b
```
$a \ominus b$

43.2.7. CirclePlus (`\oplus`)

WML link

CirclePlus[$x, y, ...$]
displays x `\oplus` y `\oplus` ...

```
>> CirclePlus[x, y, z]
```
$x \oplus y \oplus z$

```
>> a \[CirclePlus] b
```
$a \oplus b$

43.2.8. CircleTimes ($\otimes$)

WML link

CircleTimes[$x, y, ...$]
displays $x \otimes y \otimes ...$

```
>> CircleTimes[x, y, z]
```
$x \otimes y \otimes z$

```
>> a \[CircleTimes] b
```
$a \otimes b$

43.2.9. Colon (:)

WML link

Colon[$x, y, ...$]
displays $x : y : ...$

```
>> Colon[x, y, z]
```
$x : y : z$

```
>> a \[Colon] b
```
$a : b$

43.2.10. Congruent ($\equiv$)

WML link

`Congruent[`$x, y, ...$`]`
displays $x \equiv y \equiv ...$

```
>> Congruent[x, y, z]
```
$x \equiv y \equiv z$

```
>> a \[Congruent] b
```
$a \equiv b$

43.2.11. Coproduct ($\coprod$)

WML link

`Coproduct[`$x, y, ...$`]`
displays $x \coprod y \coprod ...$

```
>> Coproduct[x, y, z]
```
$x \coprod y \coprod z$

```
>> a \[Coproduct] b
```
$a \coprod b$

43.2.12. Cup ($\cup$)

WML link

`Cup[`$x, y, ...$`]`
displays $x \cup y \cup ...$

```
>> Cup[x, y, z]
```
$x \cup y \cup z$

```
>> a \[Cup] b
```
$a \cup b$

43.2.13. CupCap ($\asymp$)

WML link

`CupCap[`$x, y, ...$`]`
    displays $x \asymp y \asymp ...$

```
>> CupCap[x, y, z]
```
$x \asymp y \asymp z$

```
>> a \[CupCap] b
```
$a \asymp b$

43.2.14. Diamond ($\diamond$)

WML link

`Diamond[`$x, y, ...$`]`
    displays $x \diamond y \diamond ...$

```
>> Diamond[x, y, z]
```
$x \diamond y \diamond z$

```
>> a \[Diamond] b
```
$a \diamond b$

43.2.15. DotEqual ($\doteq$)

WML link

`DotEqual[`$x, y, ...$`]`
    displays $x \doteq y \doteq ...$

```
>> DotEqual[x, y, z]
```
$x \doteq y \doteq z$

```
>> a \[DotEqual] b
```
$a \doteq b$

43.2.16. DoubleDownArrow (⇓)

WML link

`DoubleDownArrow[`$x, y, \ldots$`]`
displays $x \Downarrow y \Downarrow \ldots$

```
>> DoubleDownArrow[x, y, z]
```
$x \Downarrow y \Downarrow z$

```
>> a \[DoubleDownArrow] b
```
$a \Downarrow b$

43.2.17. DoubleLeftArrow (⇐)

WML link

`DoubleLeftArrow[`$x, y, \ldots$`]`
displays $x \Leftarrow y \Leftarrow \ldots$

```
>> DoubleLeftArrow[x, y, z]
```
$x \Leftarrow y \Leftarrow z$

```
>> a \[DoubleLeftArrow] b
```
$a \Leftarrow b$

43.2.18. DoubleLeftRightArrow (⇔)

WML link

`DoubleLeftRightArrow[`$x, y, \ldots$`]`
displays $x \Leftrightarrow y \Leftrightarrow \ldots$

```
>> DoubleLeftRightArrow[x, y, z]
```
$x \Leftrightarrow y \Leftrightarrow z$

```
>> a \[DoubleLeftRightArrow] b
```
$a \Leftrightarrow b$

43.2.19. DoubleLeftTee (=|)

WML link

```
DoubleLeftTee[x, y, ...]
    displays x =| y =| ...
```

>> `DoubleLeftTee[x, y, z]`

$x = |y = |z$

>> `a \[DoubleLeftTee] b`

$a = |b$

43.2.20. DoubleLongLeftArrow ($\Longleftarrow$)

WML link

```
DoubleLongLeftArrow[x, y, ...]
    displays x ⟸ y ⟸ ...
```

>> `DoubleLongLeftArrow[x, y, z]`

$x \Longleftarrow y \Longleftarrow z$

>> `a \[DoubleLongLeftArrow] b`

$a \Longleftarrow b$

43.2.21. DoubleLongLeftRightArrow ($\Longleftrightarrow$)

WML link

```
DoubleLongLeftRightArrow[x, y, ...]
    displays x ⟺ y ⟺ ...
```

>> `DoubleLongLeftRightArrow[x, y, z]`

$x \Longleftrightarrow y \Longleftrightarrow z$

>> `a \[DoubleLongLeftRightArrow] b`

$a \Longleftrightarrow b$

43.2.22. DoubleLongRightArrow ($\Longrightarrow$)

WML link

```
DoubleLongRightArrow[x, y, ...]
    displays x ⟹ y ⟹ ...
```

```
>> DoubleLongRightArrow[x, y, z]
```
$x \Longrightarrow y \Longrightarrow z$

```
>> a \[DoubleLongRightArrow] b
```
$a \Longrightarrow b$

43.2.23. DoubleRightArrow (⇒)

WML link

`DoubleRightArrow[`$x, y, ...$`]`
displays $x \Rightarrow y \Rightarrow ...$

```
>> DoubleRightArrow[x, y, z]
```
$x \Rightarrow y \Rightarrow z$

```
>> a \[DoubleRightArrow] b
```
$a \Rightarrow b$

43.2.24. DoubleRightTee (⊨)

WML link

`DoubleRightTee[`$x, y, ...$`]`
displays $x \vDash y \vDash ...$

```
>> DoubleRightTee[x, y, z]
```
$x \vDash y \vDash z$

```
>> a \[DoubleRightTee] b
```
$a \vDash b$

43.2.25. DoubleUpArrow (⇑)

WML link

`DoubleUpArrow[`$x, y, ...$`]`
displays $x \Uparrow y \Uparrow ...$

```
>> DoubleUpArrow[x, y, z]
```
$x \Uparrow y \Uparrow z$

>> `a \[DoubleUpArrow] b`

$a \Uparrow b$

43.2.26. DoubleUpDownArrow ($\Updownarrow$)

WML link

`DoubleUpDownArrow[`$x, y, ...$`]`
displays $x \Updownarrow y \Updownarrow ...$

>> `DoubleUpDownArrow[x, y, z]`

$x \Updownarrow y \Updownarrow z$

>> `a \[DoubleUpDownArrow] b`

$a \Updownarrow b$

43.2.27. DoubleVerticalBar ($\|$)

WML link

`DoubleVerticalBar[`$x, y, ...$`]`
displays $x \parallel y \parallel ...$

>> `DoubleVerticalBar[x, y, z]`

$x \parallel y \parallel z$

>> `a \[DoubleVerticalBar] b`

$a \parallel b$

43.2.28. DownArrow ($\downarrow$)

WML link

`DownArrow[`$x, y, ...$`]`
displays $x \downarrow y \downarrow ...$

>> `DownArrow[x, y, z]`

$x \downarrow y \downarrow z$

>> `a \[DownArrow] b`

$a \downarrow b$

43.2.29. DownArrowBar (⤓)

WML link

DownArrowBar[x, y, ...]
    displays x ⤓ y ⤓ ...

```
>> DownArrowBar[x, y, z]
```
x⤓y⤓z

```
>> a \[DownArrowBar] b
```
a⤓b

43.2.30. DownArrowUpArrow (⇵)

WML link

DownArrowUpArrow[x, y, ...]
    displays x ⇵ y ⇵ ...

```
>> DownArrowUpArrow[x, y, z]
```
x ⇵ y ⇵ z

```
>> a \[DownArrowUpArrow] b
```
a ⇵ b

43.2.31. DownLeftRightVector (⥐)

WML link

DownLeftRightVector[x, y, ...]
    displays x ⥐ y ⥐ ...

```
>> DownLeftRightVector[x, y, z]
```
x ⥐ y ⥐ z

```
>> a \[DownLeftRightVector] b
```
a ⥐ b

43.2.32. DownLeftTeeVector (⥞)

WML link

`DownLeftTeeVector`[x, y, ...]
    displays $x \leftharpoondown | \; y \leftharpoondown |$ ...

>> `DownLeftTeeVector[x, y, z]`

$x \leftharpoondown |y \leftharpoondown |z$

>> `a \[DownLeftTeeVector] b`

$a \leftharpoondown |b$

43.2.33. DownLeftVector ($\leftharpoondown$)

WML link

`DownLeftVector`[x, y, ...]
    displays $x \leftharpoondown y \leftharpoondown$ ...

>> `DownLeftVector[x, y, z]`

$x \leftharpoondown y \leftharpoondown z$

>> `a \[DownLeftVector] b`

$a \leftharpoondown b$

43.2.34. DownLeftVectorBar (|\leftharpoondown)

WML link

`DownLeftVectorBar`[x, y, ...]
    displays x |\leftharpoondown y |\leftharpoondown ...

>> `DownLeftVectorBar[x, y, z]`

$x|\leftharpoondown y|\leftharpoondown z$

>> `a \[DownLeftVectorBar] b`

$a|\leftharpoondown b$

43.2.35. DownRightTeeVector (|\rightharpoondown)

WML link

`DownRightTeeVector`[x, y, ...]
    displays x |\rightharpoondown y |\rightharpoondown ...

>> `DownRightTeeVector[x, y, z]`

$x|\rightharpoondown y|\rightharpoondown z$

>> `a \[DownRightTeeVector] b`

$a|\rightharpoondown b$

43.2.36. DownRightVector ($\rightharpoondown$)

WML link

`DownRightVector[`$x, y, ...$`]`
displays $x \rightharpoondown y \rightharpoondown ...$

>> `DownRightVector[x, y, z]`

$x \rightharpoondown y \rightharpoondown z$

>> `a \[DownRightVector] b`

$a \rightharpoondown b$

43.2.37. DownRightVectorBar ($\rightharpoondown |$)

WML link

`DownRightVectorBar[`$x, y, ...$`]`
displays $x \rightharpoondown | \; y \rightharpoondown | \; ...$

>> `DownRightVectorBar[x, y, z]`

$x \rightharpoondown |y \rightharpoondown |z$

>> `a \[DownRightVectorBar] b`

$a \rightharpoondown |b$

43.2.38. DownTee ($\top$)

WML link

`DownTee[`$x, y, ...$`]`
displays $x \top y \top ...$

>> `DownTee[x, y, z]`

$x \top y \top z$

```
>> a \[DownTee] b
```
$a \top b$

43.2.39. DownTeeArrow (↧)

WML link

`DownTeeArrow[`$x, y, \ldots$`]`
displays x ↧ y ↧ ...

```
>> DownTeeArrow[x, y, z]
```
x↧y↧z

```
>> a \[DownTeeArrow] b
```
a↧b

43.2.40. EqualTilde (≂)

WML link

`EqualTilde[`$x, y, \ldots$`]`
displays $x \eqsim y \eqsim \ldots$

```
>> EqualTilde[x, y, z]
```
$x \eqsim y \eqsim z$

```
>> a \[EqualTilde] b
```
$a \eqsim b$

43.2.41. Equilibrium (⇌)

WML link

`Equilibrium[`$x, y, \ldots$`]`
displays $x \rightleftharpoons y \rightleftharpoons \ldots$

```
>> Equilibrium[x, y, z]
```
$x \rightleftharpoons y \rightleftharpoons z$

```
>> a \[Equilibrium] b
```
$a \rightleftharpoons b$

43.2.42. GreaterEqualLess (`a\gtreqless b`)

WML link

`GreaterEqualLess[`x, y, ...`]`
    displays x `a\gtreqless b` y `a\gtreqless b` ...

```
>> GreaterEqualLess[x, y, z]
```
$xa \gtreqless bya \gtreqless bz$

```
>> a \[GreaterEqualLess] b
```
$aa \gtreqless bb$

43.2.43. GreaterFullEqual (≧)

WML link

`GreaterFullEqual[`x, y, ...`]`
    displays $x \geqq y \geqq$ ...

```
>> GreaterFullEqual[x, y, z]
```
$x \geqq y \geqq z$

```
>> a \[GreaterFullEqual] b
```
$a \geqq b$

43.2.44. GreaterGreater (≫)

WML link

`GreaterGreater[`x, y, ...`]`
    displays $x \gg y \gg$ ...

```
>> GreaterGreater[x, y, z]
```
$x \gg y \gg z$

```
>> a \[GreaterGreater] b
```
$a \gg b$

43.2.45. GreaterLess (≷)

WML link

`GreaterLess`[x, y, ...]
: displays $x \gtrless y \gtrless \ldots$

```
>> GreaterLess[x, y, z]
```
$x \gtrless y \gtrless z$

```
>> a \[GreaterLess] b
```
$a \gtrless b$

43.2.46. GreaterSlantEqual (≥)

WML link

`GreaterSlantEqual`[x, y, ...]
: displays $x \geq y \geq \ldots$

```
>> GreaterSlantEqual[x, y, z]
```
$x \geq y \geq z$

```
>> a \[GreaterSlantEqual] b
```
$a \geq b$

43.2.47. GreaterTilde (≳)

WML link

`GreaterTilde`[x, y, ...]
: displays $x \gtrsim y \gtrsim \ldots$

```
>> GreaterTilde[x, y, z]
```
$x \gtrsim y \gtrsim z$

```
>> a \[GreaterTilde] b
```
$a \gtrsim b$

43.2.48. HumpDownHump (≎)

WML link

`HumpDownHump`[x, y, ...]
: displays $x \Bumpeq y \Bumpeq \ldots$

>> `HumpDownHump[x, y, z]`
$x \Bumpeq y \Bumpeq z$

>> `a \[HumpDownHump] b`
$a \Bumpeq b$

43.2.49. HumpEqual (≏)

WML link

`HumpEqual`[x, y, ...]
displays $x \bumpeq y \bumpeq ...$

>> `HumpEqual[x, y, z]`
$x \bumpeq y \bumpeq z$

>> `a \[HumpEqual] b`
$a \bumpeq b$

43.2.50. LeftArrow (←)

WML link

`LeftArrow`[x, y, ...]
displays $x \leftarrow y \leftarrow ...$

>> `LeftArrow[x, y, z]`
$x \leftarrow y \leftarrow z$

>> `a \[LeftArrow] b`
$a \leftarrow b$

43.2.51. LeftArrowBar (|\leftarrow)

WML link

`LeftArrowBar`[x, y, ...]
displays x `|\leftarrow` y `|\leftarrow` ...

>> `LeftArrowBar[x, y, z]`
$x|\leftarrow y|\leftarrow z$

```
>> a \[LeftArrowBar] b
```
$a|\leftarrow b$

43.2.52. LeftArrowRightArrow (⇆)

WML link

`LeftArrowRightArrow[`$x, y, ...$`]`
displays $x \leftrightarrows y \leftrightarrows ...$

```
>> LeftArrowRightArrow[x, y, z]
```
$x \leftrightarrows y \leftrightarrows z$

```
>> a \[LeftArrowRightArrow] b
```
$a \leftrightarrows b$

43.2.53. LeftDownTeeVector (⥡)

WML link

`LeftDownTeeVector[`$x, y, ...$`]`
displays x ⥡ y ⥡ ...

```
>> LeftDownTeeVector[x, y, z]
```
x⥡y⥡z

```
>> a \[LeftDownTeeVector] b
```
a⥡b

43.2.54. LeftDownVector (⇃)

WML link

`LeftDownVector[`$x, y, ...$`]`
displays $x \downharpoonleft y \downharpoonleft ...$

```
>> LeftDownVector[x, y, z]
```
$x \downharpoonleft y \downharpoonleft z$

```
>> a \[LeftDownVector] b
```
$a \downharpoonleft b$

43.2.55. LeftDownVectorBar (⥙)

WML link

LeftDownVectorBar[$x, y, ...$]
displays x ⥙ y ⥙ ...

```
>> LeftDownVectorBar[x, y, z]
```
x⥙y⥙z

```
>> a \[LeftDownVectorBar] b
```
a⥙b

43.2.56. LeftRightArrow (↔)

WML link

LeftRightArrow[$x, y, ...$]
displays $x \leftrightarrow y \leftrightarrow ...$

```
>> LeftRightArrow[x, y, z]
```
$x \leftrightarrow y \leftrightarrow z$

```
>> a \[LeftRightArrow] b
```
$a \leftrightarrow b$

43.2.57. LeftRightVector (⥎)

WML link

LeftRightVector[$x, y, ...$]
displays x ⥎ y ⥎ ...

```
>> LeftRightVector[x, y, z]
```
x ⥎ y ⥎ z

```
>> a \[LeftRightVector] b
```
a ⥎ b

43.2.58. LeftTee (⊣)

WML link

`LeftTee[`$x, y, ...$`]`
displays $x \dashv y \dashv ...$

```
>> LeftTee[x, y, z]
```
$x \dashv y \dashv z$

```
>> a \[LeftTee] b
```
$a \dashv b$

43.2.59. LeftTeeArrow (↤)

WML link

`LeftTeeArrow[`$x, y, ...$`]`
displays $x \mapsfrom y \mapsfrom ...$

```
>> LeftTeeArrow[x, y, z]
```
$x \mapsfrom y \mapsfrom z$

```
>> a \[LeftTeeArrow] b
```
$a \mapsfrom b$

43.2.60. LeftTeeVector (↼ |)

WML link

`LeftTeeVector[`$x, y, ...$`]`
displays $x \leftharpoonup | \, y \leftharpoonup | \, ...$

```
>> LeftTeeVector[x, y, z]
```
$x \leftharpoonup |y \leftharpoonup |z$

```
>> a \[LeftTeeVector] b
```
$a \leftharpoonup |b$

43.2.61. LeftTriangle (◁)

WML link

`LeftTriangle[`$x, y, ...$`]`
displays $x \lhd y \lhd ...$

```
>> LeftTriangle[x, y, z]
```
$x \lhd y \lhd z$

```
>> a \[LeftTriangle] b
```
$a \lhd b$

43.2.62. LeftTriangleBar (⧏)

WML link

`LeftTriangleBar[`$x, y, \ldots$`]`
displays x ⧏ y ⧏ ...

```
>> LeftTriangleBar[x, y, z]
```
$x \lhd |y \lhd |z$

```
>> a \[LeftTriangleBar] b
```
$a \lhd |b$

43.2.63. LeftTriangleEqual (⊴)

WML link

`LeftTriangleEqual[`$x, y, \ldots$`]`
displays $x \trianglelefteq y \trianglelefteq \ldots$

```
>> LeftTriangleEqual[x, y, z]
```
$x \trianglelefteq y \trianglelefteq z$

```
>> a \[LeftTriangleEqual] b
```
$a \trianglelefteq b$

43.2.64. LeftUpDownVector (⥑)

WML link

`LeftUpDownVector[`$x, y, \ldots$`]`
displays x ⥑ y ⥑ ...

```
>> LeftUpDownVector[x, y, z]
```

x ⥑ *y* ⥑ *z*

```
>> a \[LeftUpDownVector] b
```

a ⥑ *b*

43.2.65. LeftUpTeeVector (⥠)

WML link

> `LeftUpTeeVector`[*x*, *y*, ...]
> displays *x* ⥠ *y* ⥠ ...

```
>> LeftUpTeeVector[x, y, z]
```

x⥠*y*⥠*z*

```
>> a \[LeftUpTeeVector] b
```

a⥠*b*

43.2.66. LeftUpVector (↿)

WML link

> `LeftUpVector`[*x*, *y*, ...]
> displays *x* ↿ *y* ↿ ...

```
>> LeftUpVector[x, y, z]
```

x ↿ *y* ↿ *z*

```
>> a \[LeftUpVector] b
```

a ↿ *b*

43.2.67. LeftUpVectorBar (⥘)

WML link

> `LeftUpVectorBar`[*x*, *y*, ...]
> displays *x* ⥘ *y* ⥘ ...

>> `LeftUpVectorBar[x, y, z]`

$x \upharpoonleft\!\!\bar{} y \upharpoonleft\!\!\bar{} z$

>> `a \[LeftUpVectorBar] b`

$a \upharpoonleft\!\!\bar{} b$

43.2.68. LeftVector (↼)

WML link

`LeftVector[`x, y, ...`]`
displays $x \leftharpoonup y \leftharpoonup ...$

>> `LeftVector[x, y, z]`

$x \leftharpoonup y \leftharpoonup z$

>> `a \[LeftVector] b`

$a \leftharpoonup b$

43.2.69. LeftVectorBar (|\leftharpoonup)

WML link

`LeftVectorBar[`x, y, ...`]`
displays x |\leftharpoonup y |\leftharpoonup ...

>> `LeftVectorBar[x, y, z]`

$x| \leftharpoonup y| \leftharpoonup z$

>> `a \[LeftVectorBar] b`

$a| \leftharpoonup b$

43.2.70. LessEqualGreater (⋚)

WML link

`LessEqualGreater[`x, y, ...`]`
displays $x \lesseqgtr y \lesseqgtr ...$

>> `LessEqualGreater[x, y, z]`

$x \lesseqgtr y \lesseqgtr z$

```
>> a \[LessEqualGreater] b
```
$a \lesseqqgtr b$

43.2.71. LessFullEqual (≦)

WML link

`LessFullEqual[`$x, y, \ldots$`]`
displays $x \leqq y \leqq \ldots$

```
>> LessFullEqual[x, y, z]
```
$x \leqq y \leqq z$

```
>> a \[LessFullEqual] b
```
$a \leqq b$

43.2.72. LessGreater (≶)

WML link

`LessGreater[`$x, y, \ldots$`]`
displays $x \lessgtr y \lessgtr \ldots$

```
>> LessGreater[x, y, z]
```
$x \lessgtr y \lessgtr z$

```
>> a \[LessGreater] b
```
$a \lessgtr b$

43.2.73. LessLess (≪)

WML link

`LessLess[`$x, y, \ldots$`]`
displays $x \ll y \ll \ldots$

```
>> LessLess[x, y, z]
```
$x \ll y \ll z$

```
>> a \[LessLess] b
```
$a \ll b$

43.2.74. LessSlantEqual ($\leq$)

WML link

`LessSlantEqual[`$x, y, ...$`]`
displays $x \leq y \leq ...$

>> `LessSlantEqual[x, y, z]`
$x \leq y \leq z$

>> `a \[LessSlantEqual] b`
$a \leq b$

43.2.75. LessTilde ($\lesssim$)

WML link

`LessTilde[`$x, y, ...$`]`
displays $x \lesssim y \lesssim ...$

>> `LessTilde[x, y, z]`
$x \lesssim y \lesssim z$

>> `a \[LessTilde] b`
$a \lesssim b$

43.2.76. LongLeftArrow ($\longleftarrow$)

WML link

`LongLeftArrow[`$x, y, ...$`]`
displays $x \longleftarrow y \longleftarrow ...$

>> `LongLeftArrow[x, y, z]`
$x \longleftarrow y \longleftarrow z$

>> `a \[LongLeftArrow] b`
$a \longleftarrow b$

43.2.77. LongLeftRightArrow ($\longleftrightarrow$)

WML link

`LongLeftRightArrow[`$x, y, ...$`]`
displays $x \longleftrightarrow y \longleftrightarrow ...$

```
>> LongLeftRightArrow[x, y, z]
```
$x \longleftrightarrow y \longleftrightarrow z$

```
>> a \[LongLeftRightArrow] b
```
$a \longleftrightarrow b$

43.2.78. LongRightArrow ($\longrightarrow$)

WML link

`LongRightArrow[`$x, y, ...$`]`
displays $x \longrightarrow y \longrightarrow ...$

```
>> LongRightArrow[x, y, z]
```
$x \longrightarrow y \longrightarrow z$

```
>> a \[LongRightArrow] b
```
$a \longrightarrow b$

43.2.79. LowerLeftArrow ($\swarrow$)

WML link

`LowerLeftArrow[`$x, y, ...$`]`
displays $x \swarrow y \swarrow ...$

```
>> LowerLeftArrow[x, y, z]
```
$x \swarrow y \swarrow z$

```
>> a \[LowerLeftArrow] b
```
$a \swarrow b$

43.2.80. LowerRightArrow ($\searrow$)

WML link

`LowerRightArrow[`$x, y, ...$`]`
displays $x \searrow y \searrow ...$

```
>> LowerRightArrow[x, y, z]
```
$x \searrow y \searrow z$

```
>> a \[LowerRightArrow] b
```
$a \searrow b$

43.2.81. MinusPlus ($\mp$)

WML link

`MinusPlus[`$x, y, ...$`]`
displays $x \mp y \mp ...$

```
>> MinusPlus[x, y, z]
```
$x \mp y \mp z$

```
>> a \[MinusPlus] b
```
$a \mp b$

43.2.82. NestedGreaterGreater ($\gg$)

WML link

`NestedGreaterGreater[`$x, y, ...$`]`
displays $x \gg y \gg ...$

```
>> NestedGreaterGreater[x, y, z]
```
$x \gg y \gg z$

```
>> a \[NestedGreaterGreater] b
```
$a \gg b$

43.2.83. NestedLessLess ($\ll$)

WML link

`NestedLessLess[`$x, y, ...$`]`
displays $x \ll y \ll ...$

```
>> NestedLessLess[x, y, z]
```
$x \ll y \ll z$

```
>> a \[NestedLessLess] b
```
$a \ll b$

43.2.84. NotCongruent (≢)

WML link

`NotCongruent[`$x, y, \ldots$`]`
displays $x \not\equiv y \not\equiv \ldots$

```
>> NotCongruent[x, y, z]
```
$x \not\equiv y \not\equiv z$

```
>> a \[NotCongruent] b
```
$a \not\equiv b$

43.2.85. NotCupCap (≭)

WML link

`NotCupCap[`$x, y, \ldots$`]`
displays $x \not\asymp y \not\asymp \ldots$

```
>> NotCupCap[x, y, z]
```
$x \not\asymp y \not\asymp z$

```
>> a \[NotCupCap] b
```
$a \not\asymp b$

43.2.86. NotDoubleVerticalBar (∦)

WML link

`NotDoubleVerticalBar[`$x, y, \ldots$`]`
displays $x \nparallel y \nparallel \ldots$

```
>> NotDoubleVerticalBar[x, y, z]
```
$x \nparallel y \nparallel z$

```
>> a \[NotDoubleVerticalBar] b
```
$a \nparallel b$

43.2.87. NotGreater (≯)

WML link

`NotGreater[`$x, y, ...$`]`
displays $x \ngtr y \ngtr \ldots$

```
>> NotGreater[x, y, z]
```
$x \ngtr y \ngtr z$

```
>> a \[NotGreater] b
```
$a \ngtr b$

43.2.88. NotGreaterEqual (≱)

WML link

`NotGreaterEqual[`$x, y, ...$`]`
displays $x \ngeq y \ngeq \ldots$

```
>> NotGreaterEqual[x, y, z]
```
$x \ngeq y \ngeq z$

```
>> a \[NotGreaterEqual] b
```
$a \ngeq b$

43.2.89. NotGreaterFullEqual (≩)

WML link

`NotGreaterFullEqual[`$x, y, ...$`]`
displays $x \ngeqq y \ngeqq \ldots$

```
>> NotGreaterFullEqual[x, y, z]
```
$x \ngeqq y \ngeqq z$

```
>> a \[NotGreaterFullEqual] b
```
$a \ngeqq b$

43.2.90. NotGreaterGreater (≫̸)

WML link

NotGreaterGreater[$x, y, \ldots$]
    displays $x \not\gg y \not\gg \ldots$

>> `NotGreaterGreater[x, y, z]`

$x \not\gg y \not\gg z$

>> `a \[NotGreaterGreater] b`

$a \not\gg b$

43.2.91. NotGreaterLess ($\not\gtrless$)

WML link

NotGreaterLess[$x, y, \ldots$]
    displays $x \not\gtrless y \not\gtrless \ldots$

>> `NotGreaterLess[x, y, z]`

$x \not\gtrless y \not\gtrless z$

>> `a \[NotGreaterLess] b`

$a \not\gtrless b$

43.2.92. NotGreaterTilde ($\not\gtrsim$)

WML link

NotGreaterTilde[$x, y, \ldots$]
    displays $x \not\gtrsim y \not\gtrsim \ldots$

>> `NotGreaterTilde[x, y, z]`

$x \not\gtrsim y \not\gtrsim z$

>> `a \[NotGreaterTilde] b`

$a \not\gtrsim b$

43.2.93. NotLeftTriangle ($\ntriangleleft$)

WML link

NotLeftTriangle[$x, y, \ldots$]
    displays $x \ntriangleleft y \ntriangleleft \ldots$

```
>> NotLeftTriangle[x, y, z]
```
$x \ntriangleleft y \ntriangleleft z$

```
>> a \[NotLeftTriangle] b
```
$a \ntriangleleft b$

43.2.94. NotLeftTriangleEqual ($\ntrianglelefteq$)

WML link

`NotLeftTriangleEqual[`$x, y, ...$`]`
displays $x \ntrianglelefteq y \ntrianglelefteq ...$

```
>> NotLeftTriangleEqual[x, y, z]
```
$x \ntrianglelefteq y \ntrianglelefteq z$

```
>> a \[NotLeftTriangleEqual] b
```
$a \ntrianglelefteq b$

43.2.95. NotLess ($\nless$)

WML link

`NotLess[`$x, y, ...$`]`
displays $x \nless y \nless ...$

```
>> NotLess[x, y, z]
```
$x \nless y \nless z$

```
>> a \[NotLess] b
```
$a \nless b$

43.2.96. NotLessEqual ($\nleq$)

WML link

`NotLessEqual[`$x, y, ...$`]`
displays $x \nleq y \nleq ...$

```
>> NotLessEqual[x, y, z]
```
$x \nleq y \nleq z$

```
>> a \[NotLessEqual] b
```
$a \nleq b$

43.2.97. NotLessFullEqual (≦̸)

WML link

`NotLessFullEqual[`$x, y, ...$`]`
displays $x \nleqq y \nleqq ...$

```
>> NotLessFullEqual[x, y, z]
```
$x \nleqq y \nleqq z$

```
>> a \[NotLessFullEqual] b
```
$a \nleqq b$

43.2.98. NotLessGreater (≸)

WML link

`NotLessGreater[`$x, y, ...$`]`
displays $x \not\lessgtr y \not\lessgtr ...$

```
>> NotLessGreater[x, y, z]
```
$x \not\lessgtr y \not\lessgtr z$

```
>> a \[NotLessGreater] b
```
$a \not\lessgtr b$

43.2.99. NotLessTilde (≴)

WML link

`NotLessTilde[`$x, y, ...$`]`
displays $x \not\lesssim y \not\lesssim ...$

```
>> NotLessTilde[x, y, z]
```
$x \not\lesssim y \not\lesssim z$

```
>> a \[NotLessTilde] b
```
$a \not\lesssim b$

43.2.100. NotPrecedes ($\nprec$)

WML link

`NotPrecedes[`$x, y, ...$`]`
    displays $x \nprec y \nprec ...$

>> `NotPrecedes[x, y, z]`
    $x \nprec y \nprec z$

>> `a \[NotPrecedes] b`
    $a \nprec b$

43.2.101. NotPrecedesSlantEqual ($\npreceq$)

WML link

`NotPrecedesSlantEqual[`$x, y, ...$`]`
    displays $x \npreceq y \npreceq ...$

>> `NotPrecedesSlantEqual[x, y, z]`
    $x \npreceq y \npreceq z$

>> `a \[NotPrecedesSlantEqual] b`
    $a \npreceq b$

43.2.102. NotPrecedesTilde ($\not\precsim$)

WML link

`NotPrecedesTilde[`$x, y, ...$`]`
    displays $x \not\precsim y \not\precsim ...$

>> `NotPrecedesTilde[x, y, z]`
    $x \not\precsim y \not\precsim z$

>> `a \[NotPrecedesTilde] b`
    $a \not\precsim b$

43.2.103. NotReverseElement ($\not\ni$)

WML link

```
NotReverseElement[x, y, ...]
    displays x ∌ y ∌ ...
```

>> `NotReverseElement[x, y, z]`

$x \not\ni y \not\ni z$

>> `a \[NotReverseElement] b`

$a \not\ni b$

43.2.104. NotRightTriangle (⋫)

WML link

```
NotRightTriangle[x, y, ...]
    displays x ⋫ y ⋫ ...
```

>> `NotRightTriangle[x, y, z]`

$x \ntriangleright y \ntriangleright z$

>> `a \[NotRightTriangle] b`

$a \ntriangleright b$

43.2.105. NotRightTriangleEqual (⋭)

WML link

```
NotRightTriangleEqual[x, y, ...]
    displays x ⋭ y ⋭ ...
```

>> `NotRightTriangleEqual[x, y, z]`

$x \ntrianglerighteq y \ntrianglerighteq z$

>> `a \[NotRightTriangleEqual] b`

$a \ntrianglerighteq b$

43.2.106. NotSquareSubsetEqual (⋢)

WML link

```
NotSquareSubsetEqual[x, y, ...]
    displays x ⋢ y ⋢ ...
```

```
>> NotSquareSubsetEqual[x, y, z]
```
$x \not\sqsubseteq y \not\sqsubseteq z$

```
>> a \[NotSquareSubsetEqual] b
```
$a \not\sqsubseteq b$

43.2.107. NotSquareSupersetEqual (\[NotSquareSupersetEqual])

WML link

`NotSquareSupersetEqual[`x, y, ...`]`
displays x \[NotSquareSupersetEqual] y \[NotSquareSupersetEqual] ...

```
>> NotSquareSupersetEqual[x, y, z]
```
x\[NotSquareSupersetEqual]y\[NotSquareSupersetEqual]z

```
>> a \[NotSquareSupersetEqual] b
```
a\[NotSquareSupersetEqual]b

43.2.108. NotSubset ($\not\subset$)

WML link

`NotSubset[`x, y, ...`]`
displays $x \not\subset y \not\subset$ ...

```
>> NotSubset[x, y, z]
```
$x \not\subset y \not\subset z$

```
>> a \[NotSubset] b
```
$a \not\subset b$

43.2.109. NotSubsetEqual ($\not\subseteq$)

WML link

`NotSubsetEqual[`x, y, ...`]`
displays $x \not\subseteq y \not\subseteq$ ...

```
>> NotSubsetEqual[x, y, z]
```
$x \not\subseteq y \not\subseteq z$

```
>> a \[NotSubsetEqual] b
```
$a \nsubseteq b$

43.2.110. NotSucceeds ($\nsucc$)

WML link

`NotSucceeds[`$x, y, \ldots$`]`
displays $x \nsucc y \nsucc \ldots$

```
>> NotSucceeds[x, y, z]
```
$x \nsucc y \nsucc z$

```
>> a \[NotSucceeds] b
```
$a \nsucc b$

43.2.111. NotSucceedsSlantEqual ($\not\succcurlyeq$)

WML link

`NotSucceedsSlantEqual[`$x, y, \ldots$`]`
displays $x \not\succcurlyeq y \not\succcurlyeq \ldots$

```
>> NotSucceedsSlantEqual[x, y, z]
```
$x \not\succcurlyeq y \not\succcurlyeq z$

```
>> a \[NotSucceedsSlantEqual] b
```
$a \not\succcurlyeq b$

43.2.112. NotSucceedsTilde ($\not\succsim$)

WML link

`NotSucceedsTilde[`$x, y, \ldots$`]`
displays $x \not\succsim y \not\succsim \ldots$

```
>> NotSucceedsTilde[x, y, z]
```
$x \not\succsim y \not\succsim z$

```
>> a \[NotSucceedsTilde] b
```
$a \not\succsim b$

43.2.113. NotSuperset ($\not\supset$)

WML link

`NotSuperset[`$x, y, \ldots$`]`
    displays $x \not\supset y \not\supset \ldots$

>> **`NotSuperset[x, y, z]`**
    $x \not\supset y \not\supset z$

>> **`a \[NotSuperset] b`**
    $a \not\supset b$

43.2.114. NotSupersetEqual ($\not\supseteq$)

WML link

`NotSupersetEqual[`$x, y, \ldots$`]`
    displays $x \not\supseteq y \not\supseteq \ldots$

>> **`NotSupersetEqual[x, y, z]`**
    $x \not\supseteq y \not\supseteq z$

>> **`a \[NotSupersetEqual] b`**
    $a \not\supseteq b$

43.2.115. NotTilde ($\not\sim$)

WML link

`NotTilde[`$x, y, \ldots$`]`
    displays $x \not\sim y \not\sim \ldots$

>> **`NotTilde[x, y, z]`**
    $x \not\sim y \not\sim z$

>> **`a \[NotTilde] b`**
    $a \not\sim b$

43.2.116. NotTildeEqual ($\not\simeq$)

WML link

NotTildeEqual[$x, y, ...$]
displays $x \not\simeq y \not\simeq ...$

```
>> NotTildeEqual[x, y, z]
```
$x \not\simeq y \not\simeq z$

```
>> a \[NotTildeEqual] b
```
$a \not\simeq b$

43.2.117. NotTildeFullEqual (≇)

WML link

NotTildeFullEqual[$x, y, ...$]
displays $x \ncong y \ncong ...$

```
>> NotTildeFullEqual[x, y, z]
```
$x \ncong y \ncong z$

```
>> a \[NotTildeFullEqual] b
```
$a \ncong b$

43.2.118. NotTildeTilde (≉)

WML link

NotTildeTilde[$x, y, ...$]
displays $x \not\approx y \not\approx ...$

```
>> NotTildeTilde[x, y, z]
```
$x \not\approx y \not\approx z$

```
>> a \[NotTildeTilde] b
```
$a \not\approx b$

43.2.119. Perpendicular (⊥)

WML link

Perpendicular[$x, y, ...$]
displays $x \perp y \perp ...$

```
>> Perpendicular[x, y, z]
```
$x \perp y \perp z$

```
>> a \[Perpendicular] b
```
$a \perp b$

43.2.120. PlusMinus

WML link

`PlusMinus[`$x, y, ...$`]`
displays $x \pm y \pm ...$

```
>> PlusMinus[x, y, z]
```
$x \pm y \pm z$

```
>> a \[PlusMinus] b
```
$a \pm b$

43.2.121. Precedes ($\prec$)

WML link

`Precedes[`$x, y, ...$`]`
displays $x \prec y \prec ...$

```
>> Precedes[x, y, z]
```
$x \prec y \prec z$

```
>> a \[Precedes] b
```
$a \prec b$

43.2.122. PrecedesEqual ($\preceq$)

WML link

`PrecedesEqual[`$x, y, ...$`]`
displays $x \preceq y \preceq ...$

```
>> PrecedesEqual[x, y, z]
```
$x \preceq y \preceq z$

```
>> a \[PrecedesEqual] b
```
$a \preceq b$

43.2.123. PrecedesSlantEqual (≼)

WML link

`PrecedesSlantEqual[`$x, y, ...$`]`
displays $x \preccurlyeq y \preccurlyeq ...$

```
>> PrecedesSlantEqual[x, y, z]
```
$x \preccurlyeq y \preccurlyeq z$

```
>> a \[PrecedesSlantEqual] b
```
$a \preccurlyeq b$

43.2.124. PrecedesTilde (≾)

WML link

`PrecedesTilde[`$x, y, ...$`]`
displays $x \precsim y \precsim ...$

```
>> PrecedesTilde[x, y, z]
```
$x \precsim y \precsim z$

```
>> a \[PrecedesTilde] b
```
$a \precsim b$

43.2.125. Proportion (∷)

WML link

`Proportion[`$x, y, ...$`]`
displays $x :: y :: ...$

```
>> Proportion[x, y, z]
```
$x :: y :: z$

```
>> a \[Proportion] b
```
$a :: b$

43.2.126. Proportional (∝)

WML link

`Proportional[`$x, y, ...$`]`
displays $x \propto y \propto ...$

```
>> Proportional[x, y, z]
```
$x \propto y \propto z$

```
>> a \[Proportional] b
```
$a \propto b$

43.2.127. ReverseElement (∋)

WML link

`ReverseElement[`$x, y, ...$`]`
displays $x \ni y \ni ...$

```
>> ReverseElement[x, y, z]
```
$x \ni y \ni z$

```
>> a \[ReverseElement] b
```
$a \ni b$

43.2.128. ReverseEquilibrium (⇋)

WML link

`ReverseEquilibrium[`$x, y, ...$`]`
displays $x \leftrightharpoons y \leftrightharpoons ...$

```
>> ReverseEquilibrium[x, y, z]
```
$x \leftrightharpoons y \leftrightharpoons z$

```
>> a \[ReverseEquilibrium] b
```
$a \leftrightharpoons b$

43.2.129. ReverseUpEquilibrium (⥯)

WML link

```
ReverseUpEquilibrium[x, y, ...]
```
displays $x \downharpoonleft\upharpoonright y \downharpoonleft\upharpoonright \ldots$

>> `ReverseUpEquilibrium[x, y, z]`

$x \downharpoonleft\upharpoonright y \downharpoonleft\upharpoonright z$

>> `a \[ReverseUpEquilibrium] b`

$a \downharpoonleft\upharpoonright b$

43.2.130. RightArrow ($\rightarrow$)

WML link

```
RightArrow[x, y, ...]
```
displays $x \rightarrow y \rightarrow \ldots$

>> `RightArrow[x, y, z]`

$x \rightarrow y \rightarrow z$

>> `a \[RightArrow] b`

$a \rightarrow b$

43.2.131. RightArrowBar ($\rightarrow |$)

WML link

```
RightArrowBar[x, y, ...]
```
displays $x \rightarrow | \; y \rightarrow | \ldots$

>> `RightArrowBar[x, y, z]`

$x \rightarrow |y \rightarrow |z$

>> `a \[RightArrowBar] b`

$a \rightarrow |b$

43.2.132. RightArrowLeftArrow ($\rightleftarrows$)

WML link

```
RightArrowLeftArrow[x, y, ...]
```
displays $x \rightleftarrows y \rightleftarrows \ldots$

>> `RightArrowLeftArrow[x, y, z]`

$x \rightleftarrows y \rightleftarrows z$

>> `a \[RightArrowLeftArrow] b`

$a \rightleftarrows b$

43.2.133. RightDownTeeVector (⥡)

WML link

`RightDownTeeVector[`x, y, ...`]`
displays x ⥡ y ⥡ ...

>> `RightDownTeeVector[x, y, z]`

x⥡y⥡z

>> `a \[RightDownTeeVector] b`

a⥡b

43.2.134. RightDownVector (⇂)

WML link

`RightDownVector[`x, y, ...`]`
displays x ⇂ y ⇂ ...

>> `RightDownVector[x, y, z]`

x⇂y⇂z

>> `a \[RightDownVector] b`

a⇂b

43.2.135. RightDownVectorBar (\[RightDownVectorBar])

WML link

`RightDownVectorBar[`x, y, ...`]`
displays x \[RightDownVectorBar] y \[RightDownVectorBar] ...

>> `RightDownVectorBar[x, y, z]`

x\[RightDownVectorBar]y\[RightDownVectorBar]z

```
>> a \[RightDownVectorBar] b
```
a\[RightDownVectorBar]b

43.2.136. RightTee (⊢)

WML link

`RightTee`[x, y, ...]
    displays $x \vdash y \vdash \ldots$

```
>> RightTee[x, y, z]
```
$x \vdash y \vdash z$

```
>> a \[RightTee] b
```
$a \vdash b$

43.2.137. RightTeeArrow (⊢)

WML link

`RightTeeArrow`[x, y, ...]
    displays $x \vdash y \vdash \ldots$

```
>> RightTeeArrow[x, y, z]
```
$x \vdash y \vdash z$

```
>> a \[RightTeeArrow] b
```
$a \vdash b$

43.2.138. RightTeeVector (`|\rightharpoonup`)

WML link

`RightTeeVector`[x, y, ...]
    displays x `|\rightharpoonup` y `|\rightharpoonup` ...

```
>> RightTeeVector[x, y, z]
```
$x|\rightharpoonup y|\rightharpoonup z$

```
>> a \[RightTeeVector] b
```
$a|\rightharpoonup b$

43.2.139. RightTriangle (▷)

WML link

`RightTriangle[`$x, y, ...$`]`
    displays $x \triangleright y \triangleright ...$

>> `RightTriangle[x, y, z]`
    $x \triangleright y \triangleright z$

>> `a \[RightTriangle] b`
    $a \triangleright b$

43.2.140. RightTriangleBar (|\triangleright)

WML link

`RightTriangleBar[`$x, y, ...$`]`
    displays x `|\triangleright` y `|\triangleright` ...

>> `RightTriangleBar[x, y, z]`
    $x|\triangleright y|\triangleright z$

>> `a \[RightTriangleBar] b`
    $a|\triangleright b$

43.2.141. RightTriangleEqual (⊵)

WML link

`RightTriangleEqual[`$x, y, ...$`]`
    displays $x \trianglerighteq y \trianglerighteq ...$

>> `RightTriangleEqual[x, y, z]`
    $x \trianglerighteq y \trianglerighteq z$

>> `a \[RightTriangleEqual] b`
    $a \trianglerighteq b$

43.2.142. RightUpDownVector (⥏)

WML link

`RightUpDownVector`[x, y, ...]
displays x ⥏ y ⥏ ...

```
>> RightUpDownVector[x, y, z]
```

x ⥏ y ⥏ z

```
>> a \[RightUpDownVector] b
```

a ⥏ b

43.2.143. RightUpTeeVector (⥜)

WML link

`RightUpTeeVector`[x, y, ...]
displays x ⥜ y ⥜ ...

```
>> RightUpTeeVector[x, y, z]
```

x⥜y⥜z

```
>> a \[RightUpTeeVector] b
```

a⥜b

43.2.144. RightUpVector (↾)

WML link

`RightUpVector`[x, y, ...]
displays x ↾ y ↾ ...

```
>> RightUpVector[x, y, z]
```

x ↾ y ↾ z

```
>> a \[RightUpVector] b
```

a ↾ b

43.2.145. RightUpVectorBar ($\overline{\upharpoonright}$)

WML link

`RightUpVectorBar`[x, y, ...]
displays x $\overline{\upharpoonright}$ y $\overline{\upharpoonright}$ ...

```
>> RightUpVectorBar[x, y, z]
```
$x\overline{\upharpoonright}y\overline{\upharpoonright}z$

```
>> a \[RightUpVectorBar] b
```
$a\overline{\upharpoonright}b$

43.2.146. RightVector ($\rightharpoonup$)

WML link

`RightVector`[x, y, ...]
displays $x \rightharpoonup y \rightharpoonup$ ...

```
>> RightVector[x, y, z]
```
$x \rightharpoonup y \rightharpoonup z$

```
>> a \[RightVector] b
```
$a \rightharpoonup b$

43.2.147. RightVectorBar ($\rightharpoonup|$)

WML link

`RightVectorBar`[x, y, ...]
displays $x \rightharpoonup | \; y \rightharpoonup |$ ...

```
>> RightVectorBar[x, y, z]
```
$x \rightharpoonup |y \rightharpoonup |z$

```
>> a \[RightVectorBar] b
```
$a \rightharpoonup |b$

43.2.148. RoundImplies (RoundImplies[a, b])

WML link

`RoundImplies[`$x, y, ...$`]`
displays x RoundImplies[a, b] y RoundImplies[a, b] ...

>> **`RoundImplies[x, y, z]`**

xRoundImplies[a, b]yRoundImplies[a, b]z

>> **`a \[RoundImplies] b`**

aRoundImplies[a, b]b

43.2.149. ShortDownArrow

WML link

`ShortDownArrow[`$x, y, ...$`]`
displays $x \downarrow y \downarrow ...$

>> **`ShortDownArrow[x, y, z]`**

$x \downarrow y \downarrow z$

>> **`a \[ShortDownArrow] b`**

$a \downarrow b$

43.2.150. ShortLeftArrow

WML link

`ShortLeftArrow[`$x, y, ...$`]`
displays $x \leftarrow y \leftarrow ...$

>> **`ShortLeftArrow[x, y, z]`**

$x \leftarrow y \leftarrow z$

>> **`a \[ShortLeftArrow] b`**

$a \leftarrow b$

43.2.151. ShortRightArrow

WML link

`ShortRightArrow[`$x, y, ...$`]`
displays $x \rightarrow y \rightarrow ...$

```
>> ShortRightArrow[x, y, z]
```
$x \to y \to z$

```
>> a \[ShortRightArrow] b
```
$a \to b$

43.2.152. ShortUpArrow

WML link

`ShortUpArrow[`$x, y, ...$`]`
displays $x \uparrow y \uparrow ...$

```
>> ShortUpArrow[x, y, z]
```
$x \uparrow y \uparrow z$

```
>> a \[ShortUpArrow] b
```
$a \uparrow b$

43.2.153. SmallCircle (∘)

WML link

`SmallCircle[`$x, y, ...$`]`
displays $x \circ y \circ ...$

```
>> SmallCircle[x, y, z]
```
$x \circ y \circ z$

```
>> a \[SmallCircle] b
```
$a \circ b$

43.2.154. SquareIntersection (⊓)

WML link

`SquareIntersection[`$x, y, ...$`]`
displays $x \sqcap y \sqcap ...$

```
>> SquareIntersection[x, y, z]
```
$x \sqcap y \sqcap z$

```
>> a \[SquareIntersection] b
```
$a \sqcap b$

43.2.155. SquareSubset ($\sqsubset$)

WML link

`SquareSubset[`x, y, ...`]`
: displays $x \sqsubset y \sqsubset \ldots$

```
>> SquareSubset[x, y, z]
```
$x \sqsubset y \sqsubset z$

```
>> a \[SquareSubset] b
```
$a \sqsubset b$

43.2.156. SquareSubsetEqual ($\sqsubseteq$)

WML link

`SquareSubsetEqual[`x, y, ...`]`
: displays $x \sqsubseteq y \sqsubseteq \ldots$

```
>> SquareSubsetEqual[x, y, z]
```
$x \sqsubseteq y \sqsubseteq z$

```
>> a \[SquareSubsetEqual] b
```
$a \sqsubseteq b$

43.2.157. SquareSuperset ($\sqsupset$)

WML link

`SquareSuperset[`x, y, ...`]`
: displays $x \sqsupset y \sqsupset \ldots$

```
>> SquareSuperset[x, y, z]
```
$x \sqsupset y \sqsupset z$

```
>> a \[SquareSuperset] b
```
$a \sqsupset b$

43.2.158. SquareSupersetEqual (⊒)

WML link

`SquareSupersetEqual[`$x, y, ...$`]`
displays $x \sqsupseteq y \sqsupseteq ...$

```
>> SquareSupersetEqual[x, y, z]
```
$x \sqsupseteq y \sqsupseteq z$

```
>> a \[SquareSupersetEqual] b
```
$a \sqsupseteq b$

43.2.159. SquareUnion (⊔)

WML link

`SquareUnion[`$x, y, ...$`]`
displays $x \sqcup y \sqcup ...$

```
>> SquareUnion[x, y, z]
```
$x \sqcup y \sqcup z$

```
>> a \[SquareUnion] b
```
$a \sqcup b$

43.2.160. Star (⋆)

WML link

`Star[`$x, y, ...$`]`
displays $x \star y \star ...$

```
>> Star[x, y, z]
```
$x \star y \star z$

```
>> a \[Star] b
```
$a \star b$

43.2.161. Subset (⊂)

WML link

```
Subset[x, y, ...]
```
displays $x \subset y \subset \ldots$

```
>> Subset[x, y, z]
```
$x \subset y \subset z$

```
>> a \[Subset] b
```
$a \subset b$

43.2.162. SubsetEqual (⊆)

WML link

```
SubsetEqual[x, y, ...]
```
displays $x \subseteq y \subseteq \ldots$

```
>> SubsetEqual[x, y, z]
```
$x \subseteq y \subseteq z$

```
>> a \[SubsetEqual] b
```
$a \subseteq b$

43.2.163. Succeeds (≻)

WML link

```
Succeeds[x, y, ...]
```
displays $x \succ y \succ \ldots$

```
>> Succeeds[x, y, z]
```
$x \succ y \succ z$

```
>> a \[Succeeds] b
```
$a \succ b$

43.2.164. SucceedsEqual (⪰)

WML link

```
SucceedsEqual[x, y, ...]
```
displays $x \succeq y \succeq \ldots$

```
>> SucceedsEqual[x, y, z]
```
$x \succeq y \succeq z$

```
>> a \[SucceedsEqual] b
```
$a \succeq b$

43.2.165. SucceedsSlantEqual (≽)

WML link

`SucceedsSlantEqual[`$x, y, ...$`]`
displays $x \succcurlyeq y \succcurlyeq ...$

```
>> SucceedsSlantEqual[x, y, z]
```
$x \succcurlyeq y \succcurlyeq z$

```
>> a \[SucceedsSlantEqual] b
```
$a \succcurlyeq b$

43.2.166. SucceedsTilde (≿)

WML link

`SucceedsTilde[`$x, y, ...$`]`
displays $x \succsim y \succsim ...$

```
>> SucceedsTilde[x, y, z]
```
$x \succsim y \succsim z$

```
>> a \[SucceedsTilde] b
```
$a \succsim b$

43.2.167. SuchThat (∍)

WML link

`SuchThat[`$x, y, ...$`]`
displays $x \ni y \ni ...$

```
>> SuchThat[x, y, z]
```
$x \ni y \ni z$

```
>> a \[SuchThat] b
```
$a \ni b$

43.2.168. Superset ($\supset$)

WML link

`Superset[`$x, y, \ldots$`]`
displays $x \supset y \supset \ldots$

```
>> Superset[x, y, z]
```
$x \supset y \supset z$

```
>> a \[Superset] b
```
$a \supset b$

43.2.169. SupersetEqual ($\supseteq$)

WML link

`SupersetEqual[`$x, y, \ldots$`]`
displays $x \supseteq y \supseteq \ldots$

```
>> SupersetEqual[x, y, z]
```
$x \supseteq y \supseteq z$

```
>> a \[SupersetEqual] b
```
$a \supseteq b$

43.2.170. Therefore ($\therefore$)

WML link

`Therefore[`$x, y, \ldots$`]`
displays $x \therefore y \therefore \ldots$

```
>> Therefore[x, y, z]
```
$x \therefore y \therefore z$

```
>> a \[Therefore] b
```
$a \therefore b$

43.2.171. Tilde ($\sim$)

WML link

`Tilde[`$x, y, ...$`]`
displays $x \sim y \sim ...$

>> `Tilde[x, y, z]`
$x \sim y \sim z$

>> `a \[Tilde] b`
$a \sim b$

43.2.172. TildeEqual ($\simeq$)

WML link

`TildeEqual[`$x, y, ...$`]`
displays $x \simeq y \simeq ...$

>> `TildeEqual[x, y, z]`
$x \simeq y \simeq z$

>> `a \[TildeEqual] b`
$a \simeq b$

43.2.173. TildeFullEqual ($\cong$)

WML link

`TildeFullEqual[`$x, y, ...$`]`
displays $x \cong y \cong ...$

>> `TildeFullEqual[x, y, z]`
$x \cong y \cong z$

>> `a \[TildeFullEqual] b`
$a \cong b$

43.2.174. TildeTilde ($\approx$)

WML link

`TildeTilde[`$x, y, ...$`]`
displays $x \approx y \approx ...$

>> `TildeTilde[x, y, z]`
$x \approx y \approx z$

>> `a \[TildeTilde] b`
$a \approx b$

43.2.175. UnionPlus (⊎)

WML link

`UnionPlus[`$x, y, ...$`]`
displays $x \uplus y \uplus ...$

>> `UnionPlus[x, y, z]`
$x \uplus y \uplus z$

>> `a \[UnionPlus] b`
$a \uplus b$

43.2.176. UpArrow (↑)

WML link

`UpArrow[`$x, y, ...$`]`
displays $x \uparrow y \uparrow ...$

>> `UpArrow[x, y, z]`
$x \uparrow y \uparrow z$

>> `a \[UpArrow] b`
$a \uparrow b$

43.2.177. UpArrowBar (⤒)

WML link

`UpArrowBar[`$x, y, ...$`]`
displays x ⤒ y ⤒ $...$

```
>> UpArrowBar[x, y, z]
```
x⤒*y*⤒*z*

```
>> a \[UpArrowBar] b
```
a⤒*b*

43.2.178. UpArrowDownArrow (↕)

WML link

`UpArrowDownArrow[`*x*, *y*, ...`]`
  displays *x* ↕ *y* ↕ ...

```
>> UpArrowDownArrow[x, y, z]
```
x ↕ *y* ↕ *z*

```
>> a \[UpArrowDownArrow] b
```
a ↕ *b*

43.2.179. UpDownArrow (↕)

WML link

`UpDownArrow[`*x*, *y*, ...`]`
  displays *x* ↕ *y* ↕ ...

```
>> UpDownArrow[x, y, z]
```
x ↕ *y* ↕ *z*

```
>> a \[UpDownArrow] b
```
a ↕ *b*

43.2.180. UpEquilibrium (⥮)

WML link

`UpEquilibrium[`*x*, *y*, ...`]`
  displays *x* ⥮ *y* ⥮ ...

```
>> UpEquilibrium[x, y, z]
```
x ⥮ *y* ⥮ *z*

>> **a \[UpEquilibrium] b**
$a \upequilibrium b$

43.2.181. UpTee (⊥)

WML link

`UpTee[`$x, y, ...$`]`
displays $x \perp y \perp ...$

>> **UpTee[x, y, z]**
$x \perp y \perp z$

>> **a \[UpTee] b**
$a \perp b$

43.2.182. UpTeeArrow (↥)

WML link

`UpTeeArrow[`$x, y, ...$`]`
displays $x \underline{\uparrow} y \underline{\uparrow} ...$

>> **UpTeeArrow[x, y, z]**
$x \underline{\uparrow} y \underline{\uparrow} z$

>> **a \[UpTeeArrow] b**
$a \underline{\uparrow} b$

43.2.183. UpperLeftArrow (↖)

WML link

`UpperLeftArrow[`$x, y, ...$`]`
displays $x \nwarrow y \nwarrow ...$

>> **UpperLeftArrow[x, y, z]**
$x \nwarrow y \nwarrow z$

>> **a \[UpperLeftArrow] b**
$a \nwarrow b$

43.2.184. UpperRightArrow (↗)

WML link

`UpperRightArrow[`$x, y, \ldots$`]`
displays $x \nearrow y \nearrow \ldots$

```
>> UpperRightArrow[x, y, z]
```
$x \nearrow y \nearrow z$

```
>> a \[UpperRightArrow] b
```
$a \nearrow b$

43.2.185. Vee (∨)

WML link

`Vee[`$x, y, \ldots$`]`
displays $x \vee y \vee \ldots$

```
>> Vee[x, y, z]
```
$x \vee y \vee z$

```
>> a \[Vee] b
```
$a \vee b$

43.2.186. VerticalBar (∣)

WML link

`VerticalBar[`$x, y, \ldots$`]`
displays $x \mid y \mid \ldots$

```
>> VerticalBar[x, y, z]
```
$x \mid y \mid z$

```
>> a \[VerticalBar] b
```
$a \mid b$

43.2.187. VerticalTilde (≀)

WML link

`VerticalTilde`[x, y, ...]
    displays $x \wr y \wr \ldots$

>> `VerticalTilde[x, y, z]`
$x \wr y \wr z$

>> `a \[VerticalTilde] b`
$a \wr b$

43.2.188. Wedge (∧)

WML link

`Wedge`[x, y, ...]
    displays $x \wedge y \wedge \ldots$

>> `Wedge[x, y, z]`
$x \wedge y \wedge z$

>> `a \[Wedge] b`
$a \wedge b$

43.3. Postfix Operators without Built-in Meanings

43.3.1. InvisiblePostfixScriptBase

WML link

`InvisiblePostfixScriptBase`[x]
    displays x

>> `InvisiblePostfixScriptBase[x]`
x

>> `x \[InvisiblePostfixScriptBase]`
x

43.4. Prefix Operators without Built-in Meanings

43.4.1. CapitalDifferentialD

WML link

`CapitalDifferentialD[`*x*`]`
    displays $\mathbb{D}\ x$

```
>> CapitalDifferentialD[x]
```
$\mathbb{D}x$

```
>> \[CapitalDifferentialD]x
```
$\mathbb{D}x$

43.4.2. Del (∇)

WML link

`Del[`*x*`]`
    displays $\nabla\ x$

```
>> Del[x]
```
∇x

```
>> \[Del]x
```
∇x

43.4.3. DifferentialD (d)

WML link

`DifferentialD[`*x*`]`
    displays $d\ x$

```
>> DifferentialD[x]
```
dx

```
>> \[DifferentialD]x
```
dx

43.4.4. InvisiblePrefixScriptBase

WML link

`InvisiblePrefixScriptBase[`x`]`
    displays x

```
>> InvisiblePrefixScriptBase[x]
```
x

```
>> \[InvisiblePrefixScriptBase]x
```
x

43.4.5. Square (□)

WML link

`Square[`x`]`
    displays □ x

```
>> Square[x]
```
□x

```
>> \[Square]x
```
□x

44. Options Management

A number of functions have various options which control the behavior or the default behavior that function. Default options can be queried or set.

WMA link

Contents

44.1. All

WMA link

`All`
is an option value for a number of functions indicating to include everything.

In list functions, it indicates all levels of the list.

For example, in Part 35.3.16, `All`, extracts into a first column vector the first element of each of the list elements:

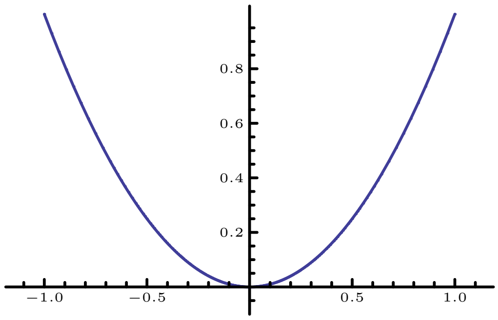
```
>> {{1, 3}, {5, 7}}[[All, 1]]
   {1,5}
```

While in Take 35.3.16, `All` extracts as a column matrix the first element as a list for each of the list elements:

```
>> Take[{{1, 3}, {5, 7}}, All, {1}]
   {{1},{5}}
```

In Plot 26.2.15, setting the Mesh 26.1.13 option to `All` will show the specific plot points:

```
>> Plot[x^2, {x, -1, 1}, MaxRecursion->5, Mesh->All]
```



44.2. Default

WMA link

`Default`[f]
: gives the default value for an omitted parameter of f.

`Default`[f, k]
: gives the default value for a parameter on the k-th position.

`Default`[f, k, n]
: gives the default value for the k-th parameter out of n.

Assign values to `Default` to specify default values.

```
>> Default[f] = 1
```
1

```
>> f[x_.] := x ^ 2
```

```
>> f[]
```
1

Default values are stored in `DefaultValues`:

```
>> DefaultValues[f]
```
{HoldPattern [Default [f]] :>1}

You can use patterns for k and n:

```
>> Default[h, k_, n_] := {k, n}
```

Note that the position of a parameter is relative to the pattern, not the matching expression:

```
>> h[] /. h[___, ___, x_., y_., ___] -> {x, y}
```
{{3,5},{4,5}}

44.3. FilterRules

WMA link

`FilterRules[`*rules*, *pattern*`]`
: gives those *rules* that have a left side that matches *pattern*.

`FilterRules[`*rules*, {$pattern_1$, $pattern_2$, ...}`]`
: gives those *rules* that have a left side that match at least one of $pattern_1$, $pattern_2$, ...

```
>> FilterRules[{x -> 100, y -> 1000}, x]
```

$\{x- > 100\}$

```
>> FilterRules[{x -> 100, y -> 1000, z -> 10000}, {a, b, x, z}]
```

$\{x- > 100, z- > 10000\}$

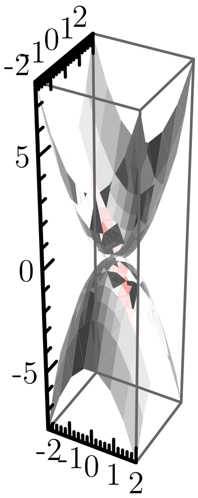
44.4. None

WMA link

`None`
: is a setting value for many options.

Plot3D shows the mesh grid between computed points by default. This the Mesh 26.1.13 However, you hide the mesh by setting the `Mesh` option value to `None`:

```
>> Plot3D[{x^2 + y^2, -x^2 - y^2}, {x, -2, 2}, {y, -2, 2}, BoxRatios->
   Automatic, Mesh->None]
```



44.5. NotOptionQ

WMA link

`NotOptionQ[`*expr*`]`
: returns `True` if *expr* does not have the form of a valid option specification.

>> `NotOptionQ[x]`
True

>> `NotOptionQ[2]`
True

>> `NotOptionQ["abc"]`
True

>> `NotOptionQ[a -> True]`
False

44.6. OptionQ

WMA link

`OptionQ[`*expr*`]`
: returns `True` if *expr* has the form of a valid option specification.

Examples of option specifications:

>> `OptionQ[a -> True]`
True

>> `OptionQ[a :> True]`
True

>> `OptionQ[{a -> True}]`
True

>> `OptionQ[{a :> True}]`
True

Options lists are flattened when are applied, so

>> `OptionQ[{a -> True, {b->1, "c"->2}}]`
True

>> `OptionQ[{a -> True, {b->1, c}}]`
False

```
>> OptionQ[{a -> True, F[b->1,c->2]}]
```
False

`OptionQ` returns `False` if its argument is not a valid option specification:

```
>> OptionQ[x]
```
False

44.7. OptionValue

WMA link

`OptionValue`[*name*]
: gives the value of the option *name* matched by `OptionsPattern`.

`OptionValue`[*f*, *name*]
: recover the value of the option *name* associated with the head *f*.

`OptionValue`[*f*, *opts*, *name*]
: recover the value of the option *name* associated with the symbol *f*, extracting the values from *optvals* if available.

`OptionValue`[..., *list*]
: recover the value of the options in *list* .

First, set up a symbol with some options using `Options`:

```
>> Options[MySetting] = {"foo" -> 5, "bar" -> 6}
```
$\{\text{foo}- > 5, \text{bar}- > 6\}$

Now get a value previously set:

```
>> OptionValue[MySetting, "bar"]
```
6

If the option does exist we get a message:

```
>> OptionValue[MySetting, "baz"]
Option name baz not found in defaults for MySetting.
```
baz

Use `OptionValue` to get the value of option `a` inside `OptionsPattern` `a->3`

```
>> f[a->3] /. f[OptionsPattern[{}]] -> {OptionValue[a]}
```
$\{3\}$

An unavailable option returns argument and does not generate a message:

```
>> f[a->3] /. f[OptionsPattern[{}]] -> {OptionValue[b]}
```
$\{b\}$

The argument of `OptionValue` must be a symbol:

```
>> f[a->3] /. f[OptionsPattern[{}]] -> {OptionValue[a+b]}
   Argument a + b at position 1 is expected to be a symbol.
```

$\{\text{OptionValue}[a+b]\}$

However, the symbol can be evaluated dynamically:

```
>> f[a->5] /. f[OptionsPattern[{}]] -> {OptionValue[Symbol["a"]]}
```

$\{5\}$

See also 'Options' 44.8 and 'OptionsPattern' 47.2.5.

44.8. Options

WMA link

`Options[`f`]`
  gives a list of optional arguments to f and their default values.

You can assign values to `Options` to specify options.

```
>> Options[f] = {n -> 2}
```

$\{n- > 2\}$

```
>> Options[f]
```

$\{n{:>}2\}$

```
>> f[x_, OptionsPattern[f]] := x ^ OptionValue[n]
```

```
>> f[x]
```

x^2

```
>> f[x, n -> 3]
```

x^3

Delayed option rules are evaluated just when the corresponding `OptionValue` is called:

```
>> f[a :> Print["value"]] /. f[OptionsPattern[{}]] :> (OptionValue[a];
   Print["between"]; OptionValue[a]);
   value
   between
   value
```

In contrast to that, normal option rules are evaluated immediately:

```
>> f[a -> Print["value"]] /. f[OptionsPattern[{}]] :> (OptionValue[a];
   Print["between"]; OptionValue[a]);
   value
   between
```

Options must be rules or delayed rules:

```
>> Options[f] = {a}
   {a} is not a valid list of option rules.
```
$\{a\}$

A single rule need not be given inside a list:

```
>> Options[f] = a -> b
```
$a- > b$

```
>> Options[f]
```
$\{a{:>}b\}$

Options can only be assigned to symbols:

```
>> Options[a + b] = {a -> b}
   Argument a + b at position 1 is expected to be a symbol.
```
$\{a- > b\}$

See also 'OptionValue' 44.7 and 'OptionsPattern' 47.2.5.

44.9. SetOptions

WMA link

`SetOptions`[*s*, name1 -> value1, name2 -> value2, ...]
    sets the specified default options for a symbol *s*. The entire set of options for *s* is returned.

One way to find the default options for a symbol is to use `SetOptions` passing no association pairs:

```
>> SetOptions[Plot]
```
$$\left\{\text{AspectRatio}- > \frac{1}{\text{GoldenRatio}}, \text{Axes}- > \text{True}, \text{AxesStyle} - > \{\}, \text{Background}- > \text{Automatic}, \text{Exclusions}- > \text{Automatic}, \text{ImageSize} - > \text{Automatic}, \text{LabelStyle}- > \{\}, \text{MaxRecursion} - > \text{Automatic}, \text{Mesh}- > \text{None}, \text{PlotPoints}- > \text{None}, \text{PlotRange} - > \text{Automatic}, \text{PlotRangePadding}- > \text{Automatic}, \text{TicksStyle}- > \{\}\right\}$$

45. Physical and Chemical data

Contents

45.1. ElementData

WMA link

`ElementData`["*name*", "*property*"]
gives the value of the *property* for the chemical specified by *name*.
`ElementData`[*n*, "*property*"]
gives the value of the *property* for the *n*-th chemical element.

```
>> ElementData[74]
```
Tungsten

```
>> ElementData["He", "AbsoluteBoilingPoint"]
```
4.22

```
>> ElementData["Carbon", "IonizationEnergies"]
```
{1086.5, 2352.6, 4620.5, 6222.7, 37831, 47277.}

```
>> ElementData[16, "ElectronConfigurationString"]
```
[Ne] 3s2 3p4

```
>> ElementData[73, "ElectronConfiguration"]
```
{{2} , {2, 6} , {2, 6, 10} , {2, 6, 10, 14} , {2, 6, 3} , {2}}

The number of known elements:

```
>> Length[ElementData[All]]
```
118

Some properties are not appropriate for certain elements:

```
>> ElementData["He", "ElectroNegativity"]
```
Missing [NotApplicable]

Some data is missing:

>> `ElementData["Tc", "SpecificHeat"]`

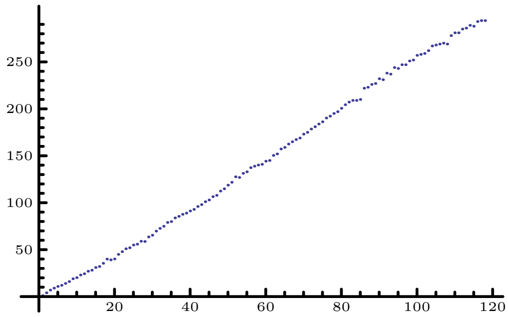
Missing [NotAvailable]

All the known properties:

>> `ElementData["Properties"]`

{Abbreviation, AbsoluteBoilingPoint, AbsoluteMeltingPoint, AtomicNumber, AtomicRadius, AtomicWeight, Block,

>> `ListPlot[Table[ElementData[z, "AtomicWeight"], {z, 118}]]`



46. Procedural Programming

Procedural programming is a programming paradigm, derived from imperative programming, based on the concept of the procedure call. This term is sometimes compared and contrasted with Functional Programming.

Procedures (a type of routine or subroutine) simply contain a series of computational steps to be carried out. Any given procedure might be called at any point during a program's execution, including by other procedures or itself.

Procedural functions are integrated into *Mathics3* symbolic programming environment.

Contents

46.1. Abort

WMA link

`Abort[]`
: aborts an evaluation completely and returns `$Aborted`.

```
>> Print["a"]; Abort[]; Print["b"]
```

a

$Aborted

46.2. Break

WMA link

`Break[]`
: exits a `For`, `While`, or `Do` loop.

```
>> n = 0;

>> While[True, If[n>10, Break[]]; n=n+1]

>> n
   11
```

46.3. Catch

WMA link

`Catch`[*expr*]
: returns the argument of the first `Throw` generated in the evaluation of *expr*.

`Catch`[*expr*, *form*]
: returns value from the first `Throw[$value$, $tag$]` for which *form* matches *tag*.

`Catch`[*expr*, *form*, *f*]
: returns *f*[*value*, *tag*].

Exit to the enclosing `Catch` as soon as `Throw` is evaluated:

```
>> Catch[r; s; Throw[t]; u; v]
   t
```

Define a function that can "throw an exception":

```
>> f[x_] := If[x > 12, Throw[overflow], x!]
```

The result of `Catch` is just what is thrown by `Throw`:

```
>> Catch[f[1] + f[15]]
   overflow

>> Catch[f[1] + f[4]]
   25
```

46.4. CheckAbort

WMA link

`CheckAbort`[*expr*, *failexpr*]
: evaluates *expr*, returning *failexpr* if an abort occurs.

```
>> CheckAbort[Abort[]; 1, 2] + x
```
$2 + x$

```
>> CheckAbort[1, 2] + x
```
$1 + x$

46.5. CompoundExpression (;)

WMA link

`CompoundExpression`[e_1, e_2, ...]
e_1 ; e_2 ; ...
: evaluates its arguments in turn, returning the last result.

```
>> a; b; c; d
```
d

If the last argument is omitted, `Null` is taken:

```
>> a;
```

46.6. Continue

WMA link

`Continue[]`
: continues with the next iteration in a `For`, `While`, or `Do` loop.

```
>> For[i=1, i<=8, i=i+1, If[Mod[i,2] == 0, Continue[]]; Print[i]]
```
```
1
3
5
7
```

46.7. Do

WMA link

Do[*expr*, {*max*}]
: evaluates *expr max* times.

Do[*expr*, {*i*, *max*}]
: evaluates *expr max* times, substituting *i* in *expr* with values from 1 to *max*.

Do[*expr*, {*i*, *min*, *max*}]
: starts with `$i$ = $max$`.

Do[*expr*, {*i*, *min*, *max*, *step*}]
: uses a step size of *step*.

Do[*expr*, {*i*, {i_1, i_2, ...}}]
: uses values i_1, i_2, ... for *i*.

Do[*expr*, {*i*, i_{min}, i_{max}}, {*j*, j_{min}, j_{max}}, ...]
: evaluates *expr* for each *j* from j_{min} to j_{max}, for each *i* from i_{min} to i_{max}, etc.

```
>> Do[Print[i], {i, 2, 4}]
2
3
4
```

```
>> Do[Print[{i, j}], {i,1,2}, {j,3,5}]
{1, 3}
{1, 4}
{1, 5}
{2, 3}
{2, 4}
{2, 5}
```

You can use `Break[]` and `Continue[]` inside `Do`:

```
>> Do[If[i > 10, Break[], If[Mod[i, 2] == 0, Continue[]]; Print[i]], {i,
   5, 20}]
5
7
9
```

46.8. For

WMA link

For[*start*, *test*, *incr*, *body*]
: evaluates *start*, and then iteratively *body* and *incr* as long as *test* evaluates to `True`.

For[*start*, *test*, *incr*]
: evaluates only *incr* and no *body*.

For[*start*, *test*]
: runs the loop without any body.

Compute the factorial of 10 using `For`:

```
>> n := 1
```

```
>> For[i=1, i<=10, i=i+1, n = n * i]
```

```
>> n
   3628800
>> n == 10!
   True
```

46.9. If

WMA link

`If[`*cond*, *pos*, *neg*`]`
: returns *pos* if *cond* evaluates to `True`, and *neg* if it evaluates to `False`.

`If[`*cond*, *pos*, *neg*, *other*`]`
: returns *other* if *cond* evaluates to neither `True` nor `False`.

`If[`*cond*, *pos*`]`
: returns `Null` if *cond* evaluates to `False`.

```
>> If[1<2, a, b]
   a
```

If the second branch is not specified, `Null` is taken:

```
>> If[1<2, a]
   a
>> If[False, a] //FullForm
   Null
```

You might use comments inside `(*` and `*)` to make the branches of `If` more readable:

```
>> If[a, (*then*)b, (*else*)c];
```

Since one or more arguments to a boolean operation could be symbolic, it is possible that an `If` cannot be evaluated. For example:

```
>> Clear[a, b]; If [a < b, a, b]
```

If $[a < b, a, b]$

To handle this, `If` takes an optional fourth parameter:

```
>> If [a < b, a, b, "I give up"]
   I give up
```

46.10. Interrupt

WMA link

`Interrupt[]`
: Interrupt an evaluation and returns `$Aborted`.

```
>> Print["a"]; Interrupt[]; Print["b"]
a
$Aborted
```

46.11. Pause

WMA link

`Pause[n]`
: pauses for at least *n* seconds.

```
>> Pause[0.5]
```

46.12. Return

WMA link

`Return[`*expr*`]`
: aborts a function call and returns *expr*.

```
>> f[x_] := (If[x < 0, Return[0]]; x)
```

```
>> f[-1]
0
```

```
>> Do[If[i > 3, Return[]]; Print[i], {i, 10}]
1
2
3
```

`Return` only exits from the innermost control flow construct.

```
>> g[x_] := (Do[If[x < 0, Return[0]], {i, {2, 1, 0, -1}}]; x)
```

```
>> g[-1]
-1
```

46.13. Switch

WMA link

`Switch`[*expr*, *pattern*$_1$, *value*$_1$, *pattern*$_2$, *value*$_2$, ...]
: yields the first *value* for which *expr* matches the corresponding *pattern*.

```
>> Switch[2, 1, x, 2, y, 3, z]
```
y

```
>> Switch[5, 1, x, 2, y]
```
$\text{Switch}\left[5,1,x,2,y\right]$

```
>> Switch[5, 1, x, 2, a, _, b]
```
b

```
>> Switch[2, 1]
Switch called with 2 arguments. Switch must be called with an odd
number of arguments.
```
Switch [2, 1]

Notice that `Switch` evaluates each pattern before it against *expr*, stopping after the first match:

```
>> a:=(Print["a->p"];p); b:=(Print["b->q"];q);
```

```
>> Switch[p,a,1,b,2]
a->p
```
1

```
>> a=.; b=.;
```

46.14. Throw

WMA link

`Throw[`value`]`
: stops evaluation and returns ‘value‘ as the value of the nearest enclosing `Catch`.

`Catch[`value`, `tag`]`
: is caught only by ‘Catch[expr,form]‘, where tag matches form.

Using Throw can affect the structure of what is returned by a function:

```
>> NestList[#^2 + 1 &, 1, 7]
```
{1, 2, 5, 26, 677, 458330, 210066388901, 44127887745906175987802}

```
>> Catch[NestList[If[# > 1000, Throw[#], #^2 + 1] &, 1, 7]]
```
458330

```
>> Throw[1]
```
Uncaught Throw[1] returned to top level.

Hold [Throw [1]]

46.15. Which

WMA link

`Which[`$cond_1$, $expr_1$, $cond_2$, $expr_2$, ...`]`
: yields $expr_1$ if $cond_1$ evaluates to `True`, $expr_2$ if $cond_2$ evaluates to `True`, etc.

```
>> n = 5;
```

```
>> Which[n == 3, x, n == 5, y]
```
y

```
>> f[x_] := Which[x < 0, -x, x == 0, 0, x > 0, x]
```

```
>> f[-3]
```
3

If no test yields `True`, `Which` returns `Null`:

```
>> Which[False, a]
```

If a test does not evaluate to `True` or `False`, evaluation stops and a `Which` expression containing the remaining cases is returned:

```
>> Which[False, a, x, b, True, c]
```
Which $[x, b, \text{True}, c]$

`Which` must be called with an even number of arguments:

```
>> Which[a, b, c]
```
Which called with 3 arguments.

Which $[a, b, c]$

46.16. While

WMA link

`While[`$test$, $body$`]`
: evaluates $body$ as long as $test$ evaluates to `True`.

`While[`$test$`]`
: runs the loop without any body.

Compute the GCD of two numbers:

```
>> {a, b} = {27, 6};

>> While[b != 0, {a, b} = {b, Mod[a, b]}];

>> a
3
```

47. Rules and Patterns

WMA link

The concept of transformation rules for arbitrary symbolic patterns is key in *Mathics3*.

Also, functions can get applied or transformed depending on whether or not functions arguments match.

Some examples:

```
>> a + b + c /. a + b -> t
```

$c + t$

```
>> a + 2 + b + c + x * y /. n_Integer + s__Symbol + rest_ -> {n, s, rest
   }
```

$\{2, a, b + c + xy\}$

```
>> f[a, b, c, d] /. f[first_, rest___] -> {first, {rest}}
```

$\{a, \{b, c, d\}\}$

Tests and Conditions:

```
>> f[4] /. f[x_?(# > 0&)] -> x ^ 2
```

16

```
>> f[4] /. f[x_] /; x > 0 -> x ^ 2
```

16

Elements in the beginning of a pattern rather match fewer elements:

```
>> f[a, b, c, d] /. f[start__, end__] -> {{start}, {end}}
```

$\{\{a\}, \{b, c, d\}\}$

Optional arguments using `Optional`:

```
>> f[a] /. f[x_, y_:3] -> {x, y}
```

$\{a, 3\}$

Options using `OptionsPattern` and `OptionValue`:

```
>> f[y, a->3] /. f[x_, OptionsPattern[{a->2, b->5}]] -> {x, OptionValue[
   a], OptionValue[b]}
```

$\{y, 3, 5\}$

The attributes `Flat`, `Orderless`, and `OneIdentity` affect pattern matching.

Contents

47.1. Basic Pattern Objects

47.1.1. Blank

WMA link

`Blank[]`
`_`
    represents any single expression in a pattern.
`Blank[`h`]`
`_$h$`
    represents any expression with head h.

```
>> MatchQ[a + b, _]
```
True

Patterns of the form _h can be used to test the types of objects:

```
>> MatchQ[42, _Integer]
```
True

```
>> MatchQ[1.0, _Integer]
```
False

```
>> {42, 1.0, x} /. {_Integer -> "integer", _Real -> "real"} // InputForm
```
{“integer”, “real”, x}

`Blank` only matches a single expression:

```
>> MatchQ[f[1, 2], f[_]]
   False
```

47.1.2. BlankNullSequence

WMA link

`BlankNullSequence[]`
`___`
: represents any sequence of expression elements in a pattern, including an empty sequence.

`BlankNullSequence` is like `BlankSequence`, except it can match an empty sequence:

```
>> MatchQ[f[], f[___]]
   True
```

47.1.3. BlankSequence

WMA link

`BlankSequence[]`
`__`
: represents any non-empty sequence of expression elements in a pattern.

`BlankSequence[`*h*`]`
`__$h$`
: represents any sequence of elements, all of which have head *h*.

Use a `BlankSequence` pattern to stand for a non-empty sequence of arguments:

```
>> MatchQ[f[1, 2, 3], f[__]]
   True
>> MatchQ[f[], f[__]]
   False
```

`__`*h* will match only if all elements have head *h*:

```
>> MatchQ[f[1, 2, 3], f[__Integer]]
   True
>> MatchQ[f[1, 2.0, 3], f[__Integer]]
   False
```

The value captured by a named `BlankSequence` pattern is a `Sequence` object:

```
>> f[1, 2, 3] /. f[x__] -> x
```
Sequence $[1,2,3]$

47.2. Composite Patterns

47.2.1. Alternatives (|)

WMA link

`Alternatives[`p_1, p_2, ..., p_i`]`
p_1 `|` p_2 `|` ... `|` p_i
: is a pattern that matches any of the patterns p_1, p_2, ..., p_i.

```
>> a+b+c+d/.(a|b)->t
```
$c + d + 2t$

Alternatives can also be used for string expressions:

```
>> StringReplace["0123 3210", "1" | "2" -> "X"]
```
0XX3 3XX0

47.2.2. Except

WMA link

`Except[`c`]`
: represents a pattern object that matches any expression except those matching c.

`Except[`c, p`]`
: represents a pattern object that matches p but not c.

```
>> Cases[{x, a, b, x, c}, Except[x]]
```
$\{a, b, c\}$

```
>> Cases[{a, 0, b, 1, c, 2, 3}, Except[1, _Integer]]
```
$\{0, 2, 3\}$

Except can also be used for string expressions:

```
>> StringReplace["Hello world!", Except[LetterCharacter] -> ""]
```
Helloworld

47.2.3. HoldPattern

WMA link

`HoldPattern[`*expr*`]`
: is equivalent to *expr* for pattern matching, but maintains it in an unevaluated form.

```
>> HoldPattern[x + x]
```
HoldPattern $[x + x]$

```
>> x /. HoldPattern[x] -> t
```
t

`HoldPattern` has attribute `HoldAll`:

```
>> Attributes[HoldPattern]
```
{HoldAll, Protected}

47.2.4. Longest

WMA link

`Longest[`*pat*`]`
: is a pattern object that matches the longest sequence consistent with the pattern *pat*.

```
>> StringCases["aabaaab", Longest["a" ~~__ ~~"b"]]
```
{aabaaab}

```
>> StringCases["aabaaab", Longest[RegularExpression["a+b"]]]
```
{aab, aaab}

47.2.5. OptionsPattern

WMA link

`OptionsPattern[`*f*`]`
: is a pattern that stands for a sequence of options given to a function, with default values taken from `Options[$f$]`. The options can be of the form `$opt$->$value$` or `$opt$:>$value$`, and might be in arbitrarily nested lists.

`OptionsPattern[{`opt_1`->`$value_1$`, ...}]`
: takes explicit default values from the given list. The list may also contain symbols f, for which `Options[$f$]` is taken into account; it may be arbitrarily nested. `OptionsPattern[{}]` does not use any default values.

The option values can be accessed using `OptionValue`.

```
>> f[x_, OptionsPattern[{n->2}]] := x ^ OptionValue[n]
```

```
>> f[x]
```
x^2

```
>> f[x, n->3]
```
x^3

Delayed rules as options:

```
>> e = f[x, n:>a]
```
x^a

```
>> a = 5;
```

```
>> e
```
x^5

Options might be given in nested lists:

```
>> f[x, {{{n->4}}}]
```
x^4

See also 'Options' 44.8 and 'OptionValue' 44.7.

47.2.6. Pattern

WMA link

`Pattern`[*symb*, *pat*]
symb : *pat*
    assigns the name *symb* to the pattern *pat*.
symb_head
    is equivalent to *symb* : *_head* (accordingly with __ and ___).
symb : *pat* : *default*
    is a pattern with name *symb* and default value *default*, equivalent to `Optional`[*pat* : *symb*, *default*].

```
>> FullForm[a_b]
```
Pattern [*a*, Blank [*b*]]

```
>> FullForm[a:_:b]
```
Optional [Pattern [*a*, Blank []] , *b*]

`Pattern` has attribute `HoldFirst`, so it does not evaluate its name:

```
>> x = 2
```
2

```
>> x_
   x_
```

Nested `Pattern` assigns multiple names to the same pattern. Still, the last parameter is the default value.

```
>> f[y] /. f[a:b,_:d] -> {a, b}
```

$f[y]$

This is equivalent to:

```
>> f[a] /. f[a:_:b] -> {a, b}
```

$\{a, b\}$

'FullForm':

```
>> FullForm[a:b:c:d:e]
```

Optional [Pattern [a, b] , Optional [Pattern [c, d] , e]]

```
>> f[] /. f[a:_:b] -> {a, b}
```

$\{b, b\}$

47.2.7. Repeated (..)

WMA link

`Repeated[`*pat*`]`
: matches one or more occurrences of *pat*.

```
>> a_Integer.. // FullForm
```

Repeated [Pattern [a, Blank [Integer]]]

```
>> 0..1//FullForm
```

Repeated [0]

```
>> {{}, {a}, {a, b}, {a, a, a}, {a, a, a, a}} /. {Repeated[x : a | b,
   3]} -> x
```

$\{\{\}, a, \{a, b\}, a, \{a, a, a, a\}\}$

```
>> f[x, 0, 0, 0] /. f[x, s:0..] -> s
```

Sequence [0, 0, 0]

47.2.8. RepeatedNull (...)

WMA link

`RepeatedNull[`*pat*`]`
: matches zero or more occurrences of *pat*.

```
>> a___Integer...//FullForm
```
RepeatedNull [Pattern [*a*, BlankNullSequence [Integer]]]

```
>> f[x] /. f[x, 0...] -> t
```
t

47.2.9. Shortest

WMA link

`Shortest[`*pat*`]`
: is a pattern object that matches the shortest sequence consistent with the pattern *pat*.

```
>> StringCases["aabaaab", Shortest["a" ~~__ ~~"b"]]
```
{aab, aaab}

```
>> StringCases["aabaaab", Shortest[RegularExpression["a+b"]]]
```
{aab, aaab}

47.2.10. Verbatim

WMA link

`Verbatim[`*expr*`]`
: prevents pattern constructs in *expr* from taking effect, allowing them to match themselves.

Create a pattern matching `Blank`:

```
>> _ /. Verbatim[_]->t
```
t

```
>> x /. Verbatim[_]->t
```
x

Without `Verbatim`, `Blank` has its normal effect:

```
>> x /. _->t
```
t

47.3. Defining, applying and compiling rules.

WMA link

Rules are a basic element in the evaluation process. Every Definition in *Mathics3* consists of a set of rules associated with a symbol. The evaluation process consists of the sequential application of rules associated with the symbols appearing in a given expression. The process iterates until no rules match the final expression.

In *Mathics3*, rules consist of a Pattern object *pat* and an Expression *repl*. When the Rule is applied to a symbolic Expression *expr*, the interpreter tries to match the pattern with subexpressions of *expr* in a top-to-bottom way. If a match is found, the subexpression is then replaced by *repl*.

If the *pat* includes named subpatterns, symbols in *repl* associated with that name are replaced by the (sub) match in the final expression.

Let us consider, for example, the `Rule`:

```
>> rule = F[u_]->g[u]
```

$F[\mathrm{u\_}] - > g[u]$

This rule associates the pattern `F[u_]` with the expression `g[u]`.

Then, using the `Replace` operator `/.` we can apply the rule to an expression

```
>> a + F[x ^ 2] /. rule
```

$a + g\left[x^2\right]$

Notice that the rule is applied from top to bottom just once:

```
>> a + F[F[x ^ 2]] /. rule
```

$a + g\left[F\left[x^2\right]\right]$

Here, the subexpression `F[F[x^2]]` matches with the pattern, and the named subpattern `u_` matches with `F[x^2]`. The original expression is then replaced by `g[u]`, and `u` is replaced with the subexpression that matches the subpattern (`F[x ^ 2]`).

Notice also that the rule is applied just once. We can apply it recursively until no further matches are found by using the `ReplaceRepeated` operator `//.`:

```
>> a + F[F[x ^ 2]] //. rule
```

$a + g\left[g\left[x^2\right]\right]$

Rules are kept as expressions until a `Replace` expression is evaluated. At that moment, `Pattern` objects are `compiled`, taking into account the attributes of the symbols involved. To make the repeated application of the same rule over different expressions faster, it is convenient to use `Dispatch` tables. These expressions store precompiled versions of a list of rules, avoiding repeating the `compilation` step each time the rules are applied.

```
>> dispatchrule = Dispatch[{rule}]
```
Dispatch [<1>]

```
>> a + F[F[x ^ 2]] //. dispatchrule
```
$a + g\left[g\left[x^2\right]\right]$

47.3.1. Dispatch

WMA link

`Dispatch[`*rulelist*`]`
: Introduced for compatibility. Currently, it just return *rulelist*. In the future, it should return an optimized DispatchRules atom, containing an optimized set of rules.

```
>> rules = {{a_,b_}->a^b, {1,2}->3., F[x_]->x^2};
```

```
>> F[2] /. rules
```
4

```
>> dispatchrules = Dispatch[rules]
```
Dispatch [<3>]

```
>> F[2] /. dispatchrules
```
4

47.3.2. Replace

WMA link

`Replace[`*expr*, *x* -> *y*`]`
: yields the result of replacing *expr* with *y* if it matches the pattern *x*.

`Replace[`*expr*, *x* -> *y*, *levelspec*`]`
: replaces only subexpressions at levels specified through *levelspec*.

`Replace[`*expr*, {*x* -> *y*, ...}`]`
: performs replacement with multiple rules, yielding a single result expression.

`Replace[`*expr*, {{*a* -> *b*, ...}, {*c* -> *d*, ...}, ...}`]`
: returns a list containing the result of performing each set of replacements.

```
>> Replace[x, {x -> 2}]
```
2

By default, only the top level is searched for matches:

```
>> Replace[1 + x, {x -> 2}]
```
$1 + x$

```
>> Replace[x, {{x -> 1}, {x -> 2}}]
   {1,2}
```

Replace stops after the first replacement:

```
>> Replace[x, {x -> {}, _List -> y}]
   {}
```

Replace replaces the deepest levels first:

```
>> Replace[x[1], {x[1] -> y, 1 -> 2}, All]
   x[2]
```

By default, heads are not replaced:

```
>> Replace[x[x[y]], x -> z, All]
   x[x[y]]
```

Heads can be replaced using the `Heads` option:

```
>> Replace[x[x[y]], x -> z, All, Heads -> True]
   z[z[y]]
```

Note that heads are handled at the level of elements:

```
>> Replace[x[x[y]], x -> z, {1}, Heads -> True]
   z[x[y]]
```

You can use Replace as an operator:

```
>> Replace[{x_ -> x + 1}][10]
   11
```

47.3.3. ReplaceAll (`/.`)

WMA link

`ReplaceAll`[*expr*, *x* -> *y*]
`$expr$ /. $x$ -> $y$`
: yields the result of replacing all subexpressions of *expr* matching the pattern *x* with *y*.

`$expr$ /. {$x$ -> $y$, ...}`
: performs replacement with multiple rules, yielding a single result expression.

`$expr$ /. {{$a$ -> $b$, ...}, {$c$ -> $d$, ...}, ...}`
: returns a list containing the result of performing each set of replacements.

```
>> a+b+c /. c->d
```
$a+b+d$

```
>> g[a+b+c,a]/.g[x_+y_,x_]->{x,y}
```
$\{a, b+c\}$

If *rules* is a list of lists, a list of all possible respective replacements is returned:

```
>> {a, b} /. {{a->x, b->y}, {a->u, b->v}}
```
$\{\{x, y\}, \{u, v\}\}$

The list can be arbitrarily nested:

```
>> {a, b} /. {{{a->x, b->y}, {a->w, b->z}}, {a->u, b->v}}
```
$\{\{\{x, y\}, \{w, z\}\}, \{u, v\}\}$

```
>> {a, b} /. {{{a->x, b->y}, a->w, b->z}, {a->u, b->v}}
Elements of {{a -> x, b -> y}, a -> w, b -> z} are a mixture of lists
and nonlists.
```
$\{\{a, b\} /. \{\{a->x, b->y\}, a->w, b->z\}, \{u, v\}\}$

ReplaceAll also can be used as an operator:

```
>> ReplaceAll[{a -> 1}][{a, b}]
```
$\{1, b\}$

ReplaceAll replaces the shallowest levels first:

```
>> ReplaceAll[x[1], {x[1] -> y, 1 -> 2}]
```
y

47.3.4. ReplaceList

WMA link

`ReplaceList[`*expr*, *rules*`]`
: returns a list of all possible results when applying *rules* to *expr*.

`ReplaceList[`*expr*, *rules*, *n*`]`
: returns a list of at most *n* results when applying *rules* to *expr*.

Get all subsequences of a list:

```
>> ReplaceList[{a, b, c}, {___, x__, ___} -> {x}]
```
$\{\{a\}, \{a, b\}, \{a, b, c\}, \{b\}, \{b, c\}, \{c\}\}$

You can specify the maximum number of items:

```
>> ReplaceList[{a, b, c}, {___, x__, ___} -> {x}, 3]
```
$\{\{a\},\{a,b\},\{a,b,c\}\}$

```
>> ReplaceList[{a, b, c}, {___, x__, ___} -> {x}, 0]
```
$\{\}$

If no rule matches, an empty list is returned:

```
>> ReplaceList[a, b->x]
```
$\{\}$

Like in `ReplaceAll`, *rules* can be a nested list:

```
>> ReplaceList[{a, b, c}, {{{___, x__, ___} -> {x}}, {{a, b, c} -> t}},
   2]
```
$\{\{\{a\},\{a,b\}\},\{t\}\}$

Possible matches for a sum:

```
>> ReplaceList[a + b + c, x_ + y_ -> {x, y}]
```
$\{\{a,b+c\},\{b,a+c\},\{c,a+b\},\{a+b,c\},\{a+c,b\},\{b+c,a\}\}$

47.3.5. ReplaceRepeated (//.)

WMA link

`ReplaceRepeated[`*expr*, *x* -> *y*`]`
`$expr$ //. $x$ -> $y$`
    repeatedly applies the rule `$x$ -> $y$` to *expr* until the result no longer changes.

```
>> a+b+c //. c->d
```
$a+b+d$

```
>> f = ReplaceRepeated[c->d];
```

```
>> f[a+b+c]
```
$a+b+d$

```
>> Clear[f];
```

Simplification of logarithms:

```
>> logrules = {Log[x_ * y_] :> Log[x] + Log[y], Log[x_ ^ y_] :> y * Log[
   x]};
```

```
>> Log[a * (b * c)^ d ^ e * f] //. logrules
```
$\mathrm{Log}[a]+\mathrm{Log}\left[f\right]+\left(\mathrm{Log}[b]+\mathrm{Log}[c]\right)d^{e}$

`ReplaceAll` just performs a single replacement:

```
>> Log[a * (b * c)^ d ^ e * f] /. logrules
```

$\text{Log}[a] + \text{Log}\left[f (bc)^{d^e}\right]$

47.3.6. RuleDelayed (`:\to`)

WMA link

`RuleDelayed`[x, y]
`$x$ :> $y$`
: represents a rule replacing x with y, with y held unevaluated.

```
>> Attributes[RuleDelayed]
```

{HoldRest, Protected, SequenceHold}

47.3.7. Rule

WMA link

`Rule`[x, y]
`$x$ -> $y$`
: represents a rule replacing x with y.

```
>> a+b+c /. c->d
```

$a + b + d$

```
>> {x,x^2,y} /. x->3
```

$\{3, 9, y\}$

```
>> a /. Rule[1, 2, 3] -> t
Rule called with 3 arguments; 2 arguments are expected.
```

a

47.4. Pattern Defaults

47.4.1. Optional

WMA link

`Optional[`*pattern*, *default*`]`
`$pattern$ : $default$`
: is a pattern which matches *pattern*, which if omitted should be replaced by *default*.

```
>> f[x_, y_:1] := {x, y}
```

```
>> f[1, 2]
```
$\{1,2\}$

```
>> f[a]
```
$\{a,1\}$

Note that `$symb$ : $pattern$` represents a `Pattern` object. However, there is no disambiguity, since *symb* has to be a symbol in this case.

```
>> x:_ // FullForm
```
$\text{Pattern}\,[x, \text{Blank}\,[]]$

```
>> _:d // FullForm
```
$\text{Optional}\,[\text{Blank}\,[]\,, d]$

```
>> x:_+y_:d // FullForm
```
$\text{Pattern}\left[x, \text{Plus}\left[\text{Blank}\,[]\,, \text{Optional}\left[\text{Pattern}\left[y, \text{Blank}\,[]\right]\,, d\right]\right]\right]$

`s_.` is equivalent to `Optional[s_]` and represents an optional parameter which, if omitted, gets its value from `Default`.

```
>> FullForm[s_.]
```
$\text{Optional}\,[\text{Pattern}\,[s, \text{Blank}\,[]]]$

```
>> Default[h, k_] := k
```

```
>> h[a] /. h[x_, y_.] -> {x, y}
```
$\{a,2\}$

47.5. Restrictions on Patterns

47.5.1. Condition (/;)

WMA link

`Condition[`*pattern*, *expr*`]`
`$pattern$ /; $expr$`
: places an additional constraint on *pattern* that only allows it to match if *expr* evaluates to `True`.

The controlling expression of a `Condition` can use variables from the pattern:

```
>> f[3] /. f[x_] /; x>0 -> t
```
t

```
>> f[-3] /. f[x_] /; x>0 -> t
```
$f[-3]$

`Condition` can be used in an assignment:

```
>> f[x_] := p[x] /; x>0
```

```
>> f[3]
```
$p[3]$

```
>> f[-3]
```
$f[-3]$

47.5.2. PatternTest (?)

WMA link

`PatternTest[`*pattern*, *test*`]`
`$pattern$ ? $test$`
constrains *pattern* to match *expr* only if the evaluation of `$test$[$expr$]` yields `True`.

```
>> MatchQ[3, _Integer?(#>0&)]
```
True

```
>> MatchQ[-3, _Integer?(#>0&)]
```
False

```
>> MatchQ[3, Pattern[3]]
First element in pattern Pattern[3] is not a valid pattern name.
```
False

48. Scoping Constructs

Contents

48.1. $Context

WMA link

`$Context`
is the current context.

>> `$Context`
Global‘

48.2. $ContextPath

WMA link

`$ContextPath`
is the search path for contexts.

>> `$ContextPath // InputForm`
{“System‘”, “Global‘”}

48.3. $ModuleNumber

WMA link

`$ModuleNumber`
: is the current "serial number" to be used for local module variables.

 `$ModuleNumber` is incremented every time `Module` or `Unique` is called. a Mathics session starts with `$ModuleNumber` set to 1. You can reset `$ModuleNumber` to a positive machine integer, but if you do so, naming conflicts may lead to inefficiencies.

48.4. Begin

WMA link

`Begin[`*context*`]`
: temporarily sets the current context to *context*.

```
>> Begin["test`"]
```

test'

```
>> End[]
```

test'

```
>> End[]
No previous context defined.
```

Global'

48.5. BeginPackage

WMA link

`BeginPackage[`*context*`]`
: starts the package given by *context*.

The *context* argument must be a valid context name. `BeginPackage` changes the values of `$Context` and `$ContextPath`, setting the current context to *context*.

48.6. Block

WMA link

`Block[{`x, y, ...}, *expr*]
: temporarily removes the definitions of the given variables, evaluates *expr*, and restores the original definitions afterwards.

`Block[{`x=x_0, y=y_0, ...}, *expr*]
: assigns temporary values to the variables during the evaluation of *expr*.

```
>> n = 10
```
10

```
>> Block[{n = 5}, n ^ 2]
```
25

```
>> n
```
10

Values assigned to block variables are evaluated at the beginning of the block. Keep in mind that the result of `Block` is evaluated again, so a returned block variable will get its original value.

```
>> Block[{x = n+2, n}, {x, n}]
```
$\{12, 10\}$

If the variable specification is not of the described form, an error message is raised.

```
>> Block[{x + y}, x]
Local variable specification contains x + y, which is not a symbol or
an assignment to a symbol.
```
x

Variable names may not appear more than once:

```
>> Block[{x, x}, x]
Duplicate local variable x found in local variable specification.
```
x

48.7. Contexts

WMA link

`Contexts[]`
: returns a list of contexts.

``Contexts[``string']'``
: returns a list of contexts that match the string.

`Contexts` allows the string patterns with the following metacharacters:

- `*` zero or more characters
- `@` one or more characters, excluding uppercase letters

Get a list of all contexts:

```
>> Contexts[]
```
{HTML', HTML'Parser', ImportExport', Internal', Pymathics', Settings', System', System'Convert'Asy', System'Con

Get a list of HTML contexts only:

```
>> Contexts["HTML*"]
```
{HTML', HTML'Parser'}

48.8. End

WMA link

`End[]`
: ends a context started by `Begin`.

48.9. EndPackage

WMA link

`EndPackage[]`
: marks the end of a package, undoing a previous `BeginPackage`.

After `EndPackage`, the values of `$Context` and `$ContextPath` at the time of the `BeginPackage` call are restored, with the new packageś context prepended to `$ContextPath`.

48.10. Module

WMA link

`Module[{`*vars*`}, `*expr*`]`
: localizes variables by giving them a temporary name of the form `name$number`, where number is the current value of `$ModuleNumber`. Each time a module is evaluated, `$ModuleNumber` is incremented.

48.11. System'Private'$ContextPathStack

WMA link

System`Private`$ContextPathStack
: is an internal variable tracking the values of `$ContextPath` saved by `Begin` and `BeginPackage`.

48.12. System‘Private‘$ContextStack

WMA link

System`Private`$ContextStack
: is an internal variable tracking the values of `$Context` saved by `Begin` and `BeginPackage`.

48.13. Unique

WMA link

`Unique[]`
: generates a new symbol and gives a name of the form `$number`.

`Unique[x]`
: generates a new symbol and gives a name of the form `x$number`.

`Unique[{x, y, ...}]`
: generates a list of new symbols.

`Unique[``xxx']'`
: generates a new symbol and gives a name of the form `xxxnumber`.

Create a unique symbol with no particular name:

```
>> Unique[]
   $3
```

Create a unique symbol whose name begins with x:

```
>> Unique["x"]
   x4
```

48.14. With

WMA link

`With`[{x=x_0, y=y_0, ...}, $expr$]
: specifies that all occurrences of the symbols x, y, ... in $expr$ should be replaced by x_0, y_0, ...

49. Solving Recurrence Equations

Contents

49.1. RSolve

WMA link

`RSolve[$eqn$, $a$[n], n]`
: solves a recurrence equation for the function `$a$[$n$]`.

Solve a difference equation:

```
>> RSolve[a[n] == a[n+1], a[n], n]
```
$\{\{a[n]->C[0]\}\}$

No boundary conditions gives two general parameters:

```
>> RSolve[{a[n + 2] == a[n]}, a, n]
```
$\{\{a->\text{Function}[\{n\}, C[0]+C[1](-1)\hat{}n]\}\}$

Include one boundary condition:

```
>> RSolve[{a[n + 2] == a[n], a[0] == 1}, a, n]
```
$\{\{a->\text{Function}[\{n\}, C[0]+(1-C[0])(-1)\hat{}n]\}\}$

Get a “pure function” solution for a with two boundary conditions:

```
>> RSolve[{a[n + 2] == a[n], a[0] == 1, a[1] == 4}, a, n]
```
$$\left\{\left\{a->\text{Function}\left[\{n\}, \frac{5}{2}-\frac{3(-1)\hat{}n}{2}\right]\right\}\right\}$$

50. Sparse Array Functions

Contents

50.1. SparseArray

WMA link

`SparseArray[`*rules*`]`
: Builds a sparse array according to the list of *rules*.

`SparseArray[`*rules, dims*`]`
: Builds a sparse array of dimensions *dims* according to the *rules*.

`SparseArray[`*list*`]`
: Builds a sparse representation of *list*.

```
>> SparseArray[{{1, 2} -> 1, {2, 1} -> 1}]
```
SparseArray $\left[\text{Automatic}, \{2,2\}, 0, \{\{1,2\} -> 1, \{2,1\} -> 1\}\right]$

```
>> SparseArray[{{1, 2} -> 1, {2, 1} -> 1}, {3, 3}]
```
SparseArray $\left[\text{Automatic}, \{3,3\}, 0, \{\{1,2\} -> 1, \{2,1\} -> 1\}\right]$

```
>> M=SparseArray[{{0, a}, {b, 0}}]
```
SparseArray $\left[\text{Automatic}, \{2,2\}, 0, \{\{1,2\} -> a, \{2,1\} -> b\}\right]$

```
>> M //Normal
```
$\{\{0,a\},\{b,0\}\}$

51. Special Functions

There are a number of functions found in mathematical physics and found in standard handbooks.

One good source is The NIST Digital Library of Mathematical Functions.

The technical literature often contains several conflicting definitions. So beware, and check for conformance with the Mathics3 documentation.

A number of special functions can be evaluated for arbitrary complex values of their arguments. However defining relations may apply only for some special choices of arguments. Here, the full function corresponds to an extension or "analytic continuation" of the defining relation.

For example, integral representations of functions are only valid when the integral exists, but the functions can usually be defined by analytic continuation.

Contents

51.1. Bessel and Related Functions

See also Chapter 10 Bessel Functions in the Digital Library of Mathematical Functions.

51.1.1. AiryAi

Airy function of the first kind (SymPy, WMA)

`AiryAi[`x`]`
    returns the Airy function $Ai(x)$.

Exact values:

```
>> AiryAi[0]
```

$$\frac{3^{\frac{1}{3}}}{3\text{Gamma}\left[\frac{2}{3}\right]}$$

`AiryAi` can be evaluated numerically:

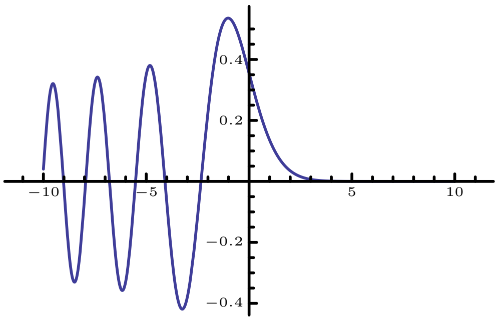
```
>> AiryAi[0.5]
```

0.231694

```
>> AiryAi[0.5 + I]
```

$0.157118 - 0.24104I$

```
>> Plot[AiryAi[x], {x, -10, 10}]
```



51.1.2. AiryAiPrime

Derivative of Airy function (Sympy, WMA link)

`AiryAiPrime[x]`
: returns the derivative of the Airy function `AiryAi[x]`.

Exact values:

```
>> AiryAiPrime[0]
```

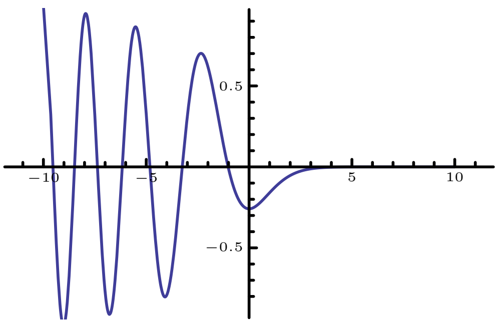
$$-\frac{3^{\frac{2}{3}}}{3\text{Gamma}\left[\frac{1}{3}\right]}$$

Numeric evaluation:

```
>> AiryAiPrime[0.5]
```

-0.224911

```
>> Plot[AiryAiPrime[x], {x, -10, 10}]
```



51.1.3. AiryAiZero

WMA link

`AiryAiZero[k]`
: returns the kth zero of the Airy function $Ai(z)$.

```
>> N[AiryAiZero[1]]
```
-2.33811

51.1.4. AiryBi

WMA link

`AiryBi[x]`
: returns the Airy function of the second kind $Bi(x)$.

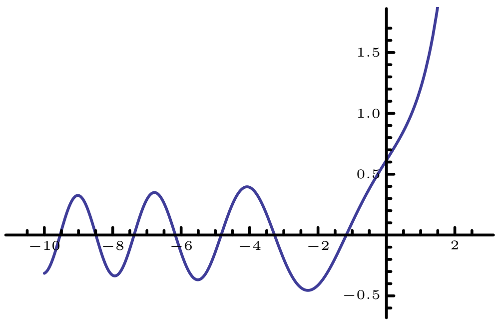
Exact values:

```
>> AiryBi[0]
```
$$\frac{3^{\frac{5}{6}}}{3\text{Gamma}\left[\frac{2}{3}\right]}$$

Numeric evaluation:

```
>> AiryBi[0.5]
```
0.854277

```
>> AiryBi[0.5 + I]
```
$0.688145 + 0.370815I$

```
>> Plot[AiryBi[x], {x, -10, 2}]
```


51.1.5. AiryBiPrime

WMA link

`AiryBiPrime[x]`
: returns the derivative of the Airy function of the second kind `AiryBi[x]`.

Exact values:

```
>> AiryBiPrime[0]
```

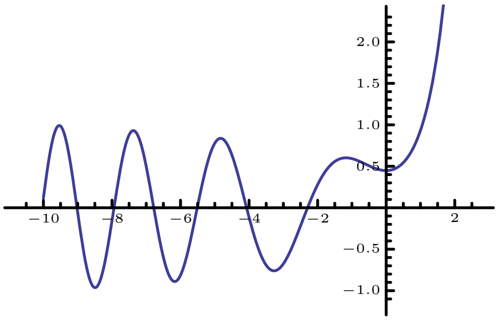
$$\frac{3^{\frac{1}{6}}}{\text{Gamma}\left[\frac{1}{3}\right]}$$

Numeric evaluation:

```
>> AiryBiPrime[0.5]
```

0.544573

```
>> Plot[AiryBiPrime[x], {x, -10, 2}]
```



51.1.6. AiryBiZero

WMA link

`AiryBiZero[`k`]`
: returns the kth zero of the Airy function $Bi(z)$.

```
>> N[AiryBiZero[1]]
```

-1.17371

51.1.7. AngerJ

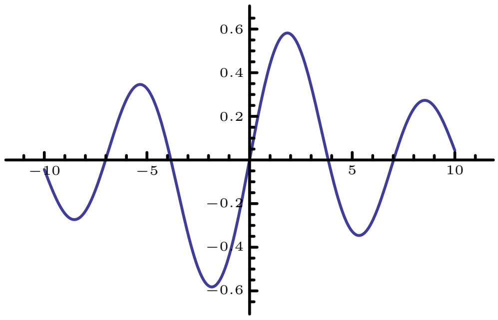
Anger function (mpmath, WMA)

`AngerJ[`n, z`]`
: returns the Anger function $J_n(z)$.

```
>> AngerJ[1.5, 3.5]
```

0.294479

```
>> Plot[AngerJ[1, x], {x, -10, 10}]
```



51.1.8. BesselI

Modified Bessel function of the first kind (Sympy, WMA)

> `BesselI`[n, z]
>
> returns the modified Bessel function of the first kind $I_n(z)$.

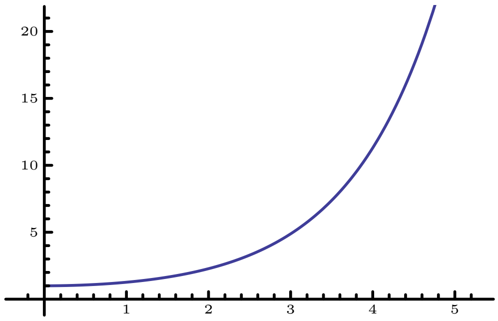
```
>> BesselI[0, 0]
```

1

```
>> BesselI[1.5, 4]
```

8.17263

```
>> Plot[BesselI[0, x], {x, 0, 5}]
```



The special case of half-integer index is expanded using Rayleigh's formulas:

```
>> BesselI[3/2, x]
```

$$\frac{\sqrt{2}\sqrt{x}\left(-\frac{\mathrm{Sinh}[x]}{x^2}+\frac{\mathrm{Cosh}[x]}{x}\right)}{\sqrt{\pi}}$$

51.1.9. BesselJ

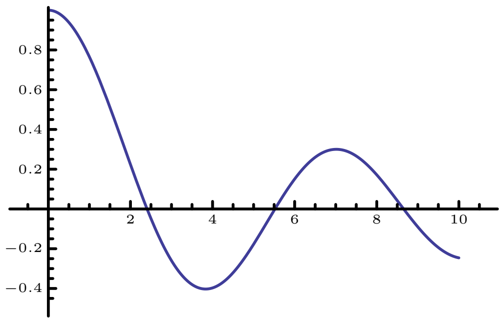
Bessel function of the first kind (SymPy, WMA)

BesselJ[n, z]
: returns the Bessel function of the first kind $J_n(z)$.

```
>> BesselJ[0, 5.2]
```
-0.11029

```
>> D[BesselJ[n, z], z]
```
$$-\frac{\text{BesselJ}[1+n,z]}{2}+\frac{\text{BesselJ}[-1+n,z]}{2}$$

```
>> BesselJ[0., 0.]
```
1.

```
>> Plot[BesselJ[0, x], {x, 0, 10}]
```


The special case of half-integer index is expanded using Rayleigh's formulas:

```
>> BesselJ[1/2, x]
```
$$\frac{\sqrt{2}\text{Sin}[x]}{\sqrt{x}\sqrt{\pi}}$$

Some integrals can be expressed in terms of Bessel functions:

```
>> Integrate[Cos[3 Sin[w]], {w, 0, Pi}]
```
$\pi\text{BesselJ}[0,3]$

51.1.10. BesselJZero

WMA link

BesselJZero[n, k]
: returns the kth zero of the Bessel function of the first kind $J_n(z)$.

```
>> N[BesselJZero[0, 1]]
```
2.40483

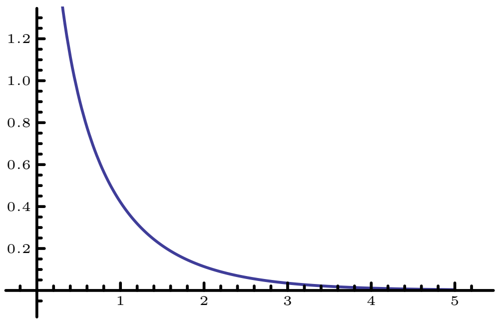
```
>> N[BesselJZero[0, 1], 10]
```
2.404825558

51.1.11. BesselK

Modified Bessel function of the second kind (SymPy, WMA)

`BesselK[`n, z`]`
: returns the modified Bessel function of the second kind $K_n(z)$.

```
>> BesselK[1.5, 4]
```
0.014347

```
>> Plot[BesselK[0, x], {x, 0, 5}]
```


The special case of half-integer index is expanded using Rayleigh's formulas:

```
>> BesselK[-3/2, x]
```
$$\frac{\sqrt{2}\sqrt{x}\sqrt{\pi}\left(\frac{E^{-x}}{x^2}+\frac{E^{-x}}{x}\right)}{2}$$

51.1.12. BesselY

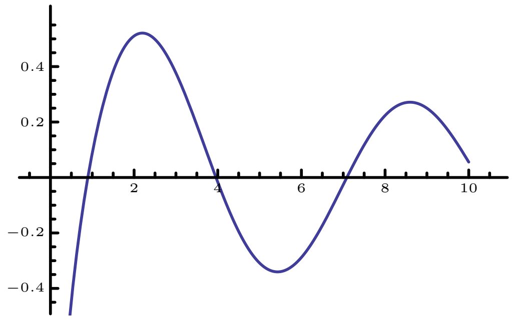
Bessel function of the second kind (SymPy, WMA)

`BesselY[`n, z`]`
: returns the Bessel function of the second kind $Y_n(z)$.

```
>> BesselY[1.5, 4]
```
0.367112

```
>> BesselY[0., 0.]
```
$-\infty$

```
>> Plot[BesselY[0, x], {x, 0, 10}]
```



The special case of half-integer index is expanded using Rayleigh's formulas:

```
>> BesselY[-3/2, x]
```

$$\frac{\sqrt{2}\sqrt{x}\left(-\frac{\mathrm{Sin}[x]}{x^2}+\frac{\mathrm{Cos}[x]}{x}\right)}{\sqrt{\pi}}$$

```
>> BesselY[0, 0]
```

$-\infty$

51.1.13. BesselYZero

WMA link

`BesselYZero`[n, k]
: returns the kth zero of the Bessel function of the second kind $Y_n(z)$.

```
>> N[BesselYZero[0, 1]]
```

0.893577

```
>> N[BesselYZero[0, 1], 10]
```

0.8935769663

51.1.14. HankelH1

WMA link

`HankelH1`[n, z]
: returns the Hankel function of the first kind $H_n^1(z)$.

```
>> HankelH1[1.5, 4]
```

$0.185286 + 0.367112I$

51.1.15. HankelH2

WMA link

`HankelH2`[n, z]
    returns the Hankel function of the second kind $H_n^2(z)$.

>> `HankelH2[1.5, 4]`
$0.185286 - 0.367112I$

51.1.16. KelvinBei

Kelvin function bei (mpmath, WMA)

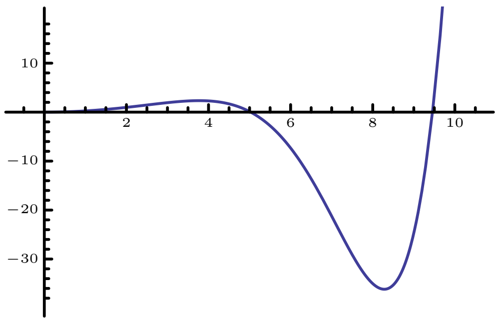
`KelvinBei`[z]
    returns the Kelvin function $bei(z)$.
`KelvinBei`[n, z]
    returns the Kelvin function $bei_n(z)$.

>> `KelvinBei[0.5]`
0.0624932

>> `KelvinBei[1.5 + I]`
$0.326323 + 0.755606I$

>> `KelvinBei[0.5, 0.25]`
0.370153

>> `Plot[KelvinBei[x], {x, 0, 10}]`



51.1.17. KelvinBer

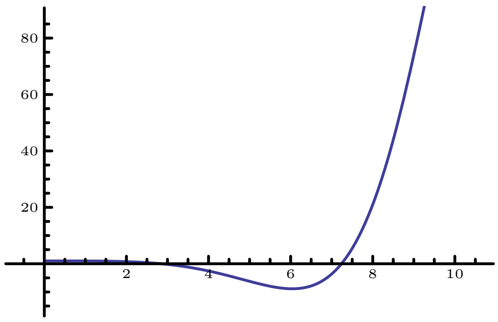
Kelvin function ber (mpmath, WMA)

`KelvinBer[`z`]`
    returns the Kelvin function $ber(z)$.
`KelvinBer[`n, z`]`
    returns the Kelvin function $ber_n(z)$.

```
>> KelvinBer[0.5]
```
0.999023

```
>> KelvinBer[1.5 + I]
```
$1.1162 - 0.117944I$

```
>> KelvinBer[0.5, 0.25]
```
0.148824

```
>> Plot[KelvinBer[x], {x, 0, 10}]
```


51.1.18. KelvinKei

Kelvin function kei (mpmath, WMA)

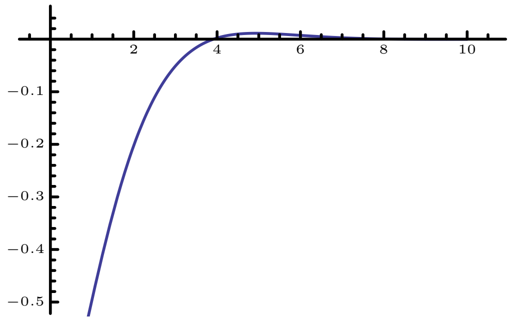
`KelvinKei[`z`]`
    returns the Kelvin function $kei(z)$.
`KelvinKei[`n, z`]`
    returns the Kelvin function $kei_n(z)$.

```
>> KelvinKei[0.5]
```
-0.671582

```
>> KelvinKei[1.5 + I]
```
$-0.248994 + 0.303326I$

```
>> KelvinKei[0.5, 0.25]
```
-2.0517

>> `Plot[KelvinKei[x], {x, 0, 10}]`



51.1.19. KelvinKer

Kelvin function ker (mpmath, WMA)

`KelvinKer[`z`]`
returns the Kelvin function $ker(z)$.

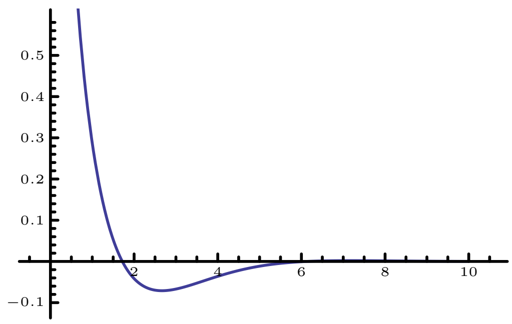
`KelvinKer[`n, z`]`
returns the Kelvin function $ker_n(z)$.

>> `KelvinKer[0.5]`
0.855906

>> `KelvinKer[1.5 + I]`
$-0.167162 - 0.184404I$

>> `KelvinKer[0.5, 0.25]`
0.450023

>> `Plot[KelvinKer[x], {x, 0, 10}]`



51.1.20. SphericalBesselJ

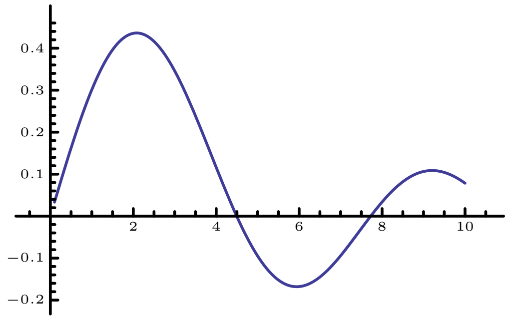
Spherical Bessel function of the first kind (Sympy, WMA)

SphericalBesselJ[n, z]
    returns the spherical Bessel function of the first kind $Y_n(z)$.

```
>> SphericalBesselJ[1, 5.2]
```

−0.122771

```
>> Plot[SphericalBesselJ[1, x], {x, 0.1, 10}]
```



51.1.21. SphericalBesselY

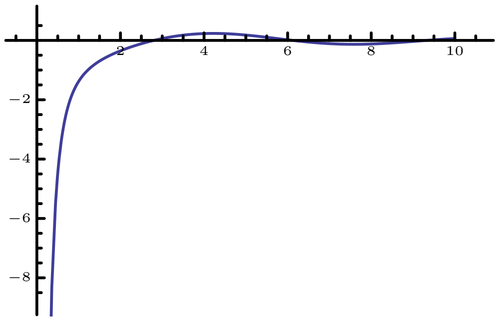
Spherical Bessel function of the first kind (Sympy, WMA)

SphericalBesselY[n, z]
    returns the spherical Bessel function of the second kind $Y_n(z)$.

```
>> SphericalBesselY[1, 5.5]
```

0.104853

```
>> Plot[SphericalBesselY[1, x], {x, 0, 10}]
```



51.1.22. SphericalHankelH1

Spherical Bessel function of the first kind (WMA link)

SphericalHankelH1[n, z]
: returns the spherical Hankel function of the first kind $h_n^{(1)}(z)$.

```
>> SphericalHankelH1[3, 1.5]
```
$0.0283246 - 3.78927I$

51.1.23. SphericalHankelH2

Spherical Bessel function of the second kind (WMA link)

SphericalHankelH1[n, z]
: returns the spherical Hankel function of the second kind $h_n^{(2)}(z)$.

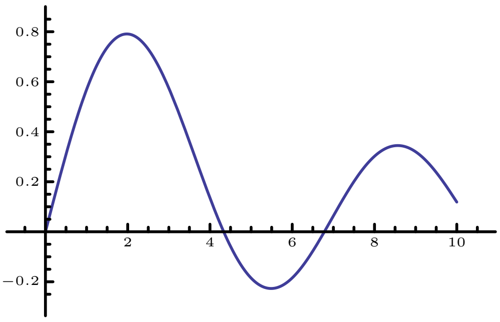
```
>> SphericalHankelH2[3, 1.5]
```
$0.0283246 + 3.78927I$

51.1.24. StruveH

Struve functions H (WMA)

StruveH[n, z]
: returns the Struve function $H_n(z)$.

```
>> StruveH[1.5, 3.5]
```
1.13192

```
>> Plot[StruveH[0, x], {x, 0, 10}]
```


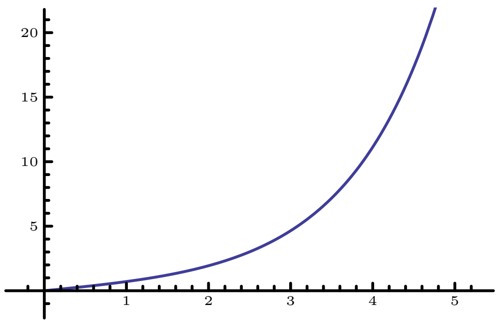
51.1.25. StruveL

Modified Struve functions L

StruveL[n, z]
    returns the modified Struve function $L_n(z)$.

>> `StruveL[1.5, 3.5]`
4.41126

>> `Plot[StruveL[0, x], {x, 0, 5}]`



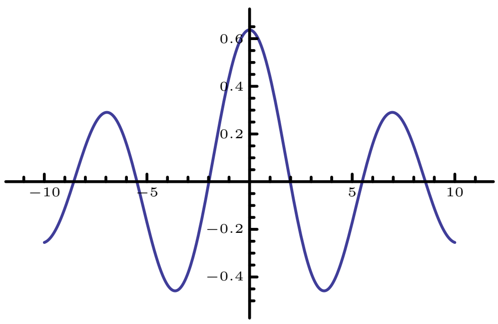
51.1.26. WeberE

WMA link

WeberE[n, z]
    returns the Weber function $E_n(z)$.

>> `WeberE[1.5, 3.5]`
-0.397256

>> `Plot[WeberE[1, x], {x, -10, 10}]`



51.2. Elliptic Integrals

In integral calculus, an elliptic integral is one of a number of related functions defined as the value of certain integral. Their name originates from their originally arising in connection with the problem of finding the arc length of an ellipse.

These functions often are used in cryptography to encode and decode messages.

See also Chapter 19 Elliptic Integrals in the Digital Library of Mathematical Functions.

51.2.1. EllipticE

Elliptic complete elliptic integral of the second kind (SymPy, WMA)

`EllipticE[`m`]`
: computes the complete elliptic integral $E(m)$.

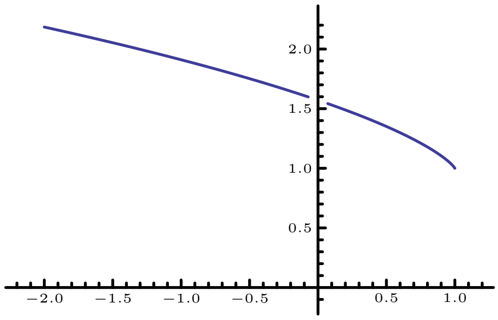
`EllipticE[`ϕ | m`]`
: computes the complete elliptic integral of the second kind $E(m|\phi)$.

Elliptic curves give Pi / 2 when evaluated at zero:

```
>> EllipticE[0]
```
$\frac{\pi}{2}$

```
>> EllipticE[0.3, 0.8]
```
0.296426

Plot over a reals centered around 0:

```
>> Plot[EllipticE[m], {m, -2, 2}]
```


51.2.2. EllipticF

Complete elliptic integral of the first kind (SymPy, WMA)

`EllipticF[`ϕ, m`]`
: computes the elliptic integral of the first kind $F(\phi|m)$.

```
>> EllipticF[0.3, 0.8]
```
0.303652

EllipticF is zero when the first argument is zero:

>> `EllipticF[0, 0.8]`

0

51.2.3. EllipticK

Complete elliptic integral of the first kind (SymPy, WMA)

`EllipticK[`m`]`
: computes the elliptic integral of the first kind $K(m)$.

>> `EllipticK[0.5]`

1.85407

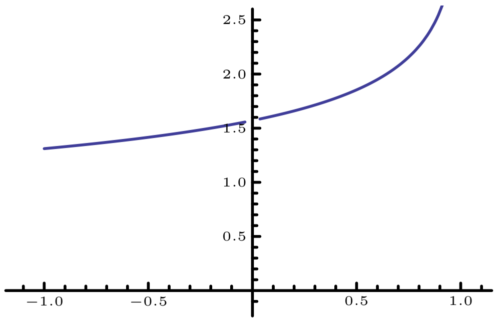
Elliptic curves give Pi / 2 when evaluated at zero:

>> `EllipticK[0]`

$\frac{\pi}{2}$

Plot over a reals around 0:

>> `Plot[EllipticK[n], {n, -1, 1}]`



51.2.4. EllipticPi

Complete elliptic integral of the third kind (SymPy, WMA)

`EllipticPi[`n, m`]`
: computes the elliptic integral of the third kind $Pi(m)$.

>> `EllipticPi[0.4, 0.6]`

2.89281

Elliptic curves give Pi / 2 when evaluated at zero:

>> EllipticPi[0, 0]

$\frac{\pi}{2}$

51.3. Error Function and Related Functions

See also Chapter 7 Error Functions, Dawson's and Fresnel Integrals.

51.3.1. Erf

Error function (SymPy, WMA)

Erf[z]
    returns the error function of z.

Erf[z_0, z_1]
    returns the result of Erf[z_1] - Erf[z_0].

Erf[x] is an odd function:

>> Erf[-x]

$-\text{Erf}[x]$

>> Erf[1.0]

0.842701

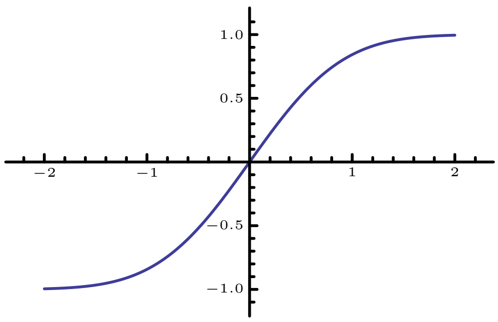
>> Erf[0]

0

>> {Erf[0, x], Erf[x, 0]}

{Erf[x], −Erf[x]}

>> Plot[Erf[x], {x, -2, 2}]



51.3.2. Erfc

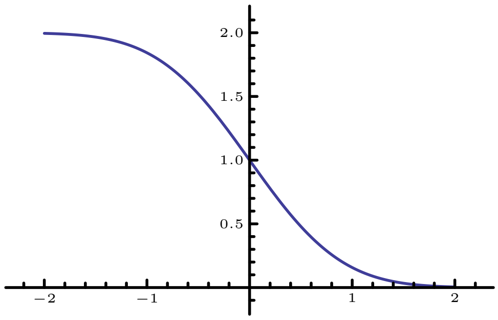
Complementary Error function (SymPy, WMA)

Erfc[z]
: returns the complementary error function of z.

```
>> Erfc[-x] / 2
```
$$\frac{2 - \text{Erfc}[x]}{2}$$

```
>> Erfc[1.0]
```
0.157299

```
>> Erfc[0]
```
1

```
>> Plot[Erfc[x], {x, -2, 2}]
```


51.3.3. FresnelC

Fresnel integral (mpmath, WMA)

FresnelC[z]
: is the Fresnel C integral $C(z)$.

```
>> FresnelC[{0, Infinity}]
```
$$\left\{0, \frac{1}{2}\right\}$$

```
>> Integrate[Cos[x^2 Pi/2], {x, 0, z}]
```
FresnelC [z]

51.3.4. FresnelS

Fresnel integral (mpmath, WMA)

FresnelS[z]
: is the Fresnel S integral $S(z)$.

```
>> FresnelS[{0, Infinity}]
```
$\left\{0, \frac{1}{2}\right\}$

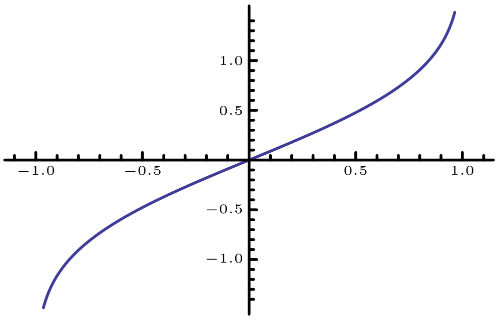
```
>> Integrate[Sin[x^2 Pi/2], {x, 0, z}]
```
FresnelS $[z]$

51.3.5. InverseErf

Inverse error function (SymPy, WMA)

`InverseErf`[z]
returns the inverse error function of z.

```
>> InverseErf /@ {-1, 0, 1}
```
$\{-\infty, 0, \infty\}$

```
>> Plot[InverseErf[x], {x, -1, 1}]
```


`InverseErf[$z$]` only returns numeric values for `-1 <= $z$ <= 1`:

```
>> InverseErf /@ {0.9, 1.0, 1.1}
```
$\{1.16309, \infty, \text{InverseErf}[1.1]\}$

51.3.6. InverseErfc

Complementary error function (SymPy, WMA)

`InverseErfc`[z]
returns the inverse complementary error function of z.

```
>> InverseErfc /@ {0, 1, 2}
```
$\{\infty, 0, -\infty\}$

51.4. Exponential Integral and Special Functions

See also Chapters 4.2-4.13 Logarithm, Exponential, Powers in the Digital Library of Mathematical Functions.

51.4.1. ExpIntegralE

WMA link

`ExpIntegralE`[n, z]
: returns the exponential integral function $E_n(z)$.

```
>> ExpIntegralE[2.0, 2.0]
```
0.0375343

51.4.2. ExpIntegralEi

WMA link

`ExpIntegralEi`[z]
: returns the exponential integral function $Ei(z)$.

```
>> ExpIntegralEi[2.0]
```
4.95423

51.4.3. LambertW

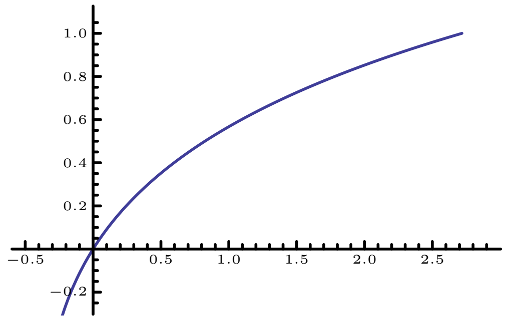
Lambert <i>W</i> Function, MathWorld

`LambertW`[k]
: alias for ProductLog[z].

`LambertW`[k, z]
: alias for ProductLog[k, z].

```
>> LambertW[k, z]
```
ProductLog [k, z]

```
>> Plot[LambertW[x], {x, -1/E, E}]
```



See also ProductLog 51.4.4.

51.4.4. ProductLog

WMA link

`ProductLog[`z`]`
: returns the principle solution for w in $z == wE^w$.

`ProductLog[`k, z`]`
: gives the k-th solution.

The defining equation:

```
>> z == ProductLog[z] * E ^ ProductLog[z]
   True
```

Some special values:

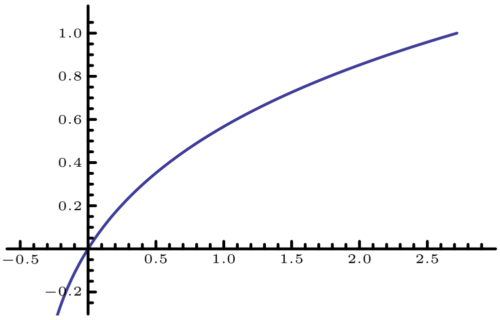
```
>> ProductLog[0]
   0
```

```
>> ProductLog[E]
   1
```

```
>> ProductLog[-1.5]
```
$-0.0327837 + 1.54964I$

The graph of `ProductLog`:

>> `Plot[ProductLog[x], {x, -1/E, E}]`



51.5. Gamma and Related Functions

See also Chapter 5 Gamma Function in the Digital Library of Mathematical Functions.

51.5.1. Beta

Euler beta function (SymPy, WMA)

`Beta`[a, b]
    is the Euler's Beta function $\mathrm{B}(a, b)$.
`Beta`[z, a, b]
    gives the incomplete Beta function $\mathrm{B}_z(a, b)$.

The Beta function satisfies the property: $\mathrm{B} = \int_0^t t^{(x-1)(1-t)^{(}y-1)} = \Gamma(a)\Gamma(b)/\Gamma(a+b)$

>> `Beta[2, 3]`

$\frac{1}{12}$

>> `12* Beta[1., 2, 3]`

1.

51.5.2. Factorial (`!`)

Factorial (SymPy, mpmath, WMA)

`Factorial`[n]
`$n$!`
    computes the factorial of n.

>> `20!`

2432902008176640000

`Factorial` handles numeric (real and complex) values using the gamma function:

```
>> 10.5!
```
1.18994*^7

```
>> (-3.0+1.5*I)!
```
$0.0427943 - 0.00461565I$

However, the value at poles is `ComplexInfinity`:

```
>> (-1.)!
```
ComplexInfinity

`Factorial` has the same operator (`!`) as `Not`, but with higher precedence:

```
>> !a! //FullForm
```
Not [Factorial [a]]

51.5.3. Factorial2 (`!!`)

WMA link

> `Factorial2[`n`]`
> `$n$!!`
> computes the double factorial of n.

The double factorial or semifactorial of a number n, is the product of all the integers from 1 up to n that have the same parity (odd or even) as n.

```
>> 5!!
```
15.

```
>> Factorial2[-3]
```
$-1.$

`Factorial2` accepts Integers, Rationals, Reals, or Complex Numbers:

```
>> I!! + 1
```
$3.71713 + 0.279527I$

Irrationals can be handled by using numeric approximation:

```
>> N[Pi!!, 6]
```
3.35237

51.5.4. Gamma

Gamma function (SymPy, mpmath, WMA)

The gamma function is one commonly used extension of the factorial function applied to complex numbers, and is defined for all complex numbers except the non-positive integers.

`Gamma[`z`]`
  is the Euler gamma function, $\Gamma(z)$ on the complex number z.
`Gamma[`a, z`]`
  is the upper incomplete gamma function, $\Gamma(a, z)$.
`Gamma[`a, z_0, z_1`]`
  is the generalized incomplete gamma function $\Gamma[a, z_0] - \Gamma(a, z_1)$.

`Gamma[$z$]` is equivalent to $(z - 1)!$:

```
>> Simplify[Gamma[z] - (z - 1)!]
```
0

Examples of using `Gamma` with exact numeric arguments:

```
>> Gamma[8]
```
5040

```
>> Gamma[1/2]
```
$\sqrt{\pi}$

```
>> Gamma[123.78]
```
4.21078*^204

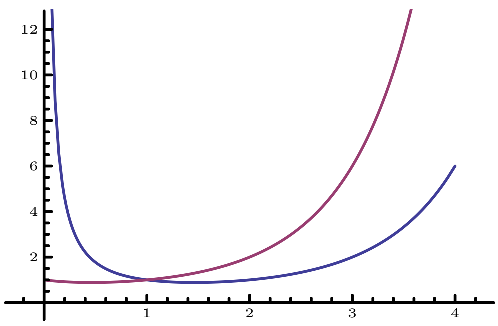
```
>> Gamma[1. + I]
```
$0.498016 - 0.15495I$

Examples of `Gamma` with symbolic arguments:

```
>> Gamma[1, x]
```
E^{-x}

```
>> Gamma[0, x]
```
ExpIntegralE $[1, x]$

Both `Gamma` and `Factorial` functions are continuous:

```
>> Plot[{Gamma[x], x!}, {x, 0, 4}]
```


51.5.5. LogGamma

log-gamma function (SymPy, WMA)

`LogGamma[`z`]`
    is the logarithm of the gamma function on the complex number z.

```
>> LogGamma[3]
```
Log [2]

LogGamma[z] has different analytical structure than Log[Gamma[z]]

```
>> LogGamma[-2.+3 I]
```
$-6.77652-4.56879I$

```
>> Log[Gamma[-2.+3 I]]
```
$-6.77652+1.71439I$

LogGamma also can be evaluated for large arguments, for which Gamma produces Overflow:

```
>> LogGamma[1.*^20]
```
4.50517*^21

```
>> Log[Gamma[1.*^20]]
Overflow occurred in computation.
```
Overflow []

51.5.6. Pochhammer

Rising factorial (SymPy, WMA)

The Pochhammer symbol or rising factorial often appears in series expansions for hypergeometric functions.

The Pochammer symbol has a definite value even when the gamma functions which appear in its definition are infinite.

`Pochhammer[`a, n`]`
    is the Pochhammer symbol a_n.

Product of the first 3 numbers:

```
>> Pochhammer[1, 3]
```
6

`Pochhammer[1, $n$]` is the same as Pochhammer[2, n-1] since 1 is a multiplicative identity.

>> `Pochhammer[1, 3] == Pochhammer[2, 2]`
True

`Pochhammer[0,` $-n$`]` for positive integer n, is $-1^n 1/|n|!$:

>> `Table[Pochhammer[0, n], {n, 0, -4, -1}]`
$\left\{1, -1, \frac{1}{2}, -\frac{1}{6}, \frac{1}{24}\right\}$

Pochhammer uses Gamma for non-Integer values of n:

>> `Pochhammer[1, 3.001]`
6.00754

>> `Pochhammer[1, 3.001] == Pochhammer[2, 2.001]`
True

>> `Pochhammer[1.001, 3] == 1.001 2.001 3.001`
True

51.5.7. PolyGamma

Polygamma function (SymPy, WMA)

PolyGamma is a meromorphic function on the complex numbers and is defined as a derivative of the logarithm of the gamma function.

PolyGamma[z]
 returns the digamma function.
PolyGamma[n,z]
 gives the n^(th) derivative of the digamma function.

>> `PolyGamma[5]`
$\frac{25}{12} - \text{EulerGamma}$

>> `PolyGamma[3, 5]`
$-\frac{22369}{3456} + \frac{\pi^4}{15}$

51.5.8. StieltjesGamma

Stieltjes constants (SymPy, WMA)

`StieltjesGamma[`n`]`
: returns the Stieltjes constant for n.

`StieltjesGamma[`n, a`]`
: gives the generalized Stieltjes constant of its parameters

51.5.9. Subfactorial

Derangement (SymPy, WMA)

`Subfactorial[`n`]`
: computes the subfactorial of n.

Here are the first few derangements:

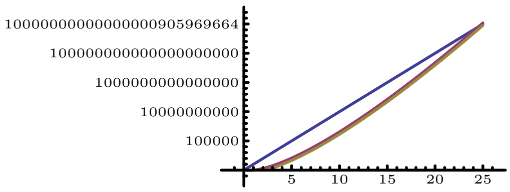
```
>> Subfactorial[{0, 1, 2, 3}]
```
$\{1, 0, 1, 2\}$

We can handle `MachineReal` numbers:

```
>> Subfactorial[6.0]
```
265

Here is how the exponential, `Factorial`, and `Subfactorial` grow in comparison:

```
>> LogPlot[{10^x, Factorial[x], Subfactorial[x]}, {x, 0, 25}, PlotPoints
   ->26]
```



51.6. Hypergeometric functions

See also Chapter 15 Hypergeometric Functions in the Digital Library of Mathematical Functions.

51.6.1. Hypergeometric1F1

Kummer confluent hypergeometric function (mpmath, WMA)

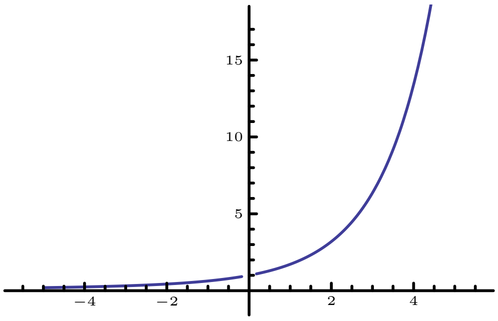
`Hypergeometric1F1[`a, b, z`]`
: returns ${}_1F_1(a; b; z)$.

Numeric evaluation:

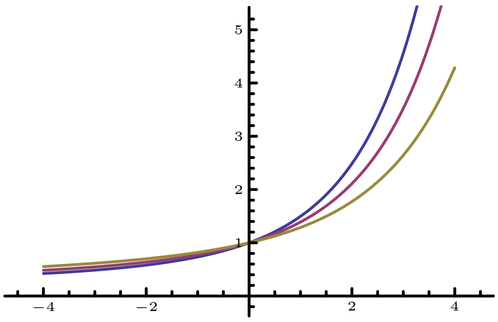
```
>> Hypergeometric1F1[1, 2, 3.0]
```
6.36185

Plot over a subset of reals:

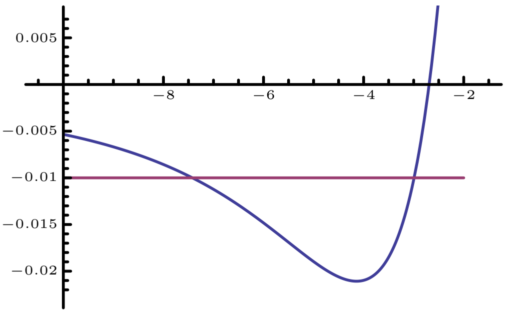
```
>> Plot[Hypergeometric1F1[1, 2, x], {x, -5, 5}]
```



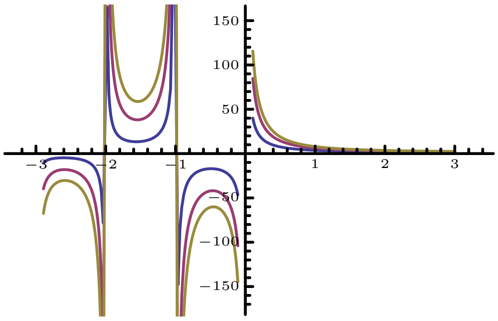

```
>> Plot[{Hypergeometric1F1[1/2, Sqrt[2], x], Hypergeometric1F1[1/2, Sqrt
   [3], x], Hypergeometric1F1[1/2, Sqrt[5], x]}, {x, -4, 4}]
```




```
>> Plot[{Hypergeometric1F1[Sqrt[3], Sqrt[2], z], -0.01}, {z, -10, -2}]
```



```
>> Plot[{Hypergeometric1F1[Sqrt[2], b, 1], Hypergeometric1F1[Sqrt[5], b,
   1], Hypergeometric1F1[Sqrt[7], b, 1]}, {b, -3, 3}]
```



Compute the elementwise values of an array:

```
>> Hypergeometric1F1[1, 1, {{1, 0}, {0, 1}}]
```

$\{\{E,1\},\{1,E\}\}$

```
>> Hypergeometric1F1[1/2, 1, x]
```

$\text{BesselI}\left[0,\frac{x}{2}\right]E^{\frac{x}{2}}$

Evaluate using complex arguments:

```
>> Hypergeometric1F1[2 + I, 2, 0.5]
```

$1.61833 + 0.379258I$

Large numbers are supported:

```
>> Hypergeometric1F1[3, 4, 10^10]
```

$-\frac{3}{500000000000000000000000000000}+\frac{1499999999700000000003E^{10000000000}}{500000000000000000000000000000}$

`Hypergeometric1F1` evaluates to simpler functions for certain parameters:

```
>> Hypergeometric1F1[1/2, 1, x]
```

$\text{BesselI}\left[0,\frac{x}{2}\right]E^{\frac{x}{2}}$

```
>> Hypergeometric1F1[2, 1, x]
```

$(1+x)E^{x}$

```
>> Hypergeometric1F1[1, 1/2, x]
```

$-\sqrt{x}\left(-\frac{E^{-x}}{\sqrt{x}}-\sqrt{\pi}\text{Erf}\left[\sqrt{x}\right]\right)E^{x}$

51.6.2. Hypergeometric2F1

Hypergeometric function (mpmath, WMA)

```
Hypergeometric2F1[a, b, c, z]
```
returns ${}_2F_1(a; b; c; z)$.

```
>> Hypergeometric2F1[2., 3., 4., 5.0]
```
$0.156542 + 0.150796I$

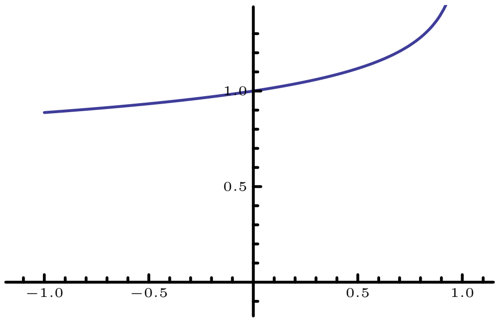
Evaluate symbolically:

```
>> Hypergeometric2F1[2, 3, 4, x]
```
$$\frac{6\mathrm{Log}\left[1 - x\right]}{x^3} + \frac{-6 + 3x}{-x^2 + x^3}$$

Evaluate using complex arguments:

```
>> Hypergeometric2F1[2 + I, -I, 3/4, 0.5 - 0.5 I]
```
$-0.972167 - 0.181659I$

Plot over a subset of the reals:

```
>> Plot[Hypergeometric2F1[1/3, 1/3, 2/3, x], {x, -1, 1}]
```


51.6.3. HypergeometricPFQ

Generalized hypergeometric function (mpmath, SymPy, WMA)

```
HypergeometricPFQ[a_1, ..., a_p, b_1, ..., b_q, z]
```
returns ${}_pF_q(a_1, ..., a_p; b_1, ..., b_q; z)$.

```
>> HypergeometricPFQ[{2}, {2}, 1]
```
E

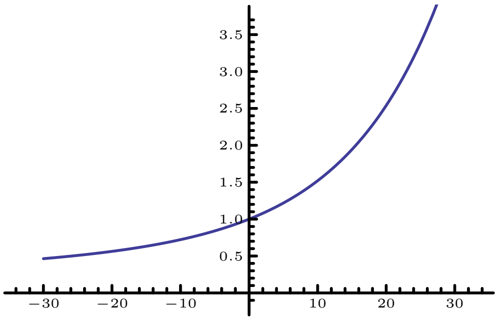
Result is symbollicaly simplified by default:

```
>> HypergeometricPFQ[{3}, {2}, 1]
```
$$\frac{3E}{2}$$

unless a numerical evaluation is explicitly requested:

```
>> HypergeometricPFQ[{3}, {2}, 1] // N
```
4.07742

```
>> HypergeometricPFQ[{3}, {2}, 1.]
```
4.07742

```
>> Plot[HypergeometricPFQ[{1, 1}, {3, 3, 3}, x], {x, -30, 30}]
```



```
>> HypergeometricPFQ[{1, 1, 2}, {3, 3}, z]
```
$$\frac{-4\text{PolyLog}[2, z]}{z^2} + \frac{4\text{Log}[1 - z]}{z^2} - \frac{4\text{Log}[1 - z]}{z} + \frac{8}{z}$$

The following special cases are handled:

```
>> HypergeometricPFQ[{}, {}, z]
```
E^z

```
>> HypergeometricPFQ[{0}, {b}, z]
```
1

```
>> HypergeometricPFQ[{1, 1, 3}, {2, 2}, x]
```
$$-\frac{\text{Log}[1 - x]}{2x} - \frac{1}{-2 + 2x}$$

`HypergeometricPFQ` evaluates to a polynomial if any of the parameters a_k is a non-positive integer:

```
>> HypergeometricPFQ[{-2, a}, {b}, x]
```
$$\frac{-2ax(1 + b) + ax^2(1 + a) + b(1 + b)}{b(1 + b)}$$

Value at origin:

```
>> HypergeometricPFQ[{a1, b2, a3}, {b1, b2, b3}, 0]
```
1

51.6.4. HypergeometricU

Confluent hypergeometric function (mpmath, WMA)

HypergeometricU[a, b, z]
returns $U(a, b, z)$.

Result is symbollicaly simplified, where possible:

```
>> HypergeometricU[3, 2, 1]
```

$$\frac{\text{MeijerG}\left[\{\{-2\},\{\}\},\{\{-1,0\},\{\}\},1\right]}{2}$$

```
>> HypergeometricU[1,4,8]
```

HypergeometricU [1, 4, 8]

unless a numerical evaluation is explicitly requested:

```
>> HypergeometricU[3, 2, 1] // N
```

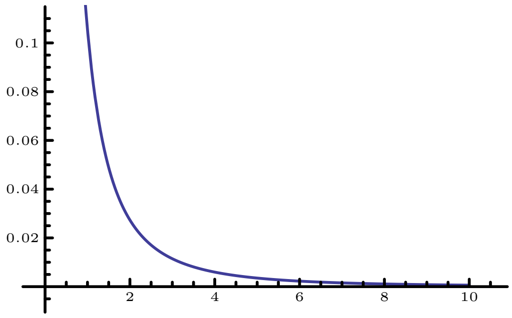
0.105479

```
>> HypergeometricU[3, 2, 1.]
```

0.105479

Plot U[3, 2, x] from 0 to 10 in steps of 0.5:

```
>> Plot[HypergeometricU[3, 2, x], {x, 0.5, 10}]
```



We handle this special case:

```
>> HypergeometricU[0, b, z]
```

1

51.6.5. MeijerG

Meijer G-function (mpmath, SymPy, WMA)

MeijerG[$a_1, ..., a_n, a_{n+1}, ..., a_p, b_1, ..., b_m, b_{m+1}, ..., a_q, z$]
returns $G^{m,n}_{p,q}(z|a_1, ..., a_p; b_1, ..., b_q)$.

Result is symbollicaly simplified by default:

```
>> MeijerG[{{1, 2}, {}}, {{3}, {}}, 1]
```
MeijerG $\left[\{\{1,2\},\{\}\},\{\{3\},\{\}\},1\right]$

unless a numerical evaluation is explicitly requested:

```
>> MeijerG[{{1, 2},{}}, {{3},{}}, 1] // N
```
0.210958

```
>> MeijerG[{{1, 2},{}}, {{3},{}}, 1.]
```
0.210958

51.7. Orthogonal Polynomials

See also Chapters 18.3 Classical Orthogonal Polynomials in the Digital Library of Mathematical Functions.

51.7.1. ChebyshevT

Chebyshev polynomial of the first kind (Sympy, WMA)

`ChebyshevT[`n, x`]`
: returns the Chebyshev polynomial of the first kind $T_n(x)$.

```
>> ChebyshevT[8, x]
```
$1 - 32x^2 + 160x^4 - 256x^6 + 128x^8$

```
>> ChebyshevT[1 - I, 0.5]
```
$0.800143 + 1.08198I$

51.7.2. ChebyshevU

Chebyshev polynomial of the second kind (Sympy, WMA)

`ChebyshevU[`n, x`]`
: returns the Chebyshev polynomial of the second kind $U_n(x)$.

```
>> ChebyshevU[8, x]
```
$1 - 40x^2 + 240x^4 - 448x^6 + 256x^8$

```
>> ChebyshevU[1 - I, 0.5]
```
$1.60029 + 0.721322I$

51.7.3. GegenbauerC

Gegenbauer polynomials (SymPy, WMA)

`GegenbauerC`[n, m, x]
    returns the Gegenbauer polynomial $C_n^{(m)}(x)$.

>> `GegenbauerC[6, 1, x]`
$-1 + 24x^2 - 80x^4 + 64x^6$

>> `GegenbauerC[4 - I, 1 + 2 I, 0.7]`
$-3.2621 - 24.9739I$

51.7.4. HermiteH

Hermite polynomial (SymPy, WMA)

`HermiteH`[n, x]
    returns the Hermite polynomial $H_n(x)$.

>> `HermiteH[8, x]`
$1680 - 13\,440x^2 + 13440x^4 - 3\,584x^6 + 256x^8$

>> `HermiteH[3, 1 + I]`
$-28 + 4I$

>> `HermiteH[4.2, 2]`
77.5291

51.7.5. JacobiP

Jacobi polynomials (SymPy, WMA)

`JacobiP`[n, a, b, x]
    returns the Jacobi polynomial $P_n^{(a,b)}(x)$.

>> `JacobiP[1, a, b, z]`
$$\frac{a}{2} - \frac{b}{2} + z\left(1 + \frac{a}{2} + \frac{b}{2}\right)$$

>> `JacobiP[3.5 + I, 3, 2, 4 - I]`
$1410.02 + 5797.3I$

51.7.6. LaguerreL

Laguerre polynomials (SymPy, WMA)

> `LaguerreL`[n, x]
>   returns the Laguerre polynomial $L_n(x)$.
> `LaguerreL`[n, a, x]
>   returns the generalised Laguerre polynomial of order n and index a, $L_n^a(x)$.

```
>> LaguerreL[8, x]
```

$$1 - 8x + 14x^2 - \frac{28x^3}{3} + \frac{35x^4}{12} - \frac{7x^5}{15} + \frac{7x^6}{180} - \frac{x^7}{630} + \frac{x^8}{40320}$$

```
>> LaguerreL[3/2, 1.7]
```

-0.947134

```
>> LaguerreL[5, 2, x]
```

$$21 - 35x + \frac{35x^2}{2} - \frac{7x^3}{2} + \frac{7x^4}{24} - \frac{x^5}{120}$$

51.7.7. LegendreP

Lengendre polynomials (SymPy, WMA)

> `LegendreP`[n, x]
>   returns the Legendre polynomial $P_n(x)$.
> `LegendreP`[n, m, x]
>   returns the associated Legendre polynomial $P_n^m(x)$.

```
>> LegendreP[4, x]
```

$$\frac{3}{8} - \frac{15x^2}{4} + \frac{35x^4}{8}$$

```
>> LegendreP[5/2, 1.5]
```

4.17762

```
>> LegendreP[1.75, 1.4, 0.53]
```

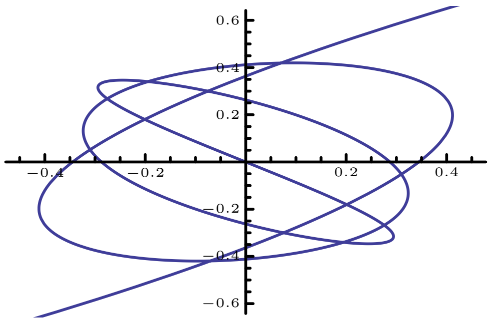
-1.32619

```
>> LegendreP[1.6, 3.1, 1.5]
```

$-0.303998 - 1.91937I$

`LegendreP` can be used to draw generalized Lissajous figures:

```
>> ParametricPlot[ {LegendreP[7, x], LegendreP[5, x]}, {x, -1, 1}]
```



51.7.8. LegendreQ

Legendre functions of the second kind (mpmath, WMA)

`LegendreQ`[n, x]
returns the Legendre function of the second kind $Q_n(x)$.

`LegendreQ`[n, m, x]
returns the associated Legendre function of the second $Q_n^m(x)$.

```
>> LegendreQ[5/2, 1.5]
```
0.036211 − 6.56219I

```
>> LegendreQ[1.75, 1.4, 0.53]
```
2.05499

```
>> LegendreQ[1.6, 3.1, 1.5]
```
− 1.71931 − 7.70273I

51.7.9. SphericalHarmonicY

Spherical Harmonic https (mpmath, WMA)

`SphericalHarmonicY`[l, m, θ, ϕ]
returns the spherical harmonic function $Y_l^m(\theta, \phi)$.

```
>> SphericalHarmonicY[3/4, 0.5, Pi/5, Pi/3]
```
0.254247 + 0.14679I

```
>> SphericalHarmonicY[3, 1, theta, phi]
```
$$\frac{\sqrt{21}\left(1 - 5\text{Cos}\,[\text{theta}]^2\right) E^{I\text{phi}}\text{Sin}\,[\text{theta}]}{8\sqrt{\pi}}$$

51.8. Zeta Functions and Polylogarithms

See also Chapters 25 Zeta and Related Functions in the Digital Library of Mathematical Functions.

51.8.1. LerchPhi

Lerch transcendent (WMA)

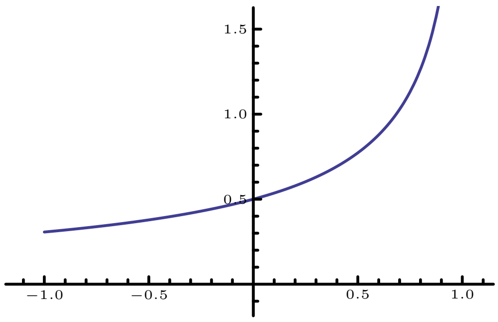
`LerchPhi[z,s,a]`
gives the Lerch transcendent $\Phi(z, s, a)$.

```
>> LerchPhi[2, 3, -1.5]
```
$19.3893 - 2.1346I$

```
>> LerchPhi[1, 2, 1/4] == 8 Catalan + Pi^2
```
True

Plot between between -1 and 1:

```
>> Plot[LerchPhi[x, 1, 2], {x, -1, 1}]
```



51.8.2. PolyLog

Polylogarithm (WMA)

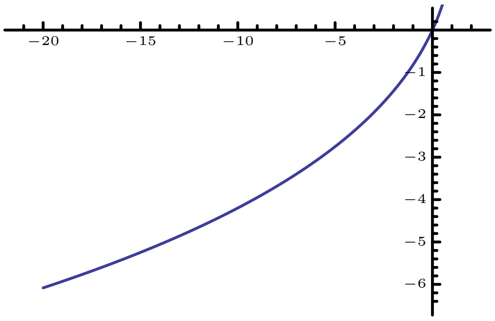
`PolyLog`$[n, z]$
returns the polylogarithm function $Li_n(z)$.

```
>> PolyLog[s, 1]
```
Zeta $[s]$

```
>> PolyLog[-7, I] //Chop
```
136.

Dilogarithm function $Li_2(x)$:

>> `Plot[PolyLog[2,x], {x, -20, 1}]`



51.8.3. Zeta

Riemenn zeta function (WMA)

`Zeta[`*z*`]`
: returns the Riemann zeta function of *z*.

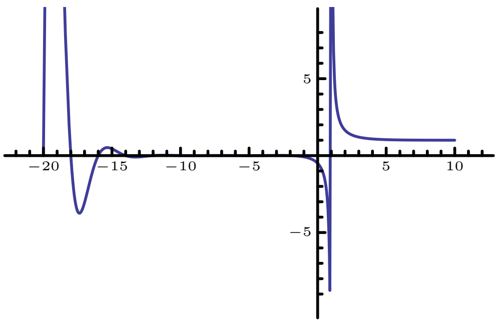
>> `Zeta[2]`

$\frac{\pi^2}{6}$

>> `Zeta[-2.5 + I]`

$0.0235936 + 0.0014078I$

>> `Plot[Zeta[z], {z, -20, 10}]`



52. Strings and Characters

Contents

52.1. Character Codes

52.1.1. FromCharacterCode

WMA link

`FromCharacterCode[`n`]`
returns the character corresponding to Unicode codepoint n.
`FromCharacterCode[`$\{n_1, n_2, ...\}$`]`
returns a string with characters corresponding to n_i.
`FromCharacterCode[`$\{\{n_{11}, n_{12}, ...\}, \{n_{21}, n_{22}, ...\}, ...\}$`]`
returns a list of strings.

```
>> FromCharacterCode[100]
   d
```

```
>> FromCharacterCode[228, "ISO8859-1"]
   ä
```

```
>> FromCharacterCode[{100, 101, 102}]
```
def

```
>> ToCharacterCode[%]
```
$\{100, 101, 102\}$

```
>> FromCharacterCode[{{97, 98, 99}, {100, 101, 102}}]
```
{abc, def}

```
>> ToCharacterCode["abc 123"] // FromCharacterCode
```
abc 123

52.1.2. ToCharacterCode

WMA link

`ToCharacterCode`[“*string*”]
  converts the string to a list of character codes (Unicode codepoints).
`ToCharacterCode`[{“$string_1$”, “$string_2$”, ...}]
  converts a list of strings to character codes.

```
>> ToCharacterCode["abc"]
```
$\{97, 98, 99\}$

```
>> FromCharacterCode[%]
```
abc

```
>> ToCharacterCode["\[Alpha]\[Beta]\[Gamma]"]
```
$\{945, 946, 947\}$

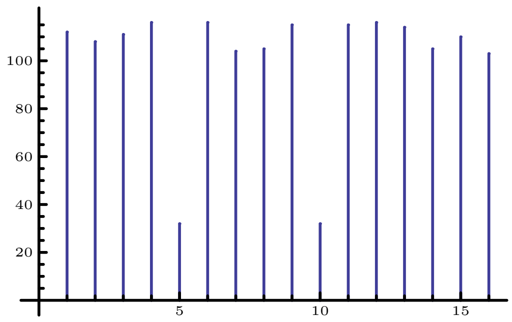
```
>> ToCharacterCode["ä", "UTF8"]
```
$\{195, 164\}$

```
>> ToCharacterCode["ä", "ISO8859-1"]
```
$\{228\}$

```
>> ToCharacterCode[{"ab", "c"}]
```
$\{\{97, 98\}, \{99\}\}$

```
>> ToCharacterCode[{"ab", x}]
String or list of strings expected at position 1 in
ToCharacterCode[{ab, x}].
```
ToCharacterCode$[\{\text{ab}, x\}]$

```
>> ListPlot[ToCharacterCode["plot this string"], Filling -> Axis]
```



52.2. Characters in Strings

52.2.1. CharacterRange

WMA link

`CharacterRange`[“*a*”, “*b*”]
: returns a list of the Unicode characters from *a* to *b* inclusive.

```
>> CharacterRange["a", "e"]
```
{a, b, c, d, e}

```
>> CharacterRange["b", "a"]
```
{}

52.2.2. Characters

WMA link

`Characters`[“*string*”]
: returns a list of the characters in *string*.

```
>> Characters["abc"]
```
{a, b, c}

52.2.3. LowerCaseQ

WMA link

`LowerCaseQ`[*s*]
: returns True if *s* consists wholly of lower case characters.

```
>> LowerCaseQ["abc"]
```
True

An empty string returns True.

```
>> LowerCaseQ[""]
```
True

52.2.4. ToLowerCase

WMA link

`ToLowerCase`[*s*]
: returns *s* in all lower case.

```
>> ToLowerCase["New York"]
```
new york

52.2.5. ToUpperCase

WMA link

`ToUpperCase`[*s*]
: returns *s* in all upper case.

```
>> ToUpperCase["New York"]
```
NEW YORK

52.2.6. UpperCaseQ

WMA link

`UpperCaseQ`[*s*]
: returns True if *s* consists wholly of upper case characters.

```
>> UpperCaseQ["ABC"]
```
True

An empty string returns True.

```
>> UpperCaseQ[""]
```
True

52.3. Operations on Strings

52.3.1. StringDrop

WMA link

`StringDrop`[“*string*”, *n*]
: gives *string* with the first *n* characters dropped.

`StringDrop`[“*string*”, -*n*]
: gives *string* with the last *n* characters dropped.

`StringDrop`[“*string*”, {*n*}]
: gives *string* with the *n*-th character dropped.

`StringDrop`[“*string*”, {*m*, *n*}]
: gives *string* with the characters *m* through *n* dropped.

```
>> StringDrop["abcde", 2]
```
cde

```
>> StringDrop["abcde", -2]
```
abc

```
>> StringDrop["abcde", {2}]
```
acde

```
>> StringDrop["abcde", {2,3}]
```
ade

```
>> StringDrop["abcd",{3,2}]
```
abcd

```
>> StringDrop["abcd",0]
```
abcd

52.3.2. StringInsert

WMA link

`StringInsert["string", "snew", n]`
: yields a string with *snew* inserted starting at position n in *string*.

`StringInsert["string", "snew", -n]`
: inserts a at position n from the end of "*string*".

`StringInsert["string", "snew", {n1, n2, ...}]`
: inserts a copy of *snew* at each position n_i in *string*; the n_i are taken before any insertion is done.

`StringInsert[{s1, s2, ...}, "snew", n]`
: gives the list of results for each of the s_i.

```
>> StringInsert["noting", "h", 4]
```
nothing

```
>> StringInsert["note", "d", -1]
```
noted

```
>> StringInsert["here", "t", -5]
```
there

```
>> StringInsert["adac", "he", {1, 5}]
```
headache

```
>> StringInsert[{"something", "sometimes"}, " ", 5]
```
{some thing, some times}

Insert dot as millar separators

```
>> StringInsert["1234567890123456", ".", Range[-16, -4, 3]]
```
1.234.567.890.123.456

52.3.3. StringJoin (<>)

WMA link

`StringJoin["s1", "s2", ...]`
: returns the concatenation of the strings s_1, s_2, .

```
>> StringJoin["a", "b", "c"]
```
abc

```
>> "a" <> "b" <> "c" // InputForm
```
"abc"

`StringJoin` flattens lists out:

```
>> StringJoin[{"a", "b"}] // InputForm
```
"ab"

```
>> Print[StringJoin[{"Hello", " ", {"world"}}, "!"]]
Hello world!
```

52.3.4. StringLength

WMA link

`StringLength["string"]`
: gives the length of *string*.

```
>> StringLength["abc"]
3
```

`StringLength` is listable:

```
>> StringLength[{"a", "bc"}]
{1,2}
```

```
>> StringLength[x]
String expected.
StringLength[x]
```

52.3.5. StringPosition

WMA link

`StringPosition["string", patt]`
: gives a list of starting and ending positions where *patt* matches "*string*".

`StringPosition["string", patt, n]`
: returns the first n matches only.

`StringPosition["string", {patt_1, patt_2, ...}, n]`
: matches multiple patterns.

`StringPosition[{s_1, s_2, ...}, patt]`
: returns a list of matches for multiple strings.

```
>> StringPosition["123ABCxyABCzzzABCABC", "ABC"]
{{4,6},{9,11},{15,17},{18,20}}
```

```
>> StringPosition["123ABCxyABCzzzABCABC", "ABC", 2]
{{4,6},{9,11}}
```

`StringPosition` can be useful for searching through text.

```
>> data = Import["ExampleData/EinsteinSzilLetter.txt", CharacterEncoding
->"UTF8"];
```

>> `StringPosition[data, "uranium"]`

$\{\{299, 305\}, \{870, 876\}, \{1538, 1544\}, \{1671, 1677\}, \{2300, 2306\}, \{2784, 2790\}, \{3093, 3099\}\}$

52.3.6. StringReplace

WMA link

`StringReplace`[“*string*”, “*a*”->“*b*”]
  replaces each occurrence of *old* with *new* in *string*.
`StringReplace`[“*string*”, {“s_1”->“sp_1”, “s_2”->“sp_2”}]
  performs multiple replacements of each *si* by the corresponding *spi* in *string*.
`StringReplace`[“*string*”, *srules*, *n*]
  only performs the first *n* replacements.
`StringReplace`[{“$string_1$”, “$string_2$”, ...}, *srules*]
  performs the replacements specified by *srules* on a list of strings.

StringReplace replaces all occurrences of one substring with another:

>> `StringReplace["xyxyxyyyxxxyyxy", "xy" -> "A"]`

AAAyyxxAyA

Multiple replacements can be supplied:

>> `StringReplace["xyzwxyzwxxyzxyzw", {"xyz" -> "A", "w" -> "BCD"}]`

ABCDABCDxAABCD

Only replace the first 2 occurrences:

>> `StringReplace["xyxyxyyyxxxyyxy", "xy" -> "A", 2]`

AAxyyyxxxyyxy

Also works for multiple rules:

>> `StringReplace["abba", {"a" -> "A", "b" -> "B"}, 2]`

ABba

StringReplace acts on lists of strings too:

>> `StringReplace[{"xyxyxxy", "yxyxyxxxyyxy"}, "xy" -> "A"]`

{AAxA, yAAxxAyA}

StringReplace also can be used as an operator:

>> `StringReplace["y" -> "ies"]["city"]`

cities

52.3.7. StringReverse

WMA link

`StringReverse["`*string*`"]`
: reverses the order of the characters in "string".

```
>> StringReverse["live"]
```
evil

52.3.8. StringRiffle

WMA link

`StringRiffle[{s1, s2, s3, ...}]`
: returns a new string by concatenating all the *si*, with spaces inserted between them.

`StringRiffle[list, sep]`
: inserts the separator *sep* between all elements in *list*.

`StringRiffle[list, {``left', ``sep'', ``right''}]'`
: use *left* and *right* as delimiters after concatenation.

```
>> StringRiffle[{"a", "b", "c", "d", "e"}]
```
a b c d e

```
>> StringRiffle[{"a", "b", "c", "d", "e"}, ", "]
```
a, b, c, d, e

```
>> StringRiffle[{"a", "b", "c", "d", "e"}, {"(", " ", ")"}]
```
(a b c d e)

52.3.9. StringSplit

WMA link

`StringSplit[`s`]`
: splits the string s at whitespace, discarding the whitespace and returning a list of strings.

`StringSplit[`s, *pattern*`]`
: splits s into substrings separated by delimiters matching the string expression *pattern*.

`StringSplit[`s, $\{p_1, p_2, ...\}$`]`
: splits s at any of the p_i patterns.

`StringSplit[`$\{s_1, s_2, ...\}$, $\{d_1, d_2, ...\}$`]`
: returns a list with the result of applying the function to each element.

```
>> StringSplit["abc,123", ","]
```
$\{abc, 123\}$

By default any number of whitespace characters are used to at a delimiter:

```
>> StringSplit[" abc 123 "]
```

{abc, 123}

However if you want instead to use only a *single* character for each delimiter, use `WhiteSpaceCharacter`:

```
>> StringSplit[" abc 123 ", WhitespaceCharacter]
```

{, , abc, , , , 123, , }

```
>> StringSplit["abc,123.456", {",", "."}]
```

{abc, 123, 456}

```
>> StringSplit["a b c", RegularExpression[" +"]]
```

{a, b, c}

```
>> StringSplit[{"a b", "c d"}, RegularExpression[" +"]]
```

{{a, b} , {c, d}}

```
>> StringSplit["x", "x"]
```

{}

Split using a delimiter that has nonzero list of 12's

```
>> StringSplit["12312123", "12"..]
```

{3, 3}

52.3.10. StringTake

WMA link

`StringTake`["*string*", *n*]
    gives the first *n* characters in *string*.
`StringTake`["*string*", -*n*]
    gives the last *n* characters in *string*.
`StringTake`["*string*", {*n*}]
    gives the *n*th character in *string*.
`StringTake`["*string*", {*m*, *n*}]
    gives characters *m* through *n* in *string*.
`StringTake`["*string*", {*m*, *n*, *s*}]
    gives characters *m* through *n* in steps of *s*.
`StringTake`[{s_1, s_2, ...} *spec*}]
    gives the list of results for each of the *si*.

```
>> StringTake["abcde", 2]
```

ab

```
>> StringTake["abcde", 0]
```

```
>> StringTake["abcde", -2]
```
de

```
>> StringTake["abcde", {2}]
```
b

```
>> StringTake["abcd", {2,3}]
```
bc

```
>> StringTake["abcdefgh", {1, 5, 2}]
```
ace

Take the last 2 characters from several strings:

```
>> StringTake[{"abcdef", "stuv", "xyzw"}, -2]
```
$\{ef, uv, zw\}$

StringTake also supports standard sequence specifications

```
>> StringTake["abcdef", All]
```
abcdef

52.3.11. StringTrim

WMA link

`StringTrim[`*s*`]`
: returns a version of *s* with whitespace removed from start and end.

```
>> StringJoin["a", StringTrim[" \tb\n "], "c"]
```
abc

```
>> StringTrim["ababaxababyaabab", RegularExpression["(ab)+"]]
```
axababya

52.4. Regular Expressions

52.4.1. RegularExpression

WMA link

`RegularExpression[``regex']'`
: represents the regex specified by the string "*regex*".

```
>> StringSplit["1.23, 4.56 7.89", RegularExpression["(\\s|,)+"]]
```
{1.23, 4.56, 7.89}

`RegularExpression` just wraps a string to be interpreted as a regular expression, but are not evaluated as stand alone expressions:

```
>> RegularExpression["[abc]"]
```
RegularExpression [[abc]]

52.5. String Patterns

52.5.1. DigitCharacter

WMA link

`DigitCharacter`
: represents the digits 0-9.

```
>> StringMatchQ["1", DigitCharacter]
```
True

```
>> StringMatchQ["a", DigitCharacter]
```
False

```
>> StringMatchQ["12", DigitCharacter]
```
False

```
>> StringMatchQ["123245", DigitCharacter..]
```
True

52.5.2. EndOfLine

WMA link

`EndOfLine`
: represents the end of a line in a string.

```
>> StringReplace["aba\nbba\na\nab", "a" ~~EndOfLine -> "c"]
```
abc
bbc
c
ab

```
>> StringSplit["abc\ndef\nhij", EndOfLine]
```
{abc,
def,
hij}

52.5.3. EndOfString

WMA link

`EndOfString`
represents the end of a string.

Test whether strings end with “e”:

```
>> StringMatchQ[#, __ ~~"e" ~~EndOfString] &/@ {"apple", "banana", "
   artichoke"}
```
{True, False, True}

```
>> StringReplace["aab\nabb", "b" ~~EndOfString -> "c"]
```
aab
abc

52.5.4. LetterCharacter

WMA link

`LetterCharacter`
represents letters.

```
>> StringMatchQ[#, LetterCharacter] & /@ {"a", "1", "A", " ", "."}
```
{True, False, True, False, False}

LetterCharacter also matches unicode characters.

```
>> StringMatchQ["\[Lambda]", LetterCharacter]
```
True

52.5.5. StartOfLine

WMA link

`StartOfLine`
  represents the start of a line in a string.

```
>> StringReplace["aba\nbba\na\nab", StartOfLine ~~"a" -> "c"]
```
cba
  bba
  c
  cb

```
>> StringSplit["abc\ndef\nhij", StartOfLine]
```
{abc
  , def
  , hij}

52.5.6. StartOfString

WMA link

`StartOfString`
  represents the start of a string.

Test whether strings start with "a":

```
>> StringMatchQ[#, StartOfString ~~"a" ~~__] &/@ {"apple", "banana", "
   artichoke"}
```
{True, False, True}

```
>> StringReplace["aba\nabb", StartOfString ~~"a" -> "c"]
```
cba
  abb

52.5.7. StringCases

WMA link

`StringCases`["*string*", *pattern*]
  gives all occurrences of *pattern* in *string*.
`StringReplace`["*string*", *pattern* -> *form*]
  gives all instances of *form* that stem from occurrences of *pattern* in *string*.
`StringCases`["*string*", {$pattern_1$, $pattern_2$, ...}]
  gives all occurrences of $pattern_1$, $pattern_2$, ...
`StringReplace`["*string*", *pattern*, *n*]
  gives only the first *n* occurrences.
`StringReplace`[{"$string_1$", "$string_2$", ...}, *pattern*]
  gives occurrences in $string_1$, $string_2$, ...

```
>> StringCases["axbaxxb", "a" ~~x_ ~~"b"]
```
{axb}

```
>> StringCases["axbaxxb", "a" ~~x__ ~~"b"]
```
{axbaxxb}

```
>> StringCases["axbaxxb", Shortest["a" ~~x__ ~~"b"]]
```
{axb, axxb}

```
>> StringCases["-abc- def -uvw- xyz", Shortest["-" ~~x__ ~~"-"] -> x]
```
{abc, uvw}

```
>> StringCases["-öhi- -abc- -.-", "-" ~~x : WordCharacter .. ~~"-" -> x]
```
{öhi, abc}

```
>> StringCases["abc-abc xyz-uvw", Shortest[x : WordCharacter .. ~~"-" ~~
   x_] -> x]
```
{abc}

```
>> StringCases["abba", {"a" -> 10, "b" -> 20}, 2]
```
{10, 20}

```
>> StringCases["a#ä_123", WordCharacter]
```
{a, ä, 1, 2, 3}

```
>> StringCases["a#ä_123", LetterCharacter]
```
{a, ä}

52.5.8. StringExpression (~~)

WMA link

`StringExpression[s_1, s_2, ...]`
  represents a sequence of strings and symbolic string objects s_i.

```
>> "a" ~~"b" // FullForm
```
"ab"

52.5.9. WhitespaceCharacter

WMA link

`WhitespaceCharacter`
  represents a single whitespace character.

```
>> StringMatchQ["\n", WhitespaceCharacter]
```
True

```
>> StringSplit["a\nb\r\nc\rd", WhitespaceCharacter]
```
{a, b, , c, d}

For sequences of whitespace characters use `Whitespace`:

```
>> StringMatchQ[" \n", WhitespaceCharacter]
```
False

```
>> StringMatchQ[" \n", Whitespace]
```
True

52.5.10. WordBoundary

WMA link

`WordBoundary`
represents the boundary between words.

```
>> StringReplace["apple banana orange artichoke", "e" ~~WordBoundary ->
   "E"]
```
applE banana orangE artichokE

52.5.11. WordCharacter

WMA link

`WordCharacter`
represents a single letter or digit character.

```
>> StringMatchQ[#, WordCharacter] &/@ {"1", "a", "A", ",", " "}
```
{True, True, True, False, False}

Test whether a string is alphanumeric:

```
>> StringMatchQ["abc123DEF", WordCharacter..]
```
True

```
>> StringMatchQ["$b;123", WordCharacter..]
```
False

53. Symbolic Execution History

In order to debug and understand program execution, the execution history can be saved.

54. Tensors

A tensor is an algebraic object that describes a (multilinear) relationship between sets of algebraic objects related to a vector space. Objects that tensors may map between include vectors and scalars, and even other tensors.

There are many types of tensors, including scalars and vectors (which are the simplest tensors), dual vectors, multilinear maps between vector spaces, and even some operations such as the dot product. Tensors are defined independent of any basis, although they are often referred to by their components in a basis related to a particular coordinate system.

Mathics3 represents tensors of vectors and matrices as lists; tensors of any rank can be handled.

Contents

54.1. ArrayDepth

WMA link

`ArrayDepth[`*a*`]`
    returns the depth of the non-ragged array *a*, defined as `Length[Dimensions[$a$]]`.

```
>> ArrayDepth[{{a,b},{c,d}}]
   2
>> ArrayDepth[x]
   0
```

54.2. ConjugateTranspose

Conjugate transpose (WMA)

`ConjugateTranspose[`*m*`]`
: gives the conjugate transpose of *m*.

```
>> ConjugateTranspose[{{0, I}, {0, 0}}]
```
$\{\{0,0\},\{-I,0\}\}$

```
>> ConjugateTranspose[{{1, 2 I, 3}, {3 + 4 I, 5, I}}]
```
$\{\{1,3-4I\},\{-2I,5\},\{3,-I\}\}$

54.3. Dimensions

WMA

`Dimensions[`*expr*`]`
: returns a list of the dimensions of the expression *expr*.

A vector of length 3:

```
>> Dimensions[{a, b, c}]
```
$\{3\}$

A 3x2 matrix:

```
>> Dimensions[{{a, b}, {c, d}, {e, f}}]
```
$\{3,2\}$

Ragged arrays are not taken into account:

```
>> Dimensions[{{a, b}, {b, c}, {c, d, e}}]
```
$\{3\}$

The expression can have any head:

```
>> Dimensions[f[f[a, b, c]]]
```
$\{1,3\}$

54.4. Dot (.)

Dot product (WMA link)

`Dot`[x, y]
`$x$ . $y$`
computes the vector dot product or matrix product x . y.

Scalar product of vectors:

```
>> {a, b, c} . {x, y, z}
```
$ax + by + cz$

Product of matrices and vectors:

```
>> {{a, b}, {c, d}} . {x, y}
```
$\{ax + by, cx + dy\}$

Matrix product:

```
>> {{a, b}, {c, d}} . {{r, s}, {t, u}}
```
$\{\{ar + bt, as + bu\}, \{cr + dt, cs + du\}\}$

```
>> a . b
```
$a.b$

54.5. Inner

WMA link

`Inner`[f, x, y, g]
computes a generalised inner product of x and y, using a multiplication function f and an addition function g.

```
>> Inner[f, {a, b}, {x, y}, g]
```
$g\left[f[a, x], f[b, y]\right]$

`Inner` can be used to compute a dot product:

```
>> Inner[Times, {a, b}, {c, d}, Plus] == {a, b} . {c, d}
```
True

The inner product of two boolean matrices:

```
>> Inner[And, {{False, False}, {False, True}}, {{True, False}, {True,
   True}}, Or]
```
{{False, False}, {True, True}}

Inner works with tensors of any depth:

```
>> Inner[f, {{{a, b}}, {{c, d}}}, {{1}, {2}}, g]
```

$\{\{\{g[f[a,1], f[b,2]]\}\}, \{\{g[f[c,1], f[d,2]]\}\}\}$

54.6. LeviCivitaTensor

Levi-Civita tensor (WMA link)

`LeviCivitaTensor[`*d*`]`
: gives the *d*-dimensional Levi-Civita totally antisymmetric tensor.

```
>> LeviCivitaTensor[3]
```

$\text{SparseArray}[\text{Automatic}, \{3,3,3\}, 0, \{\{1,2,3\} -> 1, \{1,3,2\} -> -1, \{2,1,3\} -> -1, \{2,3,1\} -> 1, \{3,1,2\} -> 1, \{3,2,1\} -> -1\}]$

```
>> LeviCivitaTensor[3, List]
```

$\{\{\{0,0,0\}, \{0,0,1\}, \{0,-1,0\}\}, \{\{0,0,-1\}, \{0,0,0\}, \{1,0,0\}\}, \{\{0,1,0\}, \{-1,0,0\}, \{0,0,0\}\}\}$

54.7. Outer

Outer product (WMA link)

`Outer[`*f*, *x*, *y*`]`
: computes a generalised outer product of *x* and *y*, using the function *f* in place of multiplication.

```
>> Outer[f, {a, b}, {1, 2, 3}]
```

$\{\{f[a,1], f[a,2], f[a,3]\}, \{f[b,1], f[b,2], f[b,3]\}\}$

Outer product of two matrices:

```
>> Outer[Times, {{a, b}, {c, d}}, {{1, 2}, {3, 4}}]
```

$\{\{\{\{a, 2a\}, \{3a, 4a\}\}, \{\{b, 2b\}, \{3b, 4b\}\}\}, \{\{\{c, 2c\}, \{3c, 4c\}\}, \{\{d, 2d\}, \{3d, 4d\}\}\}\}$

Outer product of two sparse arrays:

```
>> Outer[Times, SparseArray[{{1, 2} -> a, {2, 1} -> b}], SparseArray
   [{{1, 2} -> c, {2, 1} -> d}]]
```

$\text{SparseArray}[\text{Automatic}, \{2,2,2,2\}, 0, \{\{1,2,1,2\} -> ac, \{1,2,2,1\} -> ad, \{2,1,1,2\} -> bc, \{2,1,2,1\} -> bd\}]$

`Outer` of multiple lists:

```
>> Outer[f, {a, b}, {x, y, z}, {1, 2}]
```

$\{\{\{f[a,x,1], f[a,x,2]\}, \{f[a,y,1], f[a,y,2]\}, \{f[a,z,1], f[a,z,2]\}\}, \{\{f[b,x,1], f[b,x,2]\}, \{f[b,y,1], f[b,y,2]\}, \{f[b,z,1], f[b,z,2]\}\}\}$

`Outer` converts input sparse arrays to lists if f=!=Times, or if the input is a mixture of sparse arrays and lists:

```
>> Outer[f, SparseArray[{{1, 2} -> a, {2, 1} -> b}], SparseArray[{{1, 2}
    -> c, {2, 1} -> d}]]
```

$\{\{\{\{f[0,0], f[0,c]\}, \{f[0,d], f[0,0]\}\}, \{\{f[a,0], f[a,c]\}, \{f[a,d], f[a,0]\}\}\}, \{\{\{f[b,0], f[b,c]\}, \{f[b,d], f[b,0]\}\}, \{\{f[0,0], f[0,c]\}, \{f[0,d], f[0,0]\}\}\}\}$

```
>> Outer[Times, SparseArray[{{1, 2} -> a, {2, 1} -> b}], {c, d}]
```

$\{\{\{0,0\}, \{ac, ad\}\}, \{\{bc, bd\}, \{0,0\}\}\}$

Arrays can be ragged:

```
>> Outer[Times, {{1, 2}}, {{a, b}, {c, d, e}}]
```

$\{\{\{\{a,b\}, \{c,d,e\}\}, \{\{2a,2b\}, \{2c,2d,2e\}\}\}\}$

Word combinations:

```
>> Outer[StringJoin, {"", "re", "un"}, {"cover", "draw", "wind"}, {"", "
   ing", "s"}] // InputForm
```

{{{“cover”, “covering”, “covers”}, {“draw”, “drawing”, “draws”}, {“wind”, “winding”, “winds”}}, {{“recover”, “recovering”, “recovers”}, {“redraw”, “redrawing”, “redraws”}, {“rewind”, “rewinding”, “rewinds”}}, {{“uncover”, “uncovering”, “uncovers”}, {“undraw”, “undrawing”, “undraws”}, {“unwind”, “unwinding”, “unwinds”}}}

Compositions of trigonometric functions:

```
>> trigs = Outer[Composition, {Sin, Cos, Tan}, {ArcSin, ArcCos, ArcTan}]
```

{{Composition [Sin, ArcSin], Composition [Sin, ArcCos], Composition [Sin, ArcTan]}, {Composition [Cos, ArcSin], Composition [Cos, ArcCos], Composition [Cos, ArcTan]}, {Composition [Tan, ArcSin], Composition [Tan, ArcCos], Composition [Tan, ArcTan]}}

Evaluate at 0:

```
>> Map[#[0] &, trigs, {2}]
```

{{0, 1, 0}, {1, 0, 1}, {0, ComplexInfinity, 0}}

54.8. RotationTransform

WMA link

`RotationTransform[`*phi*`]`
    gives a rotation by *phi*.
`RotationTransform[`*phi*, *p*`]`
    gives a rotation by *phi* around the point *p*.

54.9. ScalingTransform

WMA link

`ScalingTransform[`*v*`]`
    gives a scaling transform of *v*. *v* may be a scalar or a vector.
`ScalingTransform[`*phi*, *p*`]`
    gives a scaling transform of *v* that is centered at the point *p*.

54.10. ShearingTransform

WMA link

`ShearingTransform[`*phi*, {1, 0}, {0, 1}`]`
    gives a horizontal shear by the angle *phi*.
`ShearingTransform[`*phi*, {0, 1}, {1, 0}`]`
    gives a vertical shear by the angle *phi*.
`ShearingTransform[`*phi*, *u*, *u*, *p*`]`
    gives a shear centered at the point *p*.

54.11. TransformationFunction

WMA link

`TransformationFunction[`*m*`]`
    represents a transformation.

```
>> RotationTransform[Pi].TranslationTransform[{1, -1}]
```

TransformationFunction $[\{\{-1, 0, -1\}, \{0, -1, 1\}, \{0, 0, 1\}\}]$

```
>> TranslationTransform[{1, -1}].RotationTransform[Pi]
```
TransformationFunction $\left[\{\{-1,0,1\},\{0,-1,-1\},\{0,0,1\}\}\right]$

54.12. TranslationTransform

WMA link

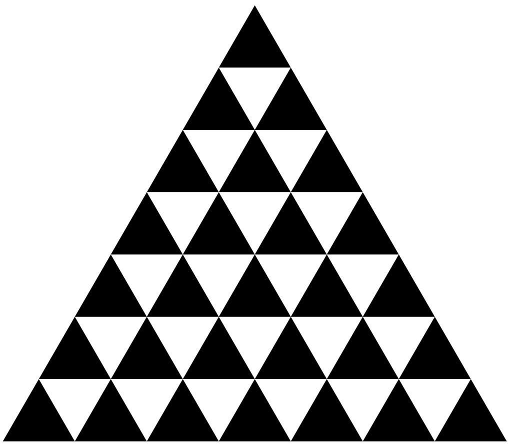
`TranslationTransform[`*v*`]`
: gives a `TransformationFunction` that translates points by vector *v*.

```
>> t = TranslationTransform[{x0, y0}]
```
TransformationFunction $\left[\{\{1,0,\mathrm{x0}\},\{0,1,\mathrm{y0}\},\{0,0,1\}\}\right]$

```
>> t[{x, y}]
```
$\{x+\mathrm{x0}, y+\mathrm{y0}\}$

From Creating a Sierpinsky gasket with the missing triangles filled in:

```
>> Show[Graphics[Table[Polygon[TranslationTransform[{Sqrt[3] (i - j/2),
   3 j/2}] /@ {{Sqrt[3]/2, -1/2}, {0, 1}, {-Sqrt[3]/2, -1/2}}], {i, 7},
   {j, i}]]]
```



54.13. Transpose

Transpose (SymPy, WMA)

`Transpose[`*list*`]`
: transposes the first two levels in *list*. The rank of *list* should be less than 4.

```
>> square = {{1, 2, 3}, {4, 5, 6}}; Transpose[square]
```
$\{\{1,4\},\{2,5\},\{3,6\}\}$

```
>> MatrixForm[%]
```
$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

```
>> matrix = {{1, 2}, {3, 4}, {5, 6}}; MatrixForm[Transpose[matrix]]
```
$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

```
>> matrix3D = {{{1, 2}, {3, 4}}, {{5, 6}, {7, 8}}}; Transpose[matrix3D]
```
$\{\{\{1,2\},\{5,6\}\},\{\{3,4\},\{7,8\}\}\}$

Transpose is its own inverse. Transposing a matrix twice will give you back the same thing you started out with:

```
>> Transpose[Transpose[matrix]] == matrix
```
True

```
>> Transpose[Transpose[matrix3D]] == matrix3D
```
True

If the rank of the list is 0 or 1, you get the list back

```
>> Transpose[{}]
```
$\{\}$

```
>> Transpose[{a, b, c}]
```
$\{a,b,c\}$

55. Testing Expressions

There are a number of functions for testing Expressions.

Functions that “ask a question” have names that end in “Q”. They return `True` for an explicit answer, and `False` otherwise.

Contents

55.1. Equality and Inequality

55.1.1. Between

WMA link

`Between`[x, {min, max}]
: equivalent to min <= x <= max.

`Between`[x, { {min_1, max_1}, {min_2, max_2}, ...]
: equivalent to min_1 <= x <= max_1 | | min_2 <= x <= max_2 ...

`Between`[$range$]
: operator form that yields `Between`[x, $range$] when applied to expression x.

Check that 6 is in range 4..10:

```
>> Between[6, {4, 10}]
```
True

Same as above in operator form:

```
>> Between[{4, 10}][6]
```
True

`Between` works with irrational numbers:

```
>> Between[2, {E, Pi}]
```
False

If more than an interval is given, `Between` returns `True` if x belongs to one of them:

```
>> {Between[3, {1, 2}, {4, 6}], Between[5, {1, 2}, {4, 6}]}
```
$\{False, True\}$

55.1.2. BooleanQ

WMA link

`BooleanQ`[$expr$]
: returns `True` if $expr$ is either `True` or `False`.

```
>> BooleanQ[True]
```
True

```
>> BooleanQ[False]
```
True

```
>> BooleanQ[a]
```
False

```
>> BooleanQ[1 < 2]
```
True

55.1.3. Equal (==)

WMA link

`Equal[`x, y`]`
`$x$ == $y$`
  is `True` if x and y are known to be equal, or `False` if x and y are known to be unequal, in which case case, `Not[$x$ == $y$]` will be `True`.
Commutative properties apply, so if $x == y$ then $y == x$.
For any expression x and y, Equal[x, y] == Not[Unequal[x, y]].
For any expression `SameQ[$x$, $y$]` implies Equal[x, y].
`$x$ == $y$ == $z$ == ...`
  express a chain of equalities.

Numerical Equalities:

```
>> 1 == 1.
```
True

```
>> 5/3 == 3/2
```
False

Comparisons are done using the lower precision:

```
>> N[E, 100] == N[E, 150]
```
True

Compare an exact numeric expression and its corresponding approximate number:

```
>> Pi == N[Pi, 20]
```
True

Symbolic constants are compared numerically:

```
>> Pi == 3.14
```
False

Compare two exact numeric expressions; a numeric test may suffice to disprove equality:

```
>> Pi ^ E == E ^ Pi
```
False

Compare an exact expression against an approximate real number:

>> `Pi == 3.1415``4`
True

Real values are considered equal if they only differ in their last digits:

>> `0.739085133215160642 == 0.739085133215160641`
True

>> `0.73908513321516064200000000 == 0.73908513321516064100000000`
False

Numeric evaluation using Equal:

>> `{Mod[6, 2] == 0, Mod[6, 4] == 0}`
{True, False}

String equalities:

>> `Equal["11", "11"]`
True

>> `Equal["121", "11"]`
False

When we have symbols without values, the values are equal only if the symbols are equal:

>> `Clear[a, b]; a == b`
a==b

>> `a == a`
True

>> `a = b; a == b`
True

Comparison to mismatched types is False:

>> `Equal[11, "11"]`
False

Lists are compared based on their elements:

>> `{{1}, {2}} == {{1}, {2}}`
True

>> `{1, 2} == {1, 2, 3}`
False

For chains of equalities, the comparison is done amongst all the pairs. The evaluation is successful only if the equality is satisfied over all the pairs:

>> `g[1] == g[1] == g[1]`
True

```
>> g[1] == g[1] == g[r]
```
$g[1]==g[1]==g[r]$

Equality can also be combined with other inequality expressions, like:

```
>> g[1] == g[2] != g[3]
```
$g[1]==g[2]\&\&g[2]!=g[3]$

```
>> g[1] == g[2] <= g[3]
```
$g[1]==g[2]\&\&g[2]<=g[3]$

`Equal` with no parameter or an empty list is `True`:

```
>> Equal[] == True
```
True

`Equal` on one parameter or list element is also `True`

```
>> {Equal[x], Equal[1], Equal["a"]}
```
{True, True, True}

This degenerate behavior is the same for `Unequal`; empty or single-element lists are both `Equal` and `Unequal`.

55.1.4. Greater (>)

WMA link

`Greater`[x, y] or `$x$ > $y$`
: yields `True` if x is known to be greater than y.

Symbolic constants are compared numerically:

```
>> E > 1
```
True

Greater operator can be chained:

```
>> a > b > c //FullForm
```
Greater $[a, b, c]$

```
>> 3 > 2 > 1
```
True

55.1.5. GreaterEqual (≥)

WMA link

`GreaterEqual`[x, y]
$x \geq y$ or `$x$ >= $y$`
: yields `True` if x is known to be greater than or equal to y.

55.1.6. Inequality

WMA link

`Inequality`
: is the head of expressions involving different inequality operators (at least temporarily). Thus, it is possible to write chains of inequalities.

```
>> a < b <= c
```
$a < b\&\&b{<=}c$

```
>> Inequality[a, Greater, b, LessEqual, c]
```
$a > b\&\&b{<=}c$

```
>> 1 < 2 <= 3
```
True

```
>> 1 < 2 > 0
```
True

```
>> 1 < 2 < -1
```
False

55.1.7. Less (<)

WMA link

`Less`[x, y] or $x < y$
: yields `True` if x is known to be less than y.

```
>> 1 < 0
```
False

LessEqual operator can be chained:

```
>> 2/18 < 1/5 < Pi/10
```
True

Using less on an undefined symbol value:

```
>> 1 < 3 < x < 2
```
$1 < 3 < x < 2$

55.1.8. LessEqual ($\leq$)

WMA link

> `LessEqual`[x, y, ...] or x <= y or $x \leq y$
> yields `True` if x is known to be less than or equal to y.

LessEqual operator can be chained:

```
>> LessEqual[1, 3, 3, 2]
```
False

```
>> 1 <= 3 <= 3
```
True

55.1.9. Max

WMA link

> `Max`[e_1, e_2, ..., e_i]
> returns the expression with the greatest value among the e_i.

Maximum of a series of values:

```
>> Max[4, -8, 1]
```
4

```
>> Max[E - Pi, Pi, E + Pi, 2 E]
```
$E + \pi$

`Max` flattens lists in its arguments:

```
>> Max[{1,2},3,{-3,3.5,-Infinity},{{1/2}}]
```
3.5

`Max` with symbolic arguments remains in symbolic form:

```
>> Max[x, y]
```
$\text{Max}\left[x, y\right]$

```
>> Max[5, x, -3, y, 40]
```
$\text{Max}\left[40, x, y\right]$

With no arguments, `Max` gives `-Infinity`:

```
>> Max[]
```
$-\infty$

Max does not compare strings or symbols:

```
>> Max[-1.37, 2, "a", b]
```
Max $[2, \mathrm{a}, b]$

55.1.10. Min

WMA link

`Min`[e_1, e_2, ..., e_i]
: returns the expression with the lowest value among the e_i.

Minimum of a series of values:

```
>> Min[4, -8, 1]
```
-8

```
>> Min[E - Pi, Pi, E + Pi, 2 E]
```
$E - \pi$

Min flattens lists in its arguments:

```
>> Min[{1,2},3,{-3,3.5,-Infinity},{{1/2}}]
```
$-\infty$

Min with symbolic arguments remains in symbolic form:

```
>> Min[x, y]
```
Min $[x, y]$

```
>> Min[5, x, -3, y, 40]
```
Min $[-3, x, y]$

With no arguments, Min gives Infinity:

```
>> Min[]
```
∞

55.1.11. SameQ (===)

WMA link

`SameQ`[x, y]
`$x$ === $y$`
: returns True if x and y are structurally identical. Commutative properties apply, so if x === y then y === x.

- `SameQ` requires exact correspondence between expressions, expect that it still considers `Real` numbers equal if they differ in their last binary digit.
- $e_1 === e_2 === e_3$ gives `True` if all the *ei*'s are identical.
- `SameQ[]` and `SameQ[$expr$]` always yield `True`.

Any object is the same as itself:

```
>> a === a
```

True

Degenerate cases of `SameQ` showing off how you can chain `===`:

```
>> SameQ[a] === SameQ[] === True
```

True

Unlike `Equal`, `SameQ` only yields `True` if x and y have the same type:

```
>> {1==1., 1===1.}
```

{True, False}

```
>> 2./9. === .2222222222222222`15.9546
```

True

The comparison consider just the lowest precision

```
>> .2222222`6 === .2222`3
```

True

Notice the extra decimal in the rhs. Because the internal representation, 0.222'3 is not equivalent to 0.2222'3:

```
>> .2222222`6 === .222`3
```

False

15.9546 is the value of `$MaxPrecision`

55.1.12. TrueQ

WMA link

`TrueQ[`*expr*`]`
: returns `True` if and only if *expr* is `True`.

```
>> TrueQ[True]
```

True

```
>> TrueQ[False]
```

False

```
>> TrueQ[a]
```
False

55.1.13. Unequal ($\neq$)

WMA link

`Unequal`[x, y] or x != y or $x \neq y$
    is `False` if x and y are known to be equal, or `True` if x and y are known to be unequal.
Commutative properties apply so if x != y then y != x.
For any expression x and y, `Unequal[$x$, $y$]` == `Not[Equal[$x$, $y$]]`.

```
>> 1 != 1.
```
False

Comparisons can be chained:

```
>> 1 != 2 != 3
```
True

```
>> 1 != 2 != x
```
1!=2!=x

Strings are allowed:

```
>> Unequal["11", "11"]
```
False

Comparison to mismatched types is True:

```
>> Unequal[11, "11"]
```
True

Lists are compared based on their elements:

```
>> {1} != {2}
```
True

```
>> {1, 2} != {1, 2}
```
False

```
>> {a} != {a}
```
False

```
>> "a" != "b"
```
True

```
>> "a" != "a"
```
False

`Unequal` using an empty parameter or list, or a list with one element is True. This is the same as 'Equal".

>> `{Unequal[], Unequal[x], Unequal[1]}`

{True, True, True}

55.1.14. UnsameQ (=!=)

WMA link

`UnsameQ`[x, y]
`$x$ =!= $y$`
: returns `True` if x and y are not structurally identical. Commutative properties apply, so if x =!= y, then y =!= x.

>> `a =!= a`

False

>> `1 =!= 1.`

True

UnsameQ accepts any number of arguments and returns True if all expressions are structurally distinct:

>> `1 =!= 2 =!= 3 =!= 4`

True

UnsameQ returns False if any expression is identical to another:

>> `1 =!= 2 =!= 1 =!= 4`

False

UnsameQ[] and UnsameQ[expr] return True:

>> `UnsameQ[]`

True

>> `UnsameQ[expr]`

True

55.2. Expression Tests

55.2.1. ListQ

WMA link

`ListQ`[*expr*]
: tests whether *expr* is a `List`.

>> `ListQ[{1, 2, 3}]`
True

>> `ListQ[{{1, 2}, {3, 4}}]`
True

>> `ListQ[x]`
False

55.2.2. MatchQ

WMA link

`MatchQ`[*expr*, *form*]
tests whether *expr* matches *form*.

>> `MatchQ[123, _Integer]`
True

>> `MatchQ[123, _Real]`
False

>> `MatchQ[_Integer][123]`
True

>> `MatchQ[3, Pattern[3]]`
`First element in pattern Pattern[3] is not a valid pattern name.`
False

See also 'Cases' 35.3.3.

55.2.3. Order

WMA link

`Order`[x, y]
returns a number indicating the canonical ordering of x and y. 1 indicates that x is before y, and -1 that y is before x. 0 indicates that there is no specific ordering. Uses the same order as `Sort`.

>> `Order[7, 11]`
1

>> `Order[100, 10]`
-1

>> `Order[x, z]`
1

```
>> Order[x, x]
```
0

55.2.4. OrderedQ

WMA link

`OrderedQ[{`*a*, *b*`}]`
: is `True` if *a* sorts before *b* according to canonical ordering.

```
>> OrderedQ[{a, b}]
```
True

```
>> OrderedQ[{b, a}]
```
False

55.2.5. PatternsOrderedQ

`PatternsOrderedQ[`*patt*1, *patt*2`]`
: returns `True` if pattern *patt*1 would be applied before *patt*2 according to canonical pattern ordering.

```
>> PatternsOrderedQ[x__, x_]
```
False

```
>> PatternsOrderedQ[x_, x__]
```
True

```
>> PatternsOrderedQ[b, a]
```
True

55.3. List-Oriented Tests

55.3.1. ArrayQ

WMA

`ArrayQ[`*expr*`]`
: tests whether *expr* is a full array.

`ArrayQ[`*expr*, *pattern*`]`
: also tests whether the array depth of *expr* matches *pattern*.

`ArrayQ[`*expr*, *pattern*, *test*`]`
: furthermore tests whether *test* yields `True` for all elements of *expr*. `ArrayQ[$expr$]` is equivalent to `ArrayQ[$expr$, _, True&]`.

>> `ArrayQ[a]`

False

>> `ArrayQ[{a}]`

True

>> `ArrayQ[{{{a}},{{b,c}}}]`

False

>> `ArrayQ[{{a, b}, {c, d}}, 2, SymbolQ]`

True

55.3.2. DisjointQ

WMA link

`DisjointQ[`*a*, *b*`]`
: gives True if *a* and *b* are disjoint, or False if *a* and *b* have any common elements.

55.3.3. IntersectingQ

WMA link

`IntersectingQ[`*a*, *b*`]`
: gives True if there are any common elements in *a* and *b*, or False if *a* and *b* are disjoint.

55.3.4. LevelQ

`LevelQ[`*expr*`]`
: tests whether *expr* is a valid level specification. This function is primarily used in function patterns for specifying type of a parameter.

>> `LevelQ[2]`

True

```
>> LevelQ[{2, 4}]
```
True

```
>> LevelQ[Infinity]
```
True

```
>> LevelQ[a + b]
```
False

We will define MyMap with the "level" parameter as a synonym for the Builtin Map equivalent:

```
>> MyMap[f_, expr_, Pattern[levelspec, _?LevelQ]] := Map[f, expr,
   levelspec]
```

```
>> MyMap[f, {{a, b}, {c, d}}, {2}]
```
$\{\{f[a], f[b]\}, \{f[c], f[d]\}\}$

```
>> Map[f, {{a, b}, {c, d}}, {2}]
```
$\{\{f[a], f[b]\}, \{f[c], f[d]\}\}$

But notice that when we pass an invalid level specification, MyMap does not match and therefore does not pass the arguments through to `Map`. So we do not see the error message that `Map` would normally produce

```
>> Map[f, {{a, b}, {c, d}}, x]
Level specification x is not of the form n, {n}, or {m, n}.
```
$\text{Map}[f, \{\{a,b\}, \{c,d\}\}, x]$

```
>> MyMap[f, {{a, b}, {c, d}}, {1, 2, 3}]
```
$\text{MyMap}[f, \{\{a,b\}, \{c,d\}\}, \{1,2,3\}]$

55.3.5. MatrixQ

WMA link

`MatrixQ[`*m*`]`
: gives `True` if *m* is a list of equal-length lists.

`MatrixQ[`*m*, *f*`]`
: gives `True` only if `$f$[$x$]` returns `True` for when applied to element *x* of the matrix *m*.

```
>> MatrixQ[{{1, 3}, {4.0, 3/2}}, NumberQ]
```
True

These are not matrices:

```
>> MatrixQ[{{1}, {1, 2}}] (* first row should have length two *)
```
False

>> `MatrixQ[Array[a, {1, 1, 2}]]`
False

Supply a test function parameter to generalize and specialize:

>> `MatrixQ[{{1, 2}, {3, 4 + 5}}, Positive]`
True

>> `MatrixQ[{{1, 2 I}, {3, 4 + 5}}, Positive]`
False

55.3.6. MemberQ

WMA link

`MemberQ[`*list*, *pattern*`]`
    returns `True` if *pattern* matches any element of *list*, or `False` otherwise.

>> `MemberQ[{a, b, c}, b]`
True

>> `MemberQ[{a, b, c}, d]`
False

>> `MemberQ[{"a", b, f[x]}, _?NumericQ]`
False

>> `MemberQ[_List][{{}}]`
True

55.3.7. NotListQ

`NotListQ[`*expr*`]`
    returns `True` if *expr* is not a list. This function is primarily used in function patterns for specifying type of a parameter.

Consider this definition for taking the deriviate `Sin` of a function:

>> `MyD[Sin[f_],x_?NotListQ] := D[f,x]*Cos[f]`

=

We use "MyD" above to distinguish it from the Builtin `D`. Now let's try it:

>> `MyD[Sin[2 x], x]`
$2\text{Cos}[2x]$

And compare it with the Builtin deriviative function `D`:

```
>> D[Sin[2 x], x]
```
2Cos [2x]

Note however the pattern only matches if the x parameter is not a list:

```
>> MyD[{Sin[2], Sin[4]}, {1, 2}]
```
MyD [{Sin [2] , Sin [4]} , {1, 2}]

55.3.8. SubsetQ

WMA link

`SubsetQ[`$list_1$, $list_2$`]`
: returns True if $list_2$ is a subset of $list_1$, and False otherwise.

```
>> SubsetQ[{1, 2, 3}, {3, 1}]
```
True

The empty list is a subset of every list:

```
>> SubsetQ[{}, {}]
```
True

```
>> SubsetQ[{1, 2, 3}, {}]
```
True

Every list is a subset of itself:

```
>> SubsetQ[{1, 2, 3}, {1, 2, 3}]
```
True

55.3.9. VectorQ

WMA link

`VectorQ[`v`]`
: returns `True` if v is a list of elements which are not themselves lists.

`VectorQ[`v, f`]`
: returns `True` if v is a vector and `$f$[$x$]` returns `True` for each element x of v.

```
>> VectorQ[{a, b, c}]
```
True

55.4. Logical Combinations

55.4.1. AllTrue

WMA link

`AllTrue`[{$expr_1$, $expr_2$, ...}, *test*]
: returns True if all applications of *test* to $expr_1$, $expr_2$, ... evaluate to True.

`AllTrue`[*list*, *test*, *level*]
: returns True if all applications of *test* to items of *list* at *level* evaluate to True.

`AllTrue`[*test*]
: gives an operator that may be applied to expressions.

```
>> AllTrue[{2, 4, 6}, EvenQ]
```
True

```
>> AllTrue[{2, 4, 7}, EvenQ]
```
False

55.4.2. And (&)

WMA link

`And`[$expr_1$, $expr_2$, ...]
`$expr_1$ && $expr_2$ && ...`
: evaluates each expression in turn, returning `False` as soon as an expression evaluates to `False.` If all expressions evaluate to `True,` `And` returns `True.`

```
>> True && True && False
```
False

If an expression does not evaluate to `True` or `False,` `And` returns a result in symbolic form:

```
>> a && b && True && c
```
a&&*b*&&*c*

55.4.3. AnyTrue

WMA link

`AnyTrue`[{$expr_1$, $expr_2$, ...}, *test*]
: returns True if any application of *test* to $expr_1$, $expr_2$, ... evaluates to True.

`AnyTrue`[*list*, *test*, *level*]
: returns True if any application of *test* to items of *list* at *level* evaluates to True.

`AnyTrue`[*test*]
: gives an operator that may be applied to expressions.

```
>> AnyTrue[{1, 3, 5}, EvenQ]
```
False

```
>> AnyTrue[{1, 4, 5}, EvenQ]
```
True

55.4.4. Equivalent (\[Equivalent])

WMA link

`Equivalent`[$expr_1$, $expr_2$, ...]
$expr_1$ \[Equivalent] $expr_2$ \[Equivalent] ...
: is equivalent to ($expr_1$ && $expr_2$ && ...) | | (!$expr_1$ && !$expr_2$ && ...)

```
>> Equivalent[True, True, False]
```
False

If all expressions do not evaluate to `True` or `False`, `Equivalent` returns a result in symbolic form:

```
>> Equivalent[a, b, c]
```
a\[Equivalent]*b*\[Equivalent]*c*

Otherwise, `Equivalent` returns a result in DNF

```
>> Equivalent[a, b, True, c]
```
a&&*b*&&*c*

55.4.5. False

WMA link

`False`
: represents the Boolean false value.

55.4.6. Implies

WMA link

`Implies`[$expr_1$, $expr_2$]
$expr_1$ $expr_2$
: evaluates each expression in turn, returning `True` as soon as the first expression evaluates to `False`. If the first expression evaluates to `True`, `Implies` returns the second expression.

>> `Implies[False, a]`
True

>> `Implies[True, a]`
a

If an expression does not evaluate to `True` or `False`, `Implies` returns a result in symbolic form:

>> `Implies[a, Implies[b, Implies[True, c]]]`
*a*Implies*b*Implies*c*

55.4.7. Nand

WMA link

`Nand`[$expr_1$, $expr_2$, ...]
$expr_1$ $expr_2$ ...
: Implements the logical NAND function. The same as `Not[And[`$expr_1$, $expr_2$, ...`]]`

>> `Nand[True, False]`
True

55.4.8. NoneTrue

WMA link

`NoneTrue`[{$expr_1$, $expr_2$, ...}, *test*]
: returns True if no application of *test* to $expr_1$, $expr_2$, ... evaluates to True.

`NoneTrue`[*list*, *test*, *level*]
: returns True if no application of *test* to items of *list* at *level* evaluates to True.

`NoneTrue`[*test*]
: gives an operator that may be applied to expressions.

>> `NoneTrue[{1, 3, 5}, EvenQ]`
True

```
>> NoneTrue[{1, 4, 5}, EvenQ]
```
False

55.4.9. Nor ($\veebar$)

WMA link

`Nor`[$expr_1$, $expr_2$, ...]
$expr_1 \veebar expr_2 \veebar$ ...
Implements the logical NOR function. The same as `Not[Or[`$expr_1$, $expr_2$, ...`]]`

```
>> Nor[True, False]
```
False

55.4.10. Not (!)

WMA link

`Not`[$expr$]
`!$expr$`
negates the logical expression $expr$.

```
>> !True
```
False

```
>> !False
```
True

```
>> !b
```
!b

55.4.11. Or (|)

WMA link

`Or`[$expr_1$, $expr_2$, ...]
`$expr_1$ || $expr_2$ || ...`
evaluates each expression in turn, returning `True` as soon as an expression evaluates to `True`. If all expressions evaluate to `False`, `Or` returns `False`.

```
>> False || True
```
True

If an expression does not evaluate to `True` or `False`, `Or` returns a result in symbolic form:

```
>> a || False || b
```
$a||b$

55.4.12. True

WMA link

`True`
: represents the Boolean true value.

55.4.13. Xor (⊕)

WMA link

`Xor`[$expr_1$, $expr_2$, ...]
$expr_1 \oplus expr_2 \oplus$ ...
: evaluates each expression in turn, returning `True` as soon as not all expressions evaluate to the same value. If all expressions evaluate to the same value, `Xor` returns `False`.

```
>> Xor[False, True]
```
True

```
>> Xor[True, True]
```
False

If an expression does not evaluate to `True` or `False`, `Xor` returns a result in symbolic form:

```
>> Xor[a, False, b]
```
a\[Xor]b

55.5. Numerical Properties

55.5.1. CoprimeQ

WMA link

`CoprimeQ`[x, y]
: tests whether x and y are coprime by computing their greatest common divisor.

>> `CoprimeQ[7, 9]`
   True

>> `CoprimeQ[-4, 9]`
   True

>> `CoprimeQ[12, 15]`
   False

For more than two arguments, CoprimeQ checks if any pair or arguments are coprime:

>> `CoprimeQ[2, 3, 5]`
   True

In this case, since 2 divides 4, the result is False:

>> `CoprimeQ[2, 4, 5]`
   False

55.5.2. EvenQ

WMA link

`EvenQ[`*x*`]`
  returns `True` if *x* is even, and `False` otherwise.

>> `EvenQ[4]`
   True

>> `EvenQ[-3]`
   False

>> `EvenQ[n]`
   False

55.5.3. ExactNumberQ

WMA link

`ExactNumberQ[`*expr*`]`
  returns `True` if *expr* is an exact real or complex number, and returns `False` otherwise.

>> `ExactNumberQ[10]`
   True

`ExactNumber[]` of a Real or MachineReal is `False`

```
>> ExactNumberQ[10.0]
   False
```

`ExactNumberQ` for complex numbers:

```
>> ExactNumberQ[I]
   True
>> ExactNumberQ[1 + I]
   True
```

but not when composed with a Real:

```
>> ExactNumberQ[1. + I]
   False
```

`ExactNumber[]` is `True` for Rational numbers:

```
>> ExactNumberQ[5/6]
   True
>> ExactNumberQ[4 * I + 5/6]
   True
```

55.5.4. InexactNumberQ

WMA link

`InexactNumberQ[`*expr*`]`
    returns `True` if *expr* is not an exact real or complex number number, and `False` otherwise.

```
>> InexactNumberQ[a]
   False
>> InexactNumberQ[3.0]
   True
>> InexactNumberQ[2/3]
   False
```

`InexactNumberQ` is `True` for complex numbers:

```
>> InexactNumberQ[4.0+I]
   True
```

55.5.5. IntegerQ

WMA link

`IntegerQ[`*expr*`]`
: returns `True` if *expr* is an integer, and `False` otherwise.

>> `IntegerQ[3]`
True

>> `IntegerQ[Pi]`
False

55.5.6. MachineNumberQ

WMA link

`MachineNumberQ[`*expr*`]`
: returns `True` if *expr* is a machine-precision real or complex number.

= True

>> `MachineNumberQ[3.14159265358979324]`
False

>> `MachineNumberQ[1.5 + 2.3 I]`
True

>> `MachineNumberQ[2.71828182845904524 + 3.14159265358979324 I]`
False

55.5.7. Negative

WMA link

`Negative[`*x*`]`
: returns `True` if *x* is a negative real number.

>> `Negative[0]`
False

>> `Negative[-3]`
True

>> `Negative[10/7]`
False

>> `Negative[1+2I]`
False

```
>> Negative[a + b]
```
Negative $[a + b]$

55.5.8. NonNegative

WMA link

`NonNegative`[x]
: returns `True` if x is a positive real number or zero.

```
>> {Positive[0], NonNegative[0]}
```
{False, True}

55.5.9. NonPositive

WMA link

`NonPositive`[x]
: returns `True` if x is a negative real number or zero.

```
>> {Negative[0], NonPositive[0]}
```
{False, True}

55.5.10. NumberQ

WMA link

`NumberQ`[$expr$]
: returns `True` if $expr$ is an explicit number, and `False` otherwise.

```
>> NumberQ[3+I]
```
True

```
>> NumberQ[5!]
```
True

```
>> NumberQ[Pi]
```
False

55.5.11. NumericQ

WMA link

`NumericQ[`*expr*`]`
tests whether *expr* represents a numeric quantity.

```
>> NumericQ[2]
   True
>> NumericQ[Sqrt[Pi]]
   True
>> NumberQ[Sqrt[Pi]]
   False
```

It is possible to set that a symbol is numeric or not by assign a boolean value to "NumericQ"

```
>> NumericQ[a]=True
   True
>> NumericQ[a]
   True
>> NumericQ[Sin[a]]
   True
```

Clear and ClearAll do not restore the default value.

```
>> Clear[a]; NumericQ[a]
   True
>> ClearAll[a]; NumericQ[a]
   True
>> NumericQ[a]=False; NumericQ[a]
   False
```

NumericQ can only set to True or False

```
>> NumericQ[a] = 37
   Cannot set NumericQ[a] to 37; the lhs argument must be a symbol and
   the rhs must be True or False.
   37
```

55.5.12. OddQ

WMA link

`OddQ[`x`]`
: returns `True` if x is odd, and `False` otherwise.

```
>> OddQ[-3]
```
True

```
>> OddQ[0]
```
False

55.5.13. Positive

WMA link

`Positive[`x`]`
: returns `True` if x is a positive real number.

```
>> Positive[1]
```
True

`Positive` returns `False` if x is zero or a complex number:

```
>> Positive[0]
```
False

```
>> Positive[1 + 2 I]
```
False

55.5.14. PossibleZeroQ

WMA link

`PossibleZeroQ[`$expr$`]`
: returns `True` if basic symbolic and numerical methods suggest that expr has value zero, and `False` otherwise.

Test whether a numeric expression is zero:

```
>> PossibleZeroQ[E^(I Pi/4)- (-1)^(1/4)]
```
True

The determination is approximate.

Test whether a symbolic expression is likely to be identically zero:

```
>> PossibleZeroQ[(x + 1)(x - 1)- x^2 + 1]
```
True

```
>> PossibleZeroQ[(E + Pi)^2 - E^2 - Pi^2 - 2 E Pi]
   True
```

Show that a numeric expression is nonzero:

```
>> PossibleZeroQ[E^Pi - Pi^E]
   False
```

```
>> PossibleZeroQ[1/x + 1/y - (x + y)/(x y)]
   True
```

Decide that a numeric expression is zero, based on approximate computations:

```
>> PossibleZeroQ[2^(2 I)- 2^(-2 I)- 2 I Sin[Log[4]]]
   True
```

```
>> PossibleZeroQ[Sqrt[x^2] - x]
   False
```

55.5.15. PrimeQ

WMA link

`PrimeQ[`*n*`]`
: returns `True` if *n* is a prime number.

For very large numbers, `PrimeQ` uses probabilistic prime testing, so it might be wrong sometimes (a number might be composite even though `PrimeQ` says it is prime). The algorithm might be changed in the future.

```
>> PrimeQ[2]
   True
```

```
>> PrimeQ[-3]
   True
```

```
>> PrimeQ[137]
   True
```

```
>> PrimeQ[2 ^ 127 - 1]
   True
```

All prime numbers between 1 and 100:

```
>> Select[Range[100], PrimeQ]
```
$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$

`PrimeQ` has attribute `Listable`:

\>\> `PrimeQ[Range[20]]`

{False, True, True, False, True, False, True, False, False, False, True, False, True, False, False, False, True, False, True, Fa

55.6. String Tests

55.6.1. DigitQ

WMA link

`DigitQ[`*string*`]`
: yields `True` if all the characters in the *string* are digits, and yields `False` otherwise.

\>\> `DigitQ["9"]`

True

\>\> `DigitQ["a"]`

False

\>\> `DigitQ["01001101011000010111010001101000011010010110001101110011"]`

True

\>\> `DigitQ["-123456789"]`

False

55.6.2. LetterQ

WMA link

`LetterQ[`*string*`]`
: yields `True` if all the characters in the *string* are letters, and yields `False` otherwise.

\>\> `LetterQ["m"]`

True

\>\> `LetterQ["9"]`

False

\>\> `LetterQ["Mathics"]`

True

\>\> `LetterQ["Welcome to Mathics"]`

False

55.6.3. StringFreeQ

WMA link

`StringFreeQ[`"*string*", *patt*`]`
: returns True if no substring in *string* matches the string expression *patt*, and returns False otherwise.

`StringFreeQ[{``s1', ``s2'', ...}, patt]'`
: returns the list of results for each element of string list.

`StringFreeQ[``string', {p1, p2, ...}]'`
: returns True if no substring matches any of the *pi*.

`StringFreeQ[patt]`
: represents an operator form of StringFreeQ that can be applied to an expression.

```
>> StringFreeQ["mathics", "m" ~~__ ~~"s"]
```
False

```
>> StringFreeQ["mathics", "a" ~~__ ~~"m"]
```
True

```
>> StringFreeQ["Mathics", "MA" , IgnoreCase -> True]
```
False

```
>> StringFreeQ[{"g", "a", "laxy", "universe", "sun"}, "u"]
```
{True, True, True, False, False}

```
>> StringFreeQ["e" ~~___ ~~"u"] /@ {"The Sun", "Mercury", "Venus", "
   Earth", "Mars", "Jupiter", "Saturn", "Uranus", "Neptune"}
```
{False, False, False, True, True, True, True, True, False}

```
>> StringFreeQ[{"A", "Galaxy", "Far", "Far", "Away"}, {"F" ~~__ ~~"r", "
   aw" ~~___}, IgnoreCase -> True]
```
{True, True, False, False, False}

55.6.4. StringMatchQ

WMA link

`StringMatchQ[`"string", *pattern*`]`
: checks is "string" matches *pattern*

```
>> StringMatchQ["abc", "abc"]
```
True

```
>> StringMatchQ["abc", "abd"]
```
False

```
>> StringMatchQ["15a94xcZ6", (DigitCharacter | LetterCharacter)..]
```
True

Use StringMatchQ as an operator

```
>> StringMatchQ[LetterCharacter]["a"]
```
True

55.6.5. StringQ

WMA link

`StringQ`[*expr*]
: returns `True` if *expr* is a `String`, or `False` otherwise.

```
>> StringQ["abc"]
```
True

```
>> StringQ[1.5]
```
False

```
>> Select[{"12", 1, 3, 5, "yz", x, y}, StringQ]
```
{12, yz}

55.6.6. SyntaxQ

WMA link

`SyntaxQ[``string']'`
: returns `True` if “string” corresponds to a syntactically correct input for a Mathics3 expression, or `False` otherwise.

```
>> SyntaxQ["a[b"]
```
False

```
>> SyntaxQ["a[b]"]
```
True

56. The Main Loop

An interactive session operates a loop, called the "main loop" in this way:

- read input
- process input
- format and print results
- repeat

As part of this loop, various global objects in this section are consulted.

There are a variety of "hooks" that allow you to insert functions to be applied to the expressions at various stages in the main loop.

If you assign a function to the global variable `$PreRead` it will be applied with the input that is read in the first step listed above.

Similarly, if you assign a function to global variable `$Pre`, it will be applied with the input before processing the input, the second step listed above.

Contents

56.1. $HistoryLength

WMA

`$HistoryLength`
    specifies the maximum number of `In` and `Out` entries.

>> **`$HistoryLength`**
100

>> **`$HistoryLength = 1;`**

>> **`42`**
42

```
>> %
42
>> %%
%3
>> $HistoryLength = 0;
>> 42
42
>> %
%7
```

56.2. $Line

WMA

`$Line`
: holds the current input line number.

```
>> $Line
1
>> $Line
2
>> $Line = 12;
>> 2 * 5
10
>> Out[13]
10
>> $Line = -1;
Non-negative integer expected.
```

56.3. $Post

WMA

`$Post`
: is a global variable whose value, if set, is applied to every output expression.

56.4. $Pre

WMA

> `$Pre`
>
> is a global variable whose value, if set, is applied to every input expression.

Set `Timing` as the `$Pre` function, stores the elapsed time in a variable, stores just the result in `Out[$Line]` and print a formatted version showing the elapsed time

```
>> $Pre := (Print["[Processing input...]"];#1)&

>> $Post := (Print["[Storing result...]"]; #1)&
   [Processing input...]
   [Storing result...]

>> $PrePrint := (Print["The result is:"]; {TimeUsed[], #1})&
   [Processing input...]
   [Storing result...]

>> 2 + 2
   [Processing input...]
   [Storing result...]
   The result is:
   {193.065, 4}

>> $Pre = .; $Post = .; $PrePrint = .; $ElapsedTime = .;
   [Processing input...]

>> 2 + 2
   4
```

56.5. $PrePrint

WMA

> `$PrePrint`
>
> is a global variable whose value, if set, is applied to every output expression before it is printed.

56.6. $PreRead

WMA

$PreRead
: is a global variable whose value, if set, is applied to the text or box form of every input expression before it is fed to the parser.

(Not implemented yet)

56.7. $SyntaxHandler

WMA

`$SyntaxHandler`
: is a global variable whose value, if set, is applied to any input string that is found to contain a syntax error.

(Not implemented yet)

56.8. In

WMA

`In[`*k*`]`
: gives the *k*-th line of input.

```
>> x = 1
```
1

```
>> x = x + 1
```
2

```
>> Do[In[2], {3}]
```

```
>> x
```
5

```
>> In[-1]
```
5

```
>> Definition[In]
```

$$\begin{array}{c} \text{Attributes}\,[\text{In}] = \{\text{Listable}, \text{Protected}\} \\ \text{In}\,[6] = \text{Definition}\,[\text{In}] \\ \text{In}\,[5] = \text{In}\,[-1] \\ \text{In}\,[4] = x \\ \text{In}\,[3] = \text{Do}\left[\text{In}\,[2], \{3\}\right] \\ \text{In}\,[2] = x = x + 1 \\ \text{In}\,[1] = x = 1 \end{array}$$

57. Tracing and Profiling

The `Trace` builtins provide a Mathics3-oriented trace of what is getting evaluated and where the time is spent in evaluation.

With this, it may be possible for both users and implementers to follow how Mathics3 arrives at its results, or guide how to speed up expression evaluation.

Python CProfile profiling is available via `PythonCProfileEvaluation`.

Contents

57.1. $TraceBuiltins

`$TraceBuiltins`
A Boolean Built-in variable when True collects function evaluation statistics.

Setting this variable True will enable statistics collection for Built-in functions that are evaluated. In contrast to `TraceBuiltins[]` statistics are accumulated and over several inputs,and are not shown after each input is evaluated.

By default, this setting is False.

```
>> $TraceBuiltins = True
   True
```

Tracing is enabled, so the expressions entered and evaluated will have statistics collected for the evaluations.

```
>> x
   x
```

To print the statistics collected, use `PrintTrace[]`:

```
>>  PrintTrace[]
    count ms Builtin name
    274  158 CompoundExpression
    230 0 Rule_
    182 0 List
    162 59 SetDelayed
    152 16 Set
    148 17 MPMathFunction
    92 8 Unprotect
    92 0 ClearAttributes
    88 7 Protect
    88 0 SetAttributes
    66 10 Power
    48 0 RuleDelayed
    44 0 Function
    38 0 Path
    38 0 RegisterImport
    38 1 Times
    34 0 Append
    34 35 Block
    34 0 Length
    34 0 SameQ
    34 26 If
    34 0 Last
    34 0 Most
    30 0 RegisterExport
    22 0 DirectedInfinity
    18 0 I_
    4 0 Map
    3 0 MakeBoxes
    2 3 TeXForm
    2 0 RowBox
    1 0 TraceBuiltinsVariable

    1 0 PrintTrace
```

To clear statistics collected use `ClearTrace[]`:

```
>>  ClearTrace[]
```

`$TraceBuiltins` cannot be set to a non-boolean value.

```
>>  $TraceBuiltins = x
    x should be True or False.
```

x

57.2. $TraceEvaluation

`$TraceEvaluation`
: A Boolean variable which when set True traces Expression evaluation calls and returns.

```
>>  $TraceEvaluation = True
```

True

```
>>  a + a
```

$2a$

Setting it to `False` again recovers the normal behaviour:

```
>>  $TraceEvaluation = False
```

False

```
>>  $TraceEvaluation
```

False

```
>>  a + a
```

$2a$

`$TraceEvaluation` cannot be set to a non-boolean value.

```
>> $TraceEvaluation = x
   x should be True or False.
   x
```

57.3. $TrackLocations

`$TrackLocations`
: specifies whether we should track source-text location information during evaluation. This can be helpful in debugging when there is a failure.

57.4. ClearTrace

`ClearTrace[]`
: Clear the statistics collected for Built-in Functions

First, set up Builtin-function tracing:

```
>> $TraceBuiltins = True
   True
```

Dump Builtin-Function statistics gathered in running that assignment:

```
>> PrintTrace[]
   count ms Builtin name
   137 76 CompoundExpression
   115 0 Rule_
   91 0 List
   81 25 SetDelayed
   76 8 Set
   74 5 MPMathFunction
   46 4 Unprotect
   46 0 ClearAttributes
   44 3 Protect
   44 0 SetAttributes
   33 4 Power
   24 0 RuleDelayed
   22 0 Function
   19 0 Path
   19 0 RegisterImport
   19 0 Times
   17 0 Append
   17 15 Block
   17 0 Length
   17 0 SameQ
   17 11 If
   17 0 Last
   17 0 Most
   15 0 RegisterExport
   11 0 DirectedInfinity
   9 0 I_
   2 0 Map
   1 0 TraceBuiltinsVariable
   1 1 TeXForm
   1 0 MakeBoxes
   1 0 RowBox

   1 0 PrintTrace
```

```
>> ClearTrace[]
```

57.5. PrintTrace

`PrintTrace[]`
: Print statistics collected for Built-in Functions

Sort Options:

- count
- name
- time

Note that in a browser the information only appears in a console.

Note: before `$TraceBuiltins` is set to `True`, `PrintTrace[]` will print an empty list.

```
>> PrintTrace[]
count ms Builtin name
28 0 MakeBoxes
2 19 PrintTrace
2 18 TableForm
2 0 DirectedInfinity
2 0 TraceBuiltinsVariable

1 0 Rule_
```

```
>> $TraceBuiltins = True
True
```

```
>> PrintTrace[SortBy -> "time"]
count ms Builtin name
137 74 CompoundExpression
81 25 SetDelayed
3 19 PrintTrace
2 18 TableForm
17 16 Block
17 11 If
76 8 Set
74 5 MPMathFunction
33 4 Power
46 4 Unprotect
44 3 Protect
1 1 TeXForm
19 0 Times
46 0 ClearAttributes
13 0 DirectedInfinity
91 0 List
44 0 SetAttributes
19 0 Path
17 0 Append
17 0 Most
22 0 Function
117 0 Rule_
19 0 RegisterImport
29 0 MakeBoxes
2 0 Map
15 0 RegisterExport
9 0 I_
17 0 Length
17 0 Last
17 0 SameQ
24 0 RuleDelayed
1 0 RowBox

2 0 TraceBuiltinsVariable
```

57.6. PythonCProfileEvaluation

Python

PythonProfileEvaluation[*expr*]
: profile *expr* with the Python's cProfiler.

57.7. TraceBuiltins

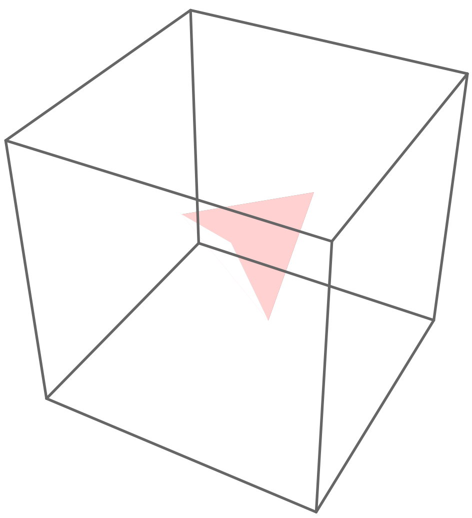
TraceBuiltins[*expr*]
: Evaluate *expr* and then print a list of the Built-in Functions called in evaluating *expr* along with the number of times is each called, and combined elapsed time in milliseconds spent in each.

Sort Options:

- count
- name
- time

```
>> TraceBuiltins[Graphics3D[Tetrahedron[]]]
count ms Builtin name
137 75 CompoundExpression
115 0 Rule_
91 0 List
81 26 SetDelayed
76 7 Set
74 5 MPMathFunction
46 4 Unprotect
46 0 ClearAttributes
44 3 Protect
44 0 SetAttributes
33 5 Power
24 0 RuleDelayed
22 0 Function
19 0 Path
19 0 RegisterImport
19 0 Times
17 0 Append
17 16 Block
17 0 Length
17 0 SameQ
17 12 If
17 0 Last
17 0 Most
15 0 RegisterExport
11 0 DirectedInfinity
9 0 I_

2 0 Map
```



By default, the output is sorted by the name:

```
>> TraceBuiltins[Times[x, x]]
count ms Builtin name
137 77 CompoundExpression
115 0 Rule_
91 0 List
81 27 SetDelayed
76 8 Set
74 6 MPMathFunction
46 4 Unprotect
46 0 ClearAttributes
44 3 Protect
44 0 SetAttributes
33 4 Power
24 0 RuleDelayed
22 0 Function
19 0 Path
19 0 RegisterImport
19 0 Times
17 0 Append
17 15 Block
17 0 Length
17 0 SameQ
17 11 If
17 0 Last
17 0 Most
15 0 RegisterExport
11 0 DirectedInfinity
9 0 I_

2 0 Map
```

x^2

By default, the output is sorted by the number of calls of the builtin from highest to lowest:

```
>> TraceBuiltins[Times[x, x], SortBy->"count"]
count ms Builtin name
137 80 CompoundExpression
115 0 Rule_
91 0 List
81 26 SetDelayed
76 8 Set
74 5 MPMathFunction
46 4 Unprotect
46 0 ClearAttributes
44 3 Protect
44 0 SetAttributes
33 4 Power
24 0 RuleDelayed
22 0 Function
19 0 Path
19 0 RegisterImport
19 0 Times
17 0 Append
17 16 Block
17 0 Length
17 0 SameQ
17 11 If
17 0 Last
17 0 Most
15 0 RegisterExport
11 0 DirectedInfinity
9 0 I_

2 0 Map
```

x^2

You can have results ordered by name, or time.

Trace an expression and list the result by time from highest to lowest.

```
>> TraceBuiltins[Times[x, x], SortBy->"time"]
```

```
count ms Builtin name
137 78 CompoundExpression
81 29 SetDelayed
17 16 Block
17 11 If
76 7 Set
74 5 MPMathFunction
33 4 Power
46 4 Unprotect
44 3 Protect
19 0 Times
46 0 ClearAttributes
44 0 SetAttributes
91 0 List
11 0 DirectedInfinity
19 0 Path
17 0 Append
17 0 Most
19 0 RegisterImport
115 0 Rule_
22 0 Function
2 0 Map
9 0 I_
15 0 RegisterExport
17 0 Length
17 0 SameQ
17 0 Last

24 0 RuleDelayed
```

x^2

57.8. TraceEvaluation

`TraceEvaluation[`*expr*, *options*`]`
: Evaluate *expr* and print each step of the evaluation.

Options adjust output and filtering behavior

`ShowTimeBySteps`
: Print the elapsed time before an evaluation occurs. default is `False`.

`ShowEvaluation`
: Show evaluation calls and returns. The default is `True`.

`ShowRewrite`
: Show the effect of rewrite rules. The default is `True`.

Note: It does not make sense to set *both* `ShowRewrite` and `ShowEvaluation` to `False`.

```
>> TraceEvaluation[(x + x)^2]
```

$4x^2$

```
>> TraceEvaluation[(x + x)^2, ShowTimeBySteps->True]
```

$4x^2$

Now consider this function which consists of a function call that involves a rewrite rule:

```
>> TraceEvaluation[BesselK[0, 0]]
```

$-\infty$

Sometimes, `TraceEvaluation` traces can get quite large. To reduce the size, it may be helpful to filter on either the evaluations or the replacement rules.

To see just the evaluations and return values, but not rewrite that occurs:

```
>>  TraceEvaluation[BesselK[0, 0], ShowRewrite-> False]
```

$-\infty$

To see just the rewrite that occurs, which tends to summarizes even more:

```
>>  TraceEvaluation[BesselK[0, 0], ShowEvaluation-> False]
```

$-\infty$

58. Units and Quantities

Contents

58.1. KnownUnitQ

WMA link

`KnownUnitQ[`*unit*`]`
  returns True if *unit* is a canonical unit, and False otherwise.

```
>> KnownUnitQ["Feet"]
```
True

```
>> KnownUnitQ["Foo"]
```
False

```
>> KnownUnitQ["meter"^2/"second"]
```
True

58.2. Quantity

WMA link

`Quantity[`*magnitude*, *unit*`]`
  represents a quantity with size *magnitude* and unit specified by *unit*.
`Quantity[`*unit*`]`
  assumes the magnitude of the specified *unit* to be 1.

```
>> Quantity["Kilogram"]
```
1 kilogram

```
>> Quantity[10, "Meters"]
```
10 meter

If the first argument is an array, then the unit is distributed on each element

```
>> Quantity[{10, 20}, "Meters"]
```
$\{10\ \text{meter}, 20\ \text{meter}\}$

If the second argument is a number, then the expression is evaluated to the product of the magnitude and that number

```
>> Quantity[2, 3/2]
```
3

Notice that units are specified as Strings. If the unit is not a Symbol or a Number, the expression is not interpreted as a Quantity object:

```
>> QuantityQ[Quantity[2, Second]]
Unable to interpret unit specification Second.
```
False

Quantities can be multiplied and raised to integer powers:

```
>> Quantity[3, "centimeter"] / Quantity[2, "second"]^2
```
$\frac{3}{4}\ \frac{\text{centimeter}}{\text{second}^2}$

Quantities of the same kind can be added:

```
>> Quantity[6, "meter"] + Quantity[3, "centimeter"]
```
603 centimeter

Quantities of different kind can not:

```
>> Quantity[6, "meter"] + Quantity[3, "second"]
second and meter are incompatible units.
```
3 second + 6 meter

See also 'QuantityQ' 58.4.

58.3. QuantityMagnitude

WMA link

`QuantityMagnitude[`*quantity*`]`
gives the amount of the specified *quantity*.
`QuantityMagnitude[`*quantity, unit*`]`
gives the value corresponding to *quantity* when converted to *unit*.

```
>> QuantityMagnitude[Quantity["Kilogram"]]
```
1

```
>> QuantityMagnitude[Quantity[10, "Meters"]]
```
10

```
>> QuantityMagnitude[Quantity[{10,20}, "Meters"]]
```
{10, 20}

58.4. QuantityQ

WMA link

`QuantityQ[`*expr*`]`
: return `True` if *expr* is a valid Quantity 58.2 with valid arguments, and `False` otherwise.

```
>> QuantityQ[Quantity[3, "Meters"]]
```
True

```
>> QuantityQ[Quantity[3, "Maters"]]
Unable to interpret unit specification Maters.
```
False

58.5. QuantityUnit

WMA link

`QuantityUnit[`*quantity*`]`
: returns the unit associated with the specified *quantity*.

```
>> QuantityUnit[Quantity["Kilogram"]]
```
kilogram

```
>> QuantityUnit[Quantity[10, "Meters"]]
```
meter

```
>> QuantityUnit[Quantity[{10,20}, "Meters"]]
```
{meter, meter}

58.6. UnitConvert

WMA link

```
UnitConvert[$quantity$, $targetunit$]
```
converts the specified *quantity* to the specified *targetunit*.

```
UnitConvert[quantity]
```
converts the specified *quantity* to its “SIBase” units.

Convert from miles to kilometers:

```
>> UnitConvert[Quantity[5.2, "miles"], "kilometers"]
```
8.36859 kilometer

Convert a Quantity object to the appropriate SI base units:

```
>> UnitConvert[Quantity[3.8, "Pounds"]]
```
1.72365 kilogram

Part III.

Mathics3 Modules

59. Graphs - Vertices and Edges

A Graph is a tuple of a set of Nodes and Edges.

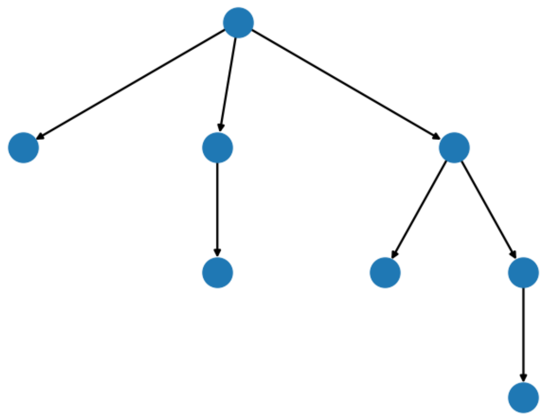
Mathics3 Module that provides functions and variables for working with graphs.

Examples:

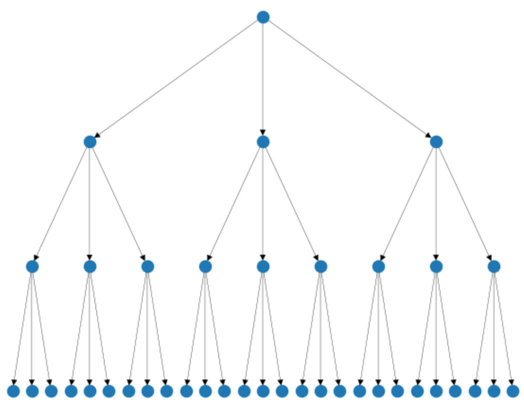
```
>>  LoadModule["pymathics.graph"]
```

pymathics.graph

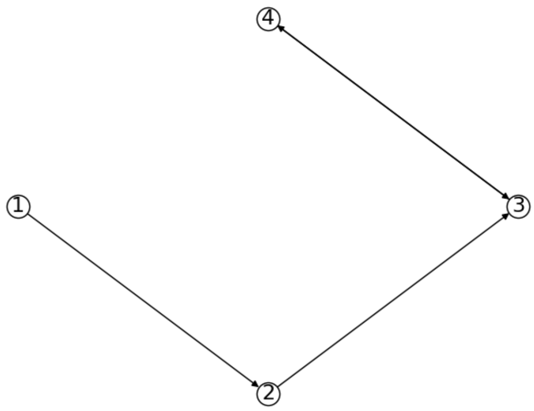
```
>>  BinomialTree[3, DirectedEdges->True]
```



```
>>  BalancedTree[3, 3]
```



```
>> g = Graph[{1 -> 2, 2 -> 3, 3 <-> 4}, VertexLabels->True]
```




```
>> ConnectedComponents[g]
```

$\{\{3,4\},\{2\},\{1\}\}$

```
>> WeaklyConnectedComponents[g]
```

$\{\{1,2,3,4\}\}$

```
>> GraphDistance[g, 1, 4]
```

3

```
>> GraphDistance[g, 3, 2]
```

∞

NetworkX does the heavy lifting here.

Contents

59.1. Centralities

Centralities

Routines to evaluate centralities of a graph.

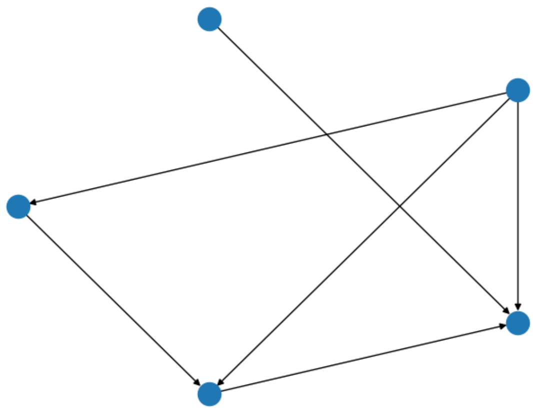
In graph theory and network analysis, the centrality is a ranking between pairs of node according some metric.

59.1.1. BetweennessCentrality

Betweenness centrality (NetworkX, WMA)

`BetweennessCentrality[`*g*`]`
gives a list of betweenness centralities for the vertices in a `Graph` or a list of edges *g*.

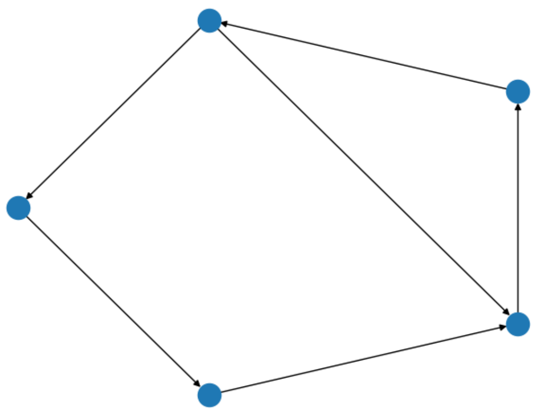
```
>> g = Graph[{a -> b, b -> c, d -> c, d -> a, e -> c, d -> b}]
```



```
>> BetweennessCentrality[g]
```

{0., 1., 0., 0., 0.}

```
>> g = Graph[{a -> b, b -> c, c -> d, d -> e, e -> c, e -> a}]
```



```
>> BetweennessCentrality[g]
```

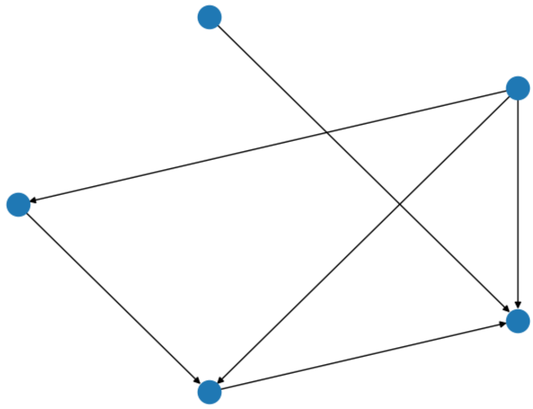
{3., 3., 6., 6., 6.}

59.1.2. ClosenessCentrality

Betweenness centrality (NetworkX, WMA)

`ClosenessCentrality[`*g*`]`
: gives a list of closeness centralities for the vertices in a `Graph` or a list of edges *g*.

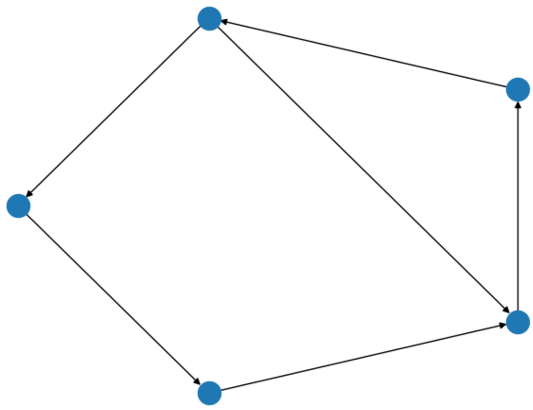
```
>> g = Graph[{a -> b, b -> c, d -> c, d -> a, e -> c, d -> b}]
```



```
>> ClosenessCentrality[g]
```

$\{0.666667, 1., 0., 1., 1.\}$

```
>> g = Graph[{a -> b, b -> c, c -> d, d -> e, e -> c, e -> a}]
```



```
>> ClosenessCentrality[g]
```

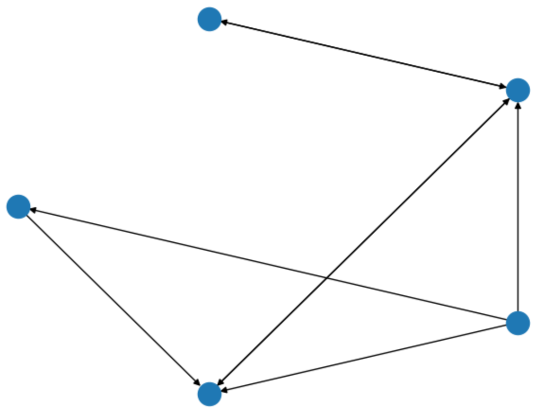
$\{0.4, 0.4, 0.4, 0.5, 0.666667\}$

59.1.3. DegreeCentrality

Degree centrality (NetworkX, WMA)

`DegreeCentrality[`*g*`]`
: gives a list of degree centralities for the vertices in a `Graph` or a list of edges *g*.

```
>> g = Graph[{a -> b, b <-> c, d -> c, d -> a, e <-> c, d -> b}]
```



```
>> DegreeCentrality[g]
```

$\{2, 4, 3, 5, 2\}$

```
>> DegreeCentrality[g, "In"]
```

$\{1, 3, 0, 3, 1\}$

```
>> DegreeCentrality[g, "Out"]
```

$\{1, 1, 3, 2, 1\}$

59.1.4. EigenvectorCentrality

Eigenvector Centrality (NetworkX,WMA)

`EigenvectorCentrality[`*g*`]`
: gives a list of eigenvector centralities for the vertices in the graph *g*.

`EigenvectorCentrality[`*g*, "In"`]`
: gives a list of eigenvector in-centralities for the vertices in the graph *g*.

`EigenvectorCentrality[`*g*, "Out"`]`
: gives a list of eigenvector out-centralities for the vertices in the graph *g*.

```
>> g = Graph[{a -> b, b -> c, c -> d, d -> e, e -> c, e -> a}];
   EigenvectorCentrality[g, "In"]
```

$\{0.16238, 0.136013, 0.276307, 0.23144, 0.193859\}$

```
>> EigenvectorCentrality[g, "Out"]
```

$\{0.136013, 0.16238, 0.193859, 0.23144, 0.276307\}$

```
>> g = Graph[{a <-> b, b <-> c, c <-> d, d <-> e, e <-> c, e <-> a}];
   EigenvectorCentrality[g]
```

$\{0.162435, 0.162435, 0.240597, 0.193937, 0.240597\}$

```
>> g = Graph[{a <-> b, b <-> c, a <-> c, d <-> e, e <-> f, f <-> d, e
   <-> d}]; EigenvectorCentrality[g]
```

$\{0.166667, 0.166667, 0.166667, 0.183013, 0.183013, 0.133975\}$

```
>> g = Graph[{a -> b, b -> c, c -> d, b -> e, a -> e, c -> a}];
   EigenvectorCentrality[g]
```

$\{0.333333, 0.333333, 0.333333, 0., 0.\}$

59.1.5. HITSCentrality

NetworkX, WMA

`HITSCentrality`[g]
  gives a list of authority and hub centralities for the vertices in the graph g.

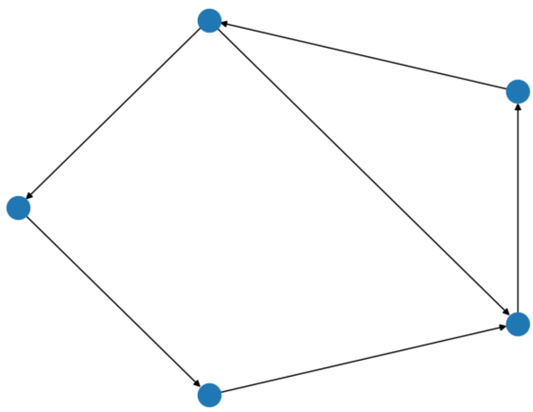
59.1.6. KatzCentrality

Katz Centrality (NetworkX, WMA)

`KatzCentrality`[g, $alpha$]
  gives a list of Katz centralities for the vertices in the graph g and weight $alpha$.

`KatzCentrality`[g, $alpha$, $beta$]
  gives a list of Katz centralities for the vertices in the graph g and weight $alpha$ and initial centralities $beta$.

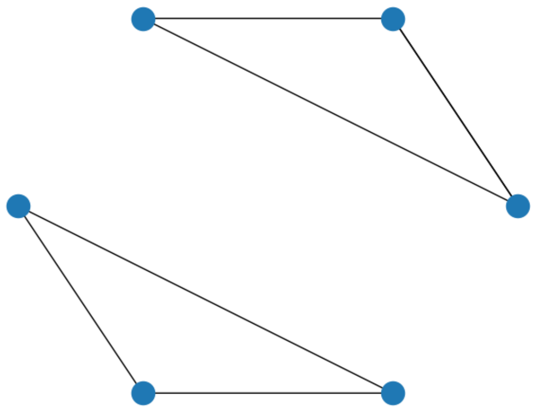
```
>> g = Graph[{a -> b, b -> c, c -> d, d -> e, e -> c, e -> a}]
```



```
>> KatzCentrality[g, 0.2]
```

$\{1.25202, 1.2504, 1.5021, 1.30042, 1.26008\}$

```
>> g = Graph[{a <-> b, b <-> c, a <-> c, d <-> e, e <-> f, f <-> d, e
   <-> d}]
```



```
>> KatzCentrality[g, 0.1]
```

$\{1.25, 1.25, 1.25, 1.41026, 1.41026, 1.28205\}$

59.1.7. PageRankCentrality

Pagerank Centrality (NetworkX, WMA)

`PageRankCentrality`[*g*, *alpha*]
  gives a list of page rank centralities for the vertices in the graph *g* and weight *alpha*.
`PageRankCentrality`[*g*, *alpha*, *beta*]
  gives a list of page rank centralities for the vertices in the graph *g* and weight *alpha* and initial centralities *beta*.

```
>> g = Graph[{a -> d, b -> c, d -> c, d -> a, e -> c, d -> c}];
   PageRankCentrality[g, 0.2]
```

$\{0.184502, 0.207565, 0.170664, 0.266605, 0.170664\}$

59.2. Core routines for working with Graphs.

59.2.1. AdjacencyList

Adjacency list (NetworkX, WMA)

`AdjacencyList`[*graph*, *v*]
  gives a list of vertices adjacent to *v* in a `Graph` or a list of edges *g*.
`AdjacencyList`[*graph*, *pattern*]
  gives a list of vertices adjacent to vertices matching *pattern*.

```
>> AdjacencyList[{1 -> 2, 2 -> 3}, 3]
```
$\{2\}$

```
>> AdjacencyList[{1 -> 2, 2 -> 3}, _?EvenQ]
```
$\{1,3\}$

```
>> AdjacencyList[{x -> 2, x -> 3, x -> 4, 2 -> 10, 2 -> 11, 4 -> 20, 4
   -> 21, 10 -> 100}, 10, 2]
```
$\{2, 11, 100, x\}$

59.2.2. DirectedEdge ($\rightarrow$)

Edge of a Directed graph (WMA)

`DirectedEdge`[u, v]
  create a directed edge from u to v.

```
>> DirectedEdge[x, y, z]
```
$x \rightarrow y \rightarrow z$

```
>> a \[DirectedEdge] b
```
$a \rightarrow b$

59.2.3. EdgeConnectivity

Edge connectivity (NetworkX, WMA)

`EdgeConnectivity`[g]
  gives the edge connectivity of the graph g.

```
>> EdgeConnectivity[{1 <-> 2, 2 <-> 3}]
```
1

```
>> EdgeConnectivity[{1 -> 2, 2 -> 3}]
```
0

```
>> EdgeConnectivity[{1 -> 2, 2 -> 3, 3 -> 1}]
```
1

```
>> EdgeConnectivity[{1 <-> 2, 2 <-> 3, 1 <-> 3}]
```
2

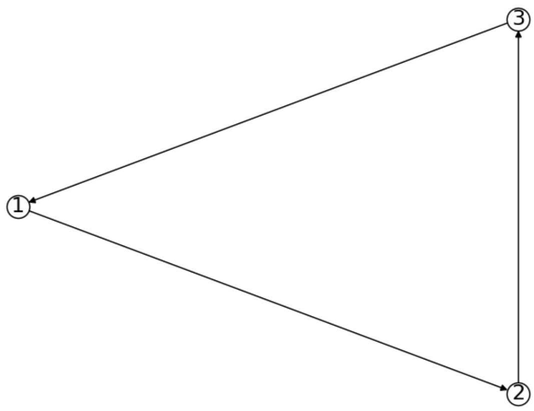
```
>> EdgeConnectivity[{1 <-> 2, 3 <-> 4}]
```
0

59.2.4. EdgeDelete

Delete an Edge (WMA)

`EdgeDelete`[*g*, *edge*]
    remove the edge *edge*.

>> `g = Graph[{1 -> 2, 2 -> 3, 3 -> 1}, VertexLabels->True]`



>> `EdgeList[EdgeDelete[g, 2 -> 3]]`

{{1,2},{3,1}}

59.2.5. EdgeIndex

WMA link

`EdgeIndex[graph, edge]`
    gives the position of the `edge` in the list of edges associated to the graph.

59.2.6. EdgeList

WMA link

`EdgeList`[*g*]
    gives the list of edges that defines *g*

59.2.7. EdgeRules

WMA link

`EdgeRules`[*g*]
gives the list of edge rules for the graph *g*.

59.2.8. FindShortestPath

Shortest path problem (NetworkX, WMA)

`FindShortestPath`[*g*, *src*, *tgt*]
List the vertices in the shortest path connecting the source *src* with the target *tgt* in the graph *g*.

```
>> FindShortestPath[{1 <-> 2, 2 <-> 3, 3 <-> 4, 2 <-> 4, 4 -> 5}, 1, 5]
```
$\{1, 2, 4, 5\}$

```
>> FindShortestPath[{1 <-> 2, 2 <-> 3, 3 <-> 4, 4 -> 2, 4 -> 5}, 1, 5]
```
$\{1, 2, 3, 4, 5\}$

```
>> FindShortestPath[{1 <-> 2, 2 <-> 3, 4 -> 3, 4 -> 2, 4 -> 5}, 1, 5]
```
$\{\}$

```
>> g = Graph[{1 -> 2, 2 -> 3, 1 -> 3}, EdgeWeight -> {0.5, a, 3}];
```

59.2.9. FindVertexCut

Minimum cut (NetworkX, WMA)

`FindVertexCut`[*g*]
finds a set of vertices of minimum cardinality that, if removed, renders *g* disconnected.
`FindVertexCut`[*g*, *s*, *t*]
finds a vertex cut that disconnects all paths from *s* to *t*.

```
>> g = Graph[{1 -> 2, 2 -> 3}]; FindVertexCut[g]
```
$\{\}$

```
>> g = Graph[{1 <-> 2, 2 <-> 3}]; FindVertexCut[g]
```
$\{2\}$

```
>> g = Graph[{1 <-> x, x <-> 2, 1 <-> y, y <-> 2, x <-> y}];
   FindVertexCut[g]
```
$\{x, y\}$

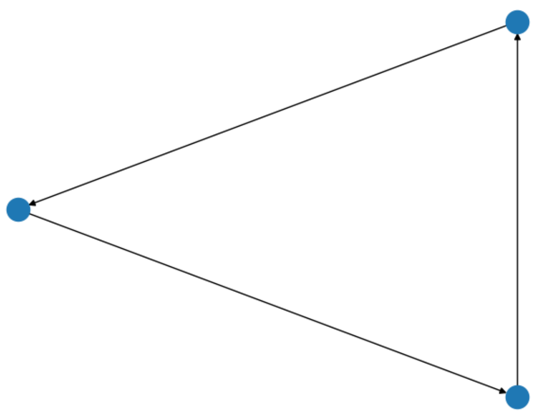
59.2.10. Graph

Graph (WMA)

`Graph[{`$e1$`,e2, ...}]`
  returns a graph with edges e_j.

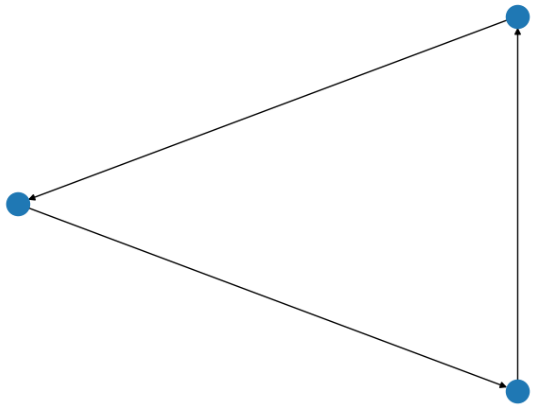
`Graph[{v1, v2, ...}, {`$e1$`,e2, ...}]`
  returns a graph with vertices v_i and edges e_j.

```
>> Graph[{1->2, 2->3, 3->1}]
```



#» Graph[{1->2, 2->3, 3->1}, EdgeStyle -> {Red, Blue, Green}] # = -Graph-

```
>> Graph[{1->2, Property[2->3, EdgeStyle -> Thick], 3->1}]
```



» Graph[{1->2, 2->3, 3->1}, VertexStyle -> {1 -> Green, 3 -> Blue}] # = -Graph-

```
>> Graph[x]
```

Graph[x]

```
>> Graph[{1}]
```
Graph $[\{1\}]$

```
>> Graph[{{1 -> 2}}]
```
Graph $[\{\{1- > 2\}\}]$

59.2.11. HighlightGraph

WMA link

`HighlightGraph`[*graph*, *what*]
    highlight in *graph* the elements enumerated in *what* by adding style marks.

59.2.12. Property

WMA link

`Property`[*item*, {*name*, *val*}]
    associate a property called *name* with value *val* to *item*.

59.2.13. PropertyValue

WMA link

`PropertyValue`[{*obj*, *item*}, *name*]
    gives the value of a property associated with the name *name* for *item* in the object *obj*.

```
>> g = Graph[{a <-> b, Property[b <-> c, SomeKey -> 123]}];
```

```
>> PropertyValue[{g, b <-> c}, SomeKey]
```
123

```
>> PropertyValue[{g, b <-> c}, SomeUnknownKey]
```
$Failed

59.2.14. UndirectedEdge (↔)

Edge of a Undirected graph WMA link

UndirectedEdge[u, v]
    create an undirected edge between u and v.

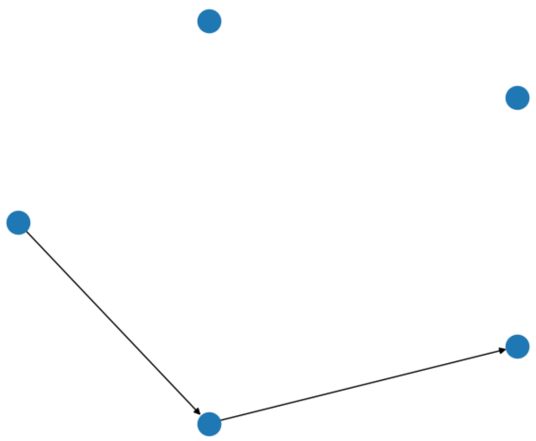
```
>> a <-> b
```
$a \leftrightarrow b$

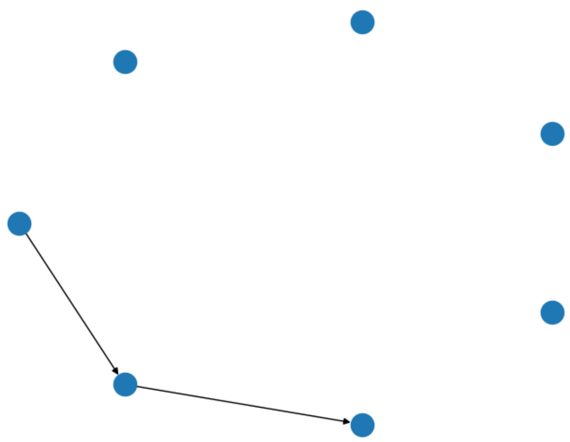
59.2.15. VertexAdd

WMA link

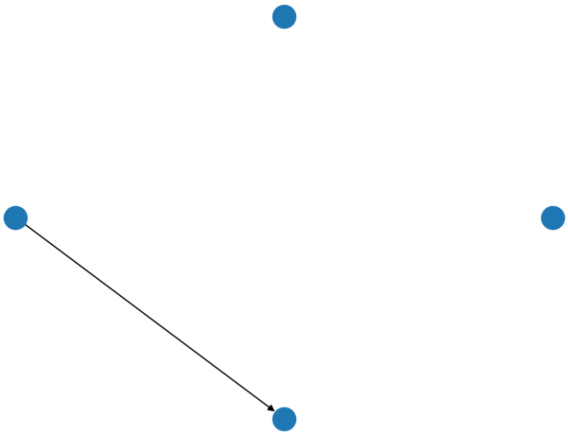
VertexAdd[g, ver]
    create a new graph from g, by adding the vertex ver.

```
>> g1 = Graph[{1 -> 2, 2 -> 3}];
```

```
>> g2 = VertexAdd[g1, 4]
```


```
>> g3 = VertexAdd[g2, {5, 10}]
```


```
>> VertexAdd[{a -> b}, c]
```



59.2.16. VertexConnectivity

WMA link

`VertexConnectivity[`*g*`]`
    gives the vertex connectivity of the graph *g*.

```
>> VertexConnectivity[{1 <-> 2, 2 <-> 3}]
   1
>> VertexConnectivity[{1 -> 2, 2 -> 3}]
   0
>> VertexConnectivity[{1 -> 2, 2 -> 3, 3 -> 1}]
   1
>> VertexConnectivity[{1 <-> 2, 2 <-> 3, 1 <-> 3}]
   2
>> VertexConnectivity[{1 <-> 2, 3 <-> 4}]
   0
```

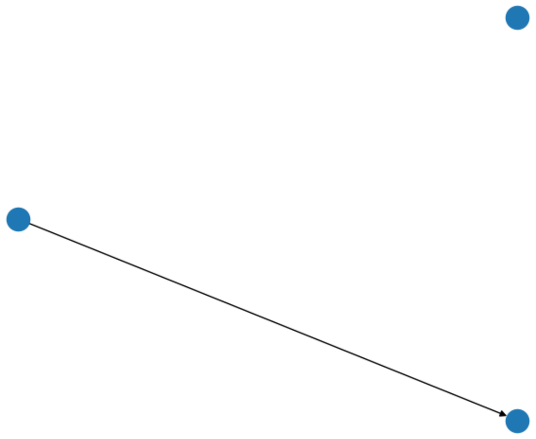
59.2.17. VertexDelete

WMA link

`VertexDelete[`*g*, *vert*`]`
    remove the vertex *vert* and their associated edges.

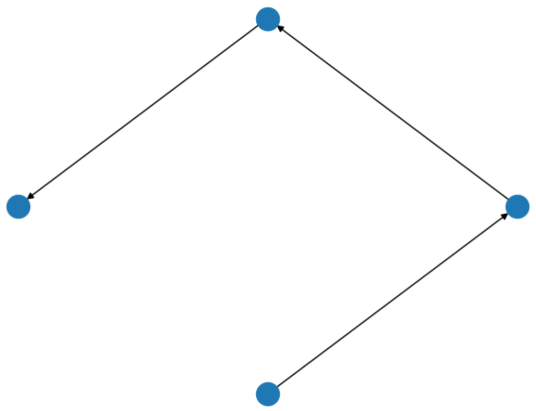
```
>> g1 = Graph[{1 -> 2, 2 -> 3, 3 -> 4}];
```

```
>>  VertexDelete[g1, 3]
```



```
>>  VertexDelete[{a -> b, b -> c, c -> d, d -> a}, {a, c}]
```

```
>>  VertexDelete[{1 -> 2, 2 -> 3, 3 -> 4, 4 -> 6, 6 -> 8, 8 -> 2}, _?OddQ
    ]
```



59.2.18. VertexIndex

WMA link

`VertexIndex[g, v]`
: gives the integer index of the vertex `v` in the graph `g`.

```
>> a=.;
```

```
>> VertexIndex[{c <-> d, d <-> a}, a]
```

3

59.2.19. VertexList

WMA link

`VertexList[`*edgelist*`]`
: list the vertices from a list of directed edges.

```
>> a=.;
```

```
>> VertexList[{1 -> 2, 2 -> 3}]
```

$\{1,2,3\}$

```
>> VertexList[{a -> c, c -> b}]
```

$\{a,c,b\}$

```
>> VertexList[{a -> c, 5 -> b}, _Integer -> 10]
```

$\{10\}$

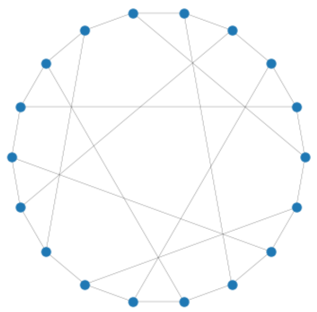
59.3. Curated Graphs

59.3.1. GraphData

WMA link

`GraphData[`*name*`]`
: Returns a graph with the specified name.

>> `GraphData["PappusGraph"]`



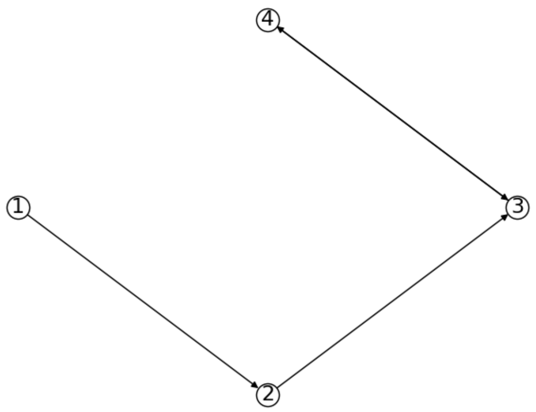
59.4. Graph Components and Connectivity

59.4.1. ConnectedComponents

Strongly connected components (NetworkX, WMA)

`ConnectedComponents[`*g*`]`
: gives the connected components of the graph *g*.

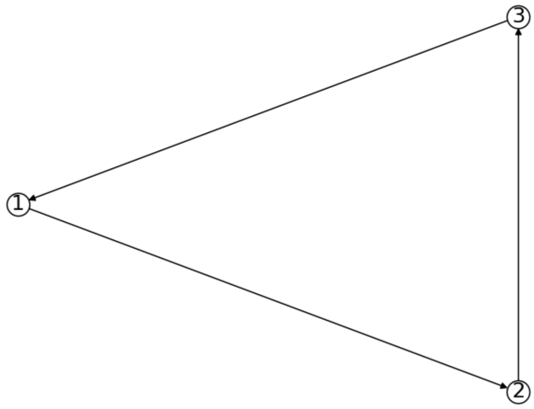
>> `g = Graph[{1 -> 2, 2 -> 3, 3 <-> 4}, VertexLabels->True]`



>> `ConnectedComponents[g]`

$\{\{3,4\},\{2\},\{1\}\}$

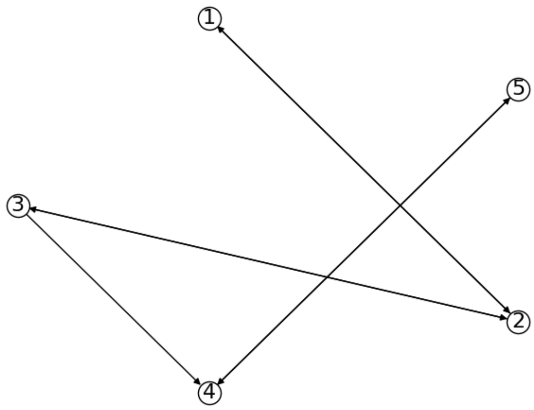
```
>> g = Graph[{1 -> 2, 2 -> 3, 3 -> 1}, VertexLabels->True]
```




```
>> ConnectedComponents[g]
```

$\{\{1,2,3\}\}$

```
>> g = Graph[{1 <-> 2, 2 <-> 3, 3 -> 4, 4 <-> 5}, VertexLabels->True]
```




```
>> ConnectedComponents[g]
```

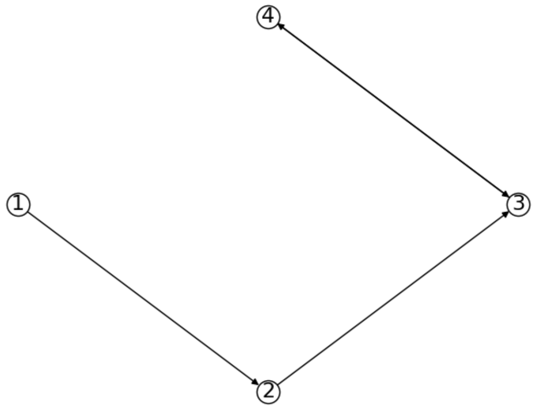
$\{\{4,5\},\{1,2,3\}\}$

59.4.2. WeaklyConnectedComponents

Weak components (NetworkX, WMA)

`WeaklyConnectedComponents[`*g*`]`
gives the weakly connected components of the graph *g*.

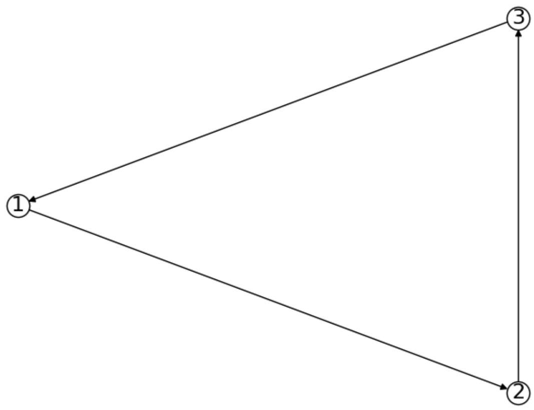
```
>> g = Graph[{1 -> 2, 2 -> 3, 3 <-> 4}, VertexLabels->True]
```




```
>> WeaklyConnectedComponents[g]
```

$\{\{1,2,3,4\}\}$

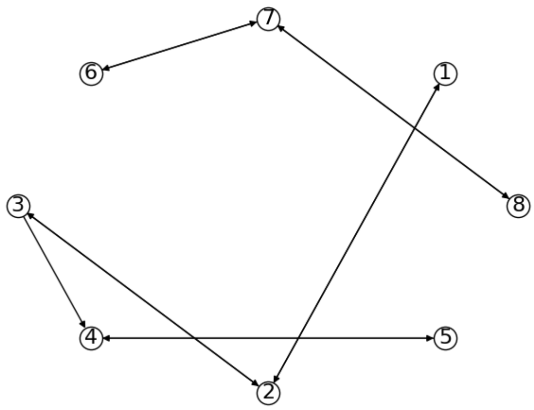
```
>> g = Graph[{1 -> 2, 2 -> 3, 3 -> 1}, VertexLabels->True]
```




```
>> WeaklyConnectedComponents[g]
```

$\{\{1,2,3\}\}$

```
>> g = Graph[{1 <-> 2, 2 <-> 3, 3 -> 4, 4 <-> 5, 6 <-> 7, 7 <-> 8},
   VertexLabels->True]
```




```
>> WeaklyConnectedComponents[g]
```

{{1,2,3,4,5},{6,7,8}}

59.5. Graph Measures and Metrics

Measures include basic measures, such as the number of vertices and edges, connectivity, degree measures, centrality, and so on.

59.5.1. EdgeCount

NetworkX, WMA

`EdgeCount[`*g*`]`
: returns a count of the number of edges in graph *g*.

`EdgeCount[`*g*, *patt*`]`
: returns the number of edges that match the pattern *patt*.

`EdgeCount[{`*v*`->`*w*`}, ...}, ...]`
: uses rules *v*`->`*w* to specify the graph *g*.

```
>> EdgeCount[{1 -> 2, 2 -> 3}]
```

2

59.5.2. GraphDistance

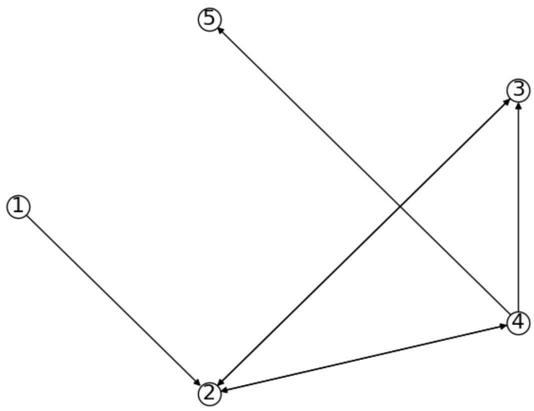
NetworkX, WMA

GraphDistance[g, s, t]
: returns the distance from source vertex s to target vertex t in the graph g.

GraphDistance[g, s]
: returns the distance from source vertex s to all vertices in the graph g.

GraphDistance[{v->w, ...}, ...]
: use rules v->w to specify the graph g.

```
>> g = Graph[{1 -> 2, 2 <-> 3, 4 -> 3, 2 <-> 4, 4 -> 5}, VertexLabels->
   True]
```




```
>> GraphDistance[g, 1, 5]
```
3

```
>> GraphDistance[g, 4, 2]
```
1

```
>> GraphDistance[g, 5, 4]
```
∞

```
>> GraphDistance[g, 5]
```
$\{\infty, \infty, \infty, \infty, 0\}$

```
>> GraphDistance[g, 3]
```
$\{\infty, 1, 2, 0, 3\}$

```
>> GraphDistance[g, 4]
```
$\{\infty, 1, 0, 1, 1\}$

59.5.3. VertexCount

NetworkX, WMA

`VertexCount`[*g*]
    returns a count of the number of vertices in graph *g*.
`VertexCount`[*g*, *patt*]
    returns the number of vertices that match the pattern *patt*.
`VertexCount`[{*v*->*w*}, ...}, ...]
    uses rules *v*->*w* to specify the graph *g*.

```
>> VertexCount[{1 -> 2, 2 -> 3}]
```
3

```
>> VertexCount[{1 -> x, x -> 3}, _Integer]
```
2

59.5.4. VertexDegree

NetworkX, WMA

`VertexDegree`[*g*]
    returns a list of the degrees of each of the vertices in graph *g*.
`EdgeCount`[*g*, *patt*]
    returns the number of edges that match the pattern *patt*.
`EdgeCount`[{*v*->*w*}, ...}, ...]
    uses rules *v*->*w* to specify the graph *g*.

```
>> VertexDegree[{1 <-> 2, 2 <-> 3, 2 <-> 4}]
```
$\{1, 3, 1, 1\}$

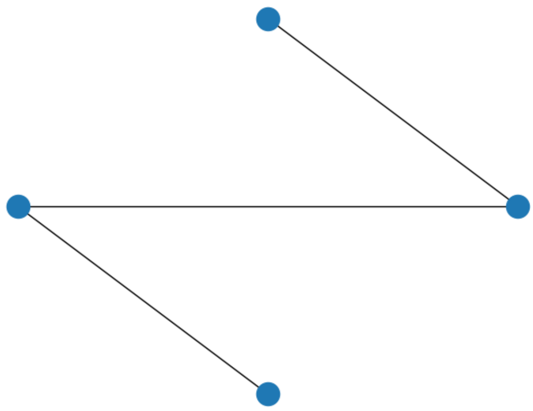
59.6. Graph Operations and Modifications

59.6.1. FindSpanningTree

Spanning Tree (NetworkX, WMA)

`FindSpanningTree`[*g*]
    finds a spanning tree of the graph *g*.

```
>> FindSpanningTree[CycleGraph[4]]
```



59.7. Graph Properties and Measurements

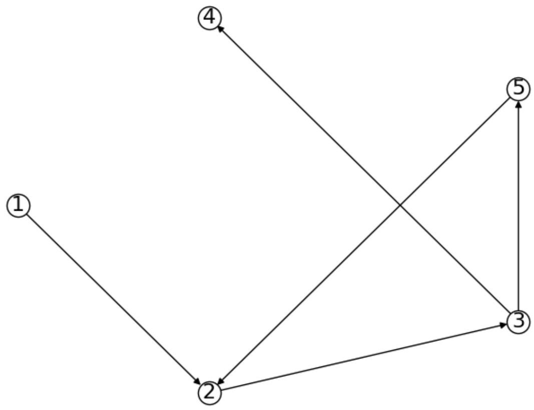
59.7.1. AcyclicGraphQ

Acyclic graph test (NetworkX, WMA)

`AcyclicGraphQ[`*graph*`]`
: check if *graph* is an acyclic graph.

Create a directed graph with a cycle in it:

```
>> g = Graph[{1 -> 2, 2 -> 3, 5 -> 2, 3 -> 4, 3 -> 5}, VertexLabels->
   True]
```




```
>> AcyclicGraphQ[g]
```

False

Remove a cycle edge:

```
>> g = EdgeDelete[g, 5 -> 2]; EdgeList[g]
```
{{1,2},{2,3},{3,4},{3,5}}

```
>> AcyclicGraphQ[g]
```
True

59.7.2. ConnectedGraphQ

Connected graph test (NetworkX, WMA)

`ConnectedGraphQ[`*graph*`]`
: check if *graph* is a connected graph.

```
>> g = Graph[{1 -> 2, 2 -> 3}]; ConnectedGraphQ[g]
```
False

```
>> g = Graph[{1 -> 2, 2 -> 3, 3 -> 1}]; ConnectedGraphQ[g]
```
True

```
>> g = Graph[{1 <-> 2, 2 <-> 3}]; ConnectedGraphQ[g]
```
True

```
>> g = Graph[{1 <-> 2, 2 <-> 3, 4 <-> 5}]; ConnectedGraphQ[g]
```
False

59.7.3. DirectedGraphQ

Directed graph test (NetworkX, WMA)

`DirectedGraphQ[`*graph*`]`
: True if *graph* is a `Graph` and all the edges are directed.

```
>> g = Graph[{1 -> 2, 2 -> 3}]; DirectedGraphQ[g]
```
True

```
>> g = Graph[{1 -> 2, 2 <-> 3}]; DirectedGraphQ[g]
```
False

59.7.4. GraphQ

WMA link

`GraphQ`[*graph*]
: True if *graph* is a `Graph`.

A graph with one one node and one self-looping edge:

```
>> GraphQ[{1 -> 2, 2 -> 3, 3 -> 1}]
```
True

```
>> GraphQ[{1, 2, 3}]
```
False

59.7.5. LoopFreeGraphQ

Loop-Free graph test (NetworkX, WMA)

`LoopFreeGraphQ`[*graph*]
: True if *graph* is a `Graph` and the edges do not close any loop.

```
>> g = Graph[{1 -> 2, 2 -> 3}]; LoopFreeGraphQ[g]
```
True

```
>> g = Graph[{1 -> 2, 2 -> 3, 1 -> 1}]; LoopFreeGraphQ[g]
```
False

59.7.6. MixedGraphQ

Mixed Graph test (WMA)

`MixedGraphQ`[*graph*]
: returns `True` if *graph* is a `Graph` with both directed and undirected edges, and `False` otherwise.

```
>> MixedGraphQ[Graph[{1 -> 2, 2 -> 3}]]
```
False

```
>> MixedGraphQ[Graph[{1 -> 2, 2 <-> 3}]]
```
True

```
>> MixedGraphQ[Graph[{}]]
```
False

```
>> MixedGraphQ["abc"]
```
False

59.7.7. MultigraphQ

Multigraph test (NetworkX, WMA)

`MultigraphQ[`*graph*`]`
: True if *graph* is a `Graph` and there vertices connected by more than one edge.

```
>> g = Graph[{1 -> 2, 2 -> 3}]; MultigraphQ[g]
```
False

```
>> g = Graph[{1 -> 2, 2 -> 3, 1 -> 2}]; MultigraphQ[g]
```
True

59.7.8. PathGraphQ

Path graph test (WMA)

`LoopFreeGraphQ[`*graph*`]`
: True if *graph* is a `Graph` and it becomes disconnected by removing a single edge.

```
>> PathGraphQ[Graph[{1 -> 2, 2 -> 3}]]
```
True

```
>> PathGraphQ[Graph[{1 -> 2, 2 <-> 3}]]
```
False

```
>> PathGraphQ[Graph[{1 -> 2, 3 -> 2}]]
```
False

```
>> PathGraphQ[Graph[{1 -> 2, 2 -> 3, 2 -> 4}]]
```
False

```
>> PathGraphQ[Graph[{1 -> 2, 3 -> 2, 2 -> 4}]]
```
False

```
>> PathGraphQ[Graph[{1 -> 2, 2 -> 3, 2 -> 3}]]
```
False

59.7.9. PlanarGraphQ

Planar Graph test (NetworkX, WMA)

`PlanarGraphQ[`*g*`]`
: Returns True if *g* is a planar graph and False otherwise.

```
>> PlanarGraphQ[CycleGraph[4]]
```
True

```
>> PlanarGraphQ[CompleteGraph[5]]
```
False

```
>> PlanarGraphQ["abc"]
Expected a graph at position 1 in PlanarGraphQ[abc].
```
False

59.7.10. SimpleGraphQ

Simple (not multigraph) graph test (WMA)

`SimpleGraphQ[`*graph*`]`
: True if *graph* is a `Graph`, loop-free and each pair of vertices are connected at most by a single edge.

```
>> g = Graph[{1 -> 2, 2 -> 3, 3 <-> 4}]; SimpleGraphQ[g]
```
True

```
>> g = Graph[{1 -> 2, 2 -> 3, 1 -> 1}]; SimpleGraphQ[g]
```
False

```
>> g = Graph[{1 -> 2, 2 -> 3, 1 -> 2}]; SimpleGraphQ[g]
```
False

59.8. Parametric Graphs

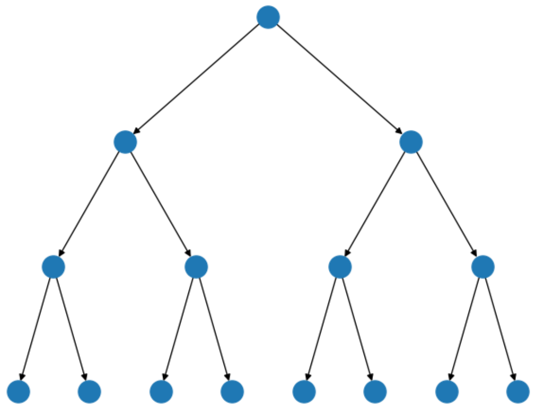
59.8.1. BalancedTree

WMA

`BalancedTree[`*r*, *h*`]`
: Returns the perfectly balanced *r*-ary tree of height *h*.

In this tree produced, all non-leaf nodes will have *r* children and the height of the path from root *r* to any leaf will be *h*.

```
>> BalancedTree[2, 3]
```

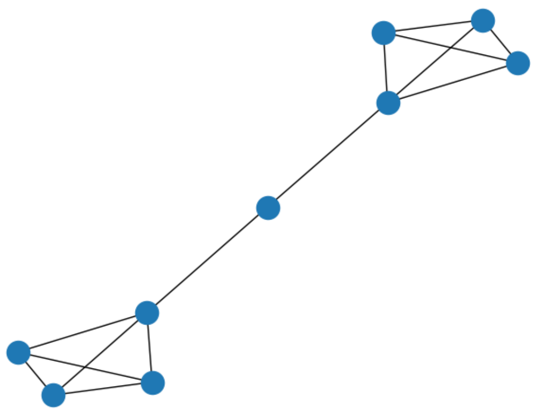


59.8.2. BarbellGraph

Barbell graph (NetworkX, Wolfram MathWorld)

`BarbellGraph`[m_1, m_2]
Barbell Graph: two complete graphs connected by a path.

```
>> BarbellGraph[4, 1]
```



59.8.3. BinomialTree

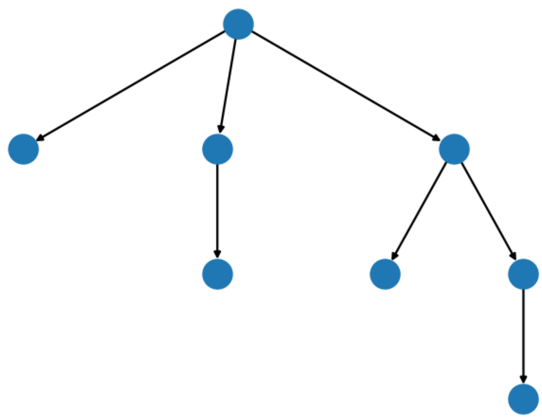
Binomial tree (NetworkX, WMA)

`BinomialTree[`n`]`

Returns the Binomial Tree of order n.

The binomial tree of order n with root R is defined as:
If k=0, `B[$k$]` = `B[0]` = {R}. i.e., the binomial tree of order zero consists of a single node, R.
If k>0, `B[$k$]` = {R, `B[0]` , `B[1]` ... `B[$k$]`}, i.e., the binomial tree of order k>0 comprises the root R, and k binomial subtrees, `B[0]` to `B[$k$]` .
Binomial trees are the underlying data structure in Binomial heaps.

```
>> BinomialTree[3]
```



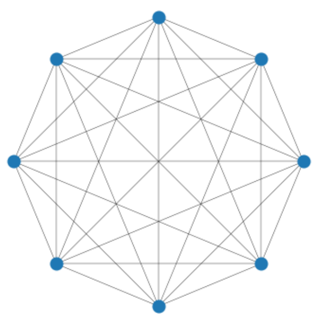
59.8.4. CompleteGraph

Complete Multipartite Graph (NetworkX, WMA)

`CompleteGraph[`n`]`

Returns the complete graph with n vertices, K_n.

```
>> CompleteGraph[8]
```



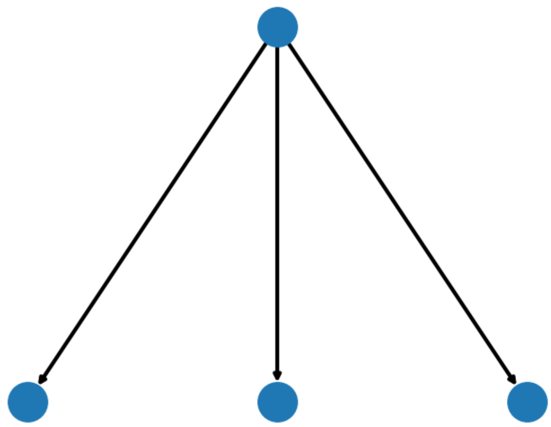
59.8.5. CompleteKaryTree

M-ary Tree (NetworkX, WMA)

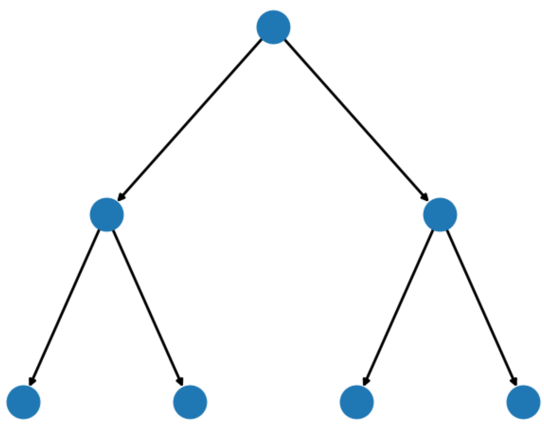
`CompleteKaryTree`[n, k]
  Creates a complete k-ary tree of n levels.

In the returned tree, with n nodes, the from root R to any leaf be k.

```
>> CompleteKaryTree[2, 3]
```



```
>> CompleteKaryTree[3]
```

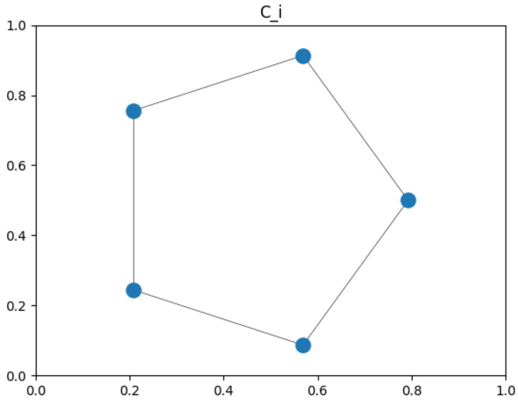


59.8.6. CycleGraph

Cycle Graph (WMA)

`CycleGraph`[n]
  Returns the cycle graph with n vertices C_n.

>> `CycleGraph[5, PlotLabel -> "C_i"]`

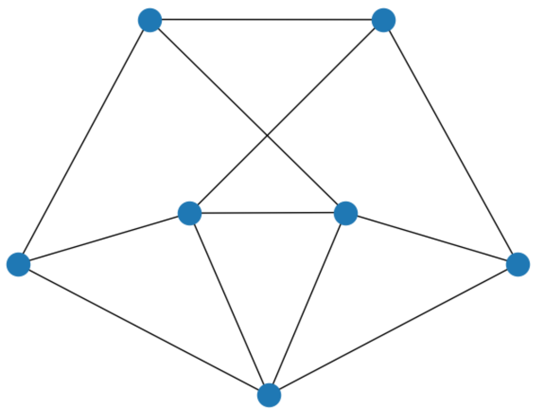


59.8.7. GraphAtlas

NetworkX

`GraphAtlas[`*n*`]`
: Returns graph number *i* from the NetworkX's Graph Atlas. There are about 1200 of them and get large as *i* increases.

>> `GraphAtlas[1000]`



59.8.8. HknHararyGraph

NetworkX, WMA

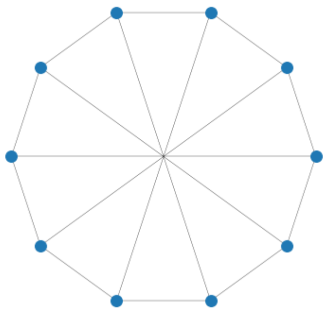
HknHararyGraph[k, n]

Returns the Harary graph with given node connectivity and node number.

This second generator gives the Harary graph that minimizes the number of edges in the graph with given node connectivity and number of nodes.

Harary, F. The Maximum Connectivity of a Graph. Proc. Nat. Acad. Sci. USA 48, 1142-1146, 1962.

```
>> HknHararyGraph[3, 10]
```



59.8.9. HmnHararyGraph

NetworkX, WMA

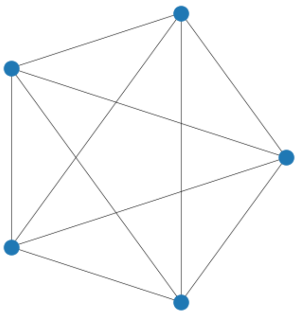
HmnHararyGraph[m, n]

Returns the Harary graph with given numbers of nodes and edges.

This generator gives the Harary graph that maximizes the node connectivity with given number of nodes and given number of edges.

Harary, F. The Maximum Connectivity of a Graph. Proc. Nat. Acad. Sci. USA 48, 1142-1146, 1962.

```
>> HmnHararyGraph[5, 10]
```



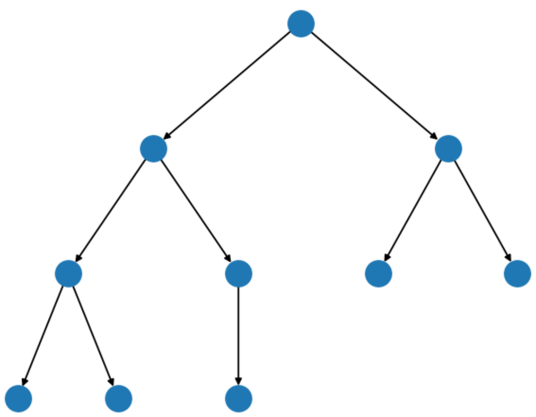
59.8.10. KaryTree

M-ary Tree

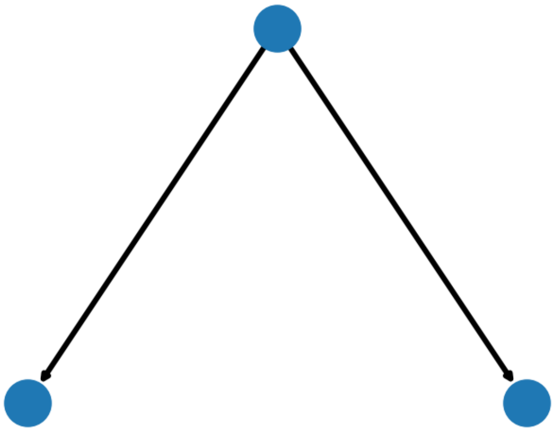
`KaryTree`[r, n]
: Creates binary tree of n vertices.

`KaryTree`[n, k]
: Creates k-ary tree with n vertices.

```
>> KaryTree[10]
```



```
>> KaryTree[3, 10]
```

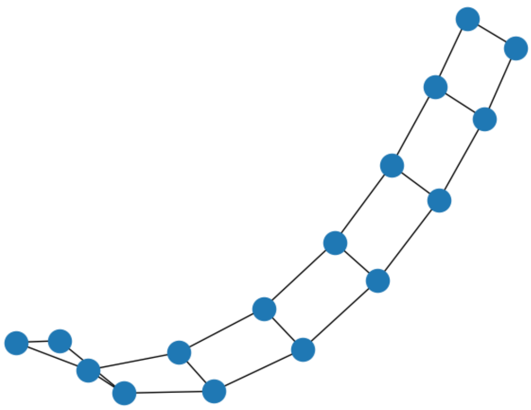


59.8.11. LadderGraph

Ladder graph (NetworkX)

`LadderGraph[n]`
: Returns the Ladder graph of length n.

```
>> LadderGraph[8]
```

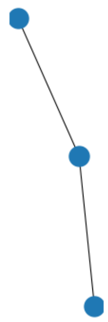


59.8.12. PathGraph

Path graph (WMA)

`PathGraph[{v1, v2, ...}]`
: Returns a Graph with a path with vertices v_i and edges between $v - i$ and $v_i + 1$.

```
>> PathGraph[{1, 2, 3}]
```



59.8.13. RandomTree

NetworkX, WMA

`RandomTree[n]`
: Returns a uniformly random tree on n nodes.

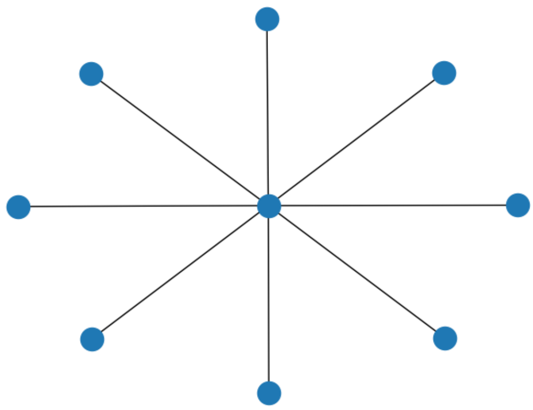
```
>> RandomTree[3]
```

59.8.14. StarGraph

Star graph(NetworkX, WMA)

`StarGraph[n]`
: Returns a star graph with n vertices.

```
>> StarGraph[8]
```



59.9. Random Graphs

59.9.1. RandomGraph

WMA link

`RandomGraph`[{n, m}]
    Returns a pseudorandom graph with n vertices and m edges.
`RandomGraph`[{n, m}, k]
    Returns list of k RandomGraph[{n, m}].

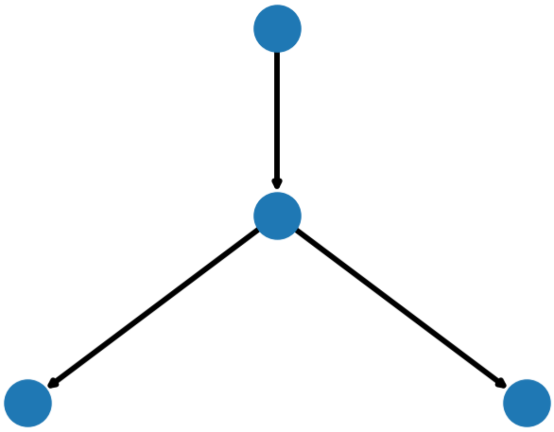
59.10. Trees

59.10.1. TreeGraph

Tree Graph (WMA)

`TreeGraph`[*edges*]
    Build a Tree-like graph from the list of edges *edges*.
`TreeGraph`[*vert*, *edges*]
    build a Tree-like graph from the list of vertices *vert* and edges *edges*.

```
>> TreeGraph[{1->2, 2->3, 2->4}]
```



If the *edges* does not match with a tree-like pattern, the evaluation fails:

```
>> TreeGraph[{1->2, 2->3, 3->1}]
   Graph is not a tree.
```

$\text{TreeGraph}\left[\{1->2, 2->3, 3->1\}\right]$

59.10.2. TreeGraphQ

Tree Graph (WMA)

`TreeGraphQ`[*g*]
    returns *True* if the graph *g* is a tree and *False* otherwise.

```
>> TreeGraphQ[StarGraph[3]]
```

True

```
>> TreeGraphQ[CompleteGraph[2]]
```

True

```
>> TreeGraphQ[CompleteGraph[3]]
```

False

60. ICU — International Components for Unicode

Functions which provide information from the Python ICU library icu library.

Examples:

Load in Mathics3 Module:

>> `LoadModule["pymathics.icu"]`
pymathics.icu

Show the language in effect:

>> `$Language`
English

Get the alphabet for that language:

>> `Alphabet[]`
{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}

Get the alphabet for that locale “es” (Spanish):

>> `Alphabet["es"]`
{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, á, é, í, ñ, ó, ú, ü}

You can also specify an alphabet using a name:

>> `Alphabet["Ukrainian"]`
{ , }

Contents

60.1. Languages - Human-Language Alphabets and Locales via PyICU.

60.1.1. Alphabet

Basic lowercase alphabet via Unicode and PyICU

`Alphabet[]`
: gives the list of lowercase letters a-z in the English alphabet.

`Alphabet[$type$]`
: gives the alphabet for the language or class *type*.

```
>> Alphabet["Ukrainian"]
```
{ , }

The alphabet when nothing is specified, "English" is used:

```
>> Alphabet[]
```
{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}

Instead of a language name, you can give a local value:

```
>> Alphabet["es"]
```
{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, á, é, í, ñ, ó, ú, ü}

Many locales are the same basic set of letters.

```
>> Alphabet["en_NZ"] == Alphabet["en"]
```
True

60.1.2. AlphabeticOrder

WMA

`AlphabetOrder`[*string*$_1$, *string*$_2$]
: gives 1 if *string*$_1$ appears before *string*$_2$ in alphabetical order, -1 if it is after, and 0 if it is identical.

```
>> AlphabeticOrder["apple", "banana"]
```
1

```
>> AlphabeticOrder["parrot", "parrot"]
```
0

When words are the same but only differ in case, usually lowercase letters come first:

```
>> AlphabeticOrder["A", "a"]
```
-1

Longer words follow their prefixes:

```
>> AlphabeticOrder["Papagayo", "Papa", "Spanish"]
```
-1

But accented letters usually appear at the end of the alphabet:

```
>> AlphabeticOrder["Papá", "Papa", "Spanish"]
```
-1

```
>> AlphabeticOrder["Papá", "Papagayo", "Spanish"]
```
1

60.1.3. $Language

WMA link

`$Language`
: is a settable global variable for the default language used in Mathics3.

See the language in effect used for functions like `Alphabet[]`:

By setting its value, The letters of `Alphabet[]` are changed:

```
>> $Language = "German"; Alphabet[]
```
{a, ä, b, c, d, e, f, g, h, i, j, k, l, m, n, o, ö, p, q, r, s, ß, t, u, ü, v, w, x, y, z}

See also Alphabet 60.1.1.

61. Natural Language Processing

Mathics3 Module module provides functions and variables to work with expressions in natural language, using the Python libraries:

- spacy for parsing natural languages</url>
- nltk for functions using WordNet-related builtins
- pyenchant and pycountry for language identification

Examples:

```
>> LoadModule["pymathics.natlang"]
```
pymathics.natlang

```
>> Pluralize["try"]
```
tries

```
>> LanguageIdentify["eins zwei drei"]
```
German

```
>> WordFrequency["Apple Tree and apple", "apple", IgnoreCase -> True]
```
0.5

```
>> TextCases["I was in London last year.", "Pronoun"]
```
{I}

```
>> DeleteStopwords["There was an Old Man of Apulia, whose conduct was
   very peculiar"]
```
Old Man Apulia, conduct peculiar

Contents

61.1. Linguistic Data

See the corresponding WMA guide.

61.1.1. DictionaryLookup

WMA link

`DictionaryLookup[`*word*`]`
    lookup words that match the given *word* or pattern.
`DictionaryLookup[`*word*, *n*`]`
    lookup first *n* words that match the given *word* or pattern.

>> `DictionaryLookup["baker" ~~___]`

{baker, baker's dozen, baker's eczema, baker's yeast, bakersfield, bakery}

>> `DictionaryLookup["baker" ~~___, 3]`

{baker, baker's dozen, baker's eczema}

61.1.2. DictionaryWordQ

WMA link

`DictionaryWordQ[`*word*`]`
    returns True if *word* is a word usually found in dictionaries, and False otherwise.

>> `DictionaryWordQ["couch"]`

True

>> `DictionaryWordQ["meep-meep"]`

False

61.1.3. RandomWord

WMA link

`RandomWord[]`
: returns a random word.

`RandomWord[`*type*`]`
: returns a random word of the given *type*, e.g. of type “Noun” or “Adverb”.

`RandomWord[`*type*, *n*`]`
: returns *n* random words of the given *type*.

>> **`RandomWord["Noun"]`**
eastern desert

>> **`RandomWord["Noun", 3]`**
{ground-controlled approach, garbage heap, valence}

61.1.4. WordData

WMA link

`WordData[`*word*`]`
: returns a list of possible senses of a word.

`WordData[`*word*, *property*`]`
: returns detailed information about a word regarding *property*, e.g. “Definitions” or “Examples”.

The following are valid properties:

- Definitions, Examples
- InflectedForms
- Synonyms, Antonyms
- BroaderTerms, NarrowerTerms
- WholeTerms, PartTerms, MaterialTerms
- EntailedTerms, CausesTerms
- UsageField
- WordNetID
- Lookup

>> **`WordData["riverside", "Definitions"]`**
{{riverside, Noun, Bank} − > the bank of a river}

>> **`WordData[{"fish", "Verb", "Angle"}, "Examples"]`**
{{fish, Verb, Angle} − > {fish for compliments}}

61.1.5. WordDefinition

WMA link

`WordDefinition`[*word*]
    returns a definition of *word* or Missing["Available"] if *word* is not known.

```
>> WordDefinition["gram"]
```
{a metric unit of weight equal to one thousandth of a kilogram}

61.1.6. WordList

WMA link

`WordList[]`
    returns a list of common words.

`WordList`[*type*]
    returns a list of common words of type *type*.

Evaluate the average length over all the words in the dictionary:

```
>> N[Mean[StringLength /@ WordList[]], 3]
```
11.6

Now, restricted to adjectives:

```
>> N[Mean[StringLength /@ WordList["Adjective"]], 2]
```
9.3

61.2. Text Analysis

See the corresponding WMA guide.

61.2.1. Containing

WMA link

`Containing`[*outer*, *inner*]
    represents an object of the type outer containing objects of type inner.

`Containing` can be used as the second parameter in `TextCases` and `TextPosition`.

Supported *outer* strings are in {"Word", "Sentence", "Paragraph", "Line", "URL", "EmailAddress"}.

Supported *inner* strings are in {"Person", "Company", "Quantity", "Number", "CurrencyAmount", "Country", "City"}.

The implementation of this symbol is based on 'spacy'.

```
>> TextCases["This is a pencil. This is another pencil from England.",
   Containing["Sentence", "Country"]]
```

{This is another pencil from England.}

```
>> TextPosition["This is a pencil. This is another pencil from England
   .", Containing["Sentence", "Country"]]
```

$\{\{19, 54\}\}$

61.2.2. SpellingCorrectionList

WMA link

`SpellingCorrectionList[`*word*`]`
returns a list of suggestions for spelling corrected versions of *word*.

Results may differ depending on which dictionaries can be found by enchant.

```
>> SpellingCorrectionList["hipopotamus"]
```

{hippopotamus, hippopotamus's, hippopotamuses}

61.2.3. WordCount

WMA link

`WordCount[`*string*`]`
returns the number of words in *string*.

```
>> WordCount["A long time ago"]
```

4

61.2.4. WordFrequency

WMA link

`WordFrequency[`*text, word*`]`
returns the relative frequency of *word* in *text*.

word may also specify multiple words using *a* | *b* | ...

```
>> text = "I have a dairy cow, it's not just any cow. She gives me
   milkshake, oh what a salty cow. She is the best cow in the county.";
```

```
>> WordFrequency[text, "a" | "the"]
```

0.121212

```
>> WordFrequency["Apple Tree", "apple", IgnoreCase -> True]
```

0.5

61.2.5. WordSimilarity

WMA link

`WordSimilarity`[$text_1$, $text_2$]
: returns a real-valued measure of semantic similarity of two texts or words.

`WordSimilarity`[{$text_1$, $i1$}, {$text_2$, j_1}]
: returns a measure of similarity of two words within two texts.

`WordSimilarity`[{$text_1$, {$i1$, $i2$, ...}}, {$text_2$, {j_1, j_2, ...}}]
: returns a measure of similarity of multiple words within two texts.

```
>> NumberForm[WordSimilarity["car", "train"], 3]
```

0.169

```
>> NumberForm[WordSimilarity["car", "hedgehog"], 3]
```

0.0173

```
>> NumberForm[WordSimilarity[{"An ocean full of water.", {2, 2}}, { "A
   desert full of sand.", {2, 5}}], 3]
```

$\{0.127, 0.256\}$

61.2.6. WordStem

WMA link

`WordStem`[*word*]
: returns a stemmed form of *word*, thereby reducing an inflected form to its root.

`WordStem`[{$word_1$, $word_2$, ...}]
: returns a stemmed form for list of *word*, thereby reducing an inflected form to its root.

```
>> WordStem["towers"]
```

tower

```
>> WordStem[{"heroes", "roses", "knights", "queens"}]
```

{hero, rose, knight, queen}

61.3. Text Normalization

See the corresponding WMA guide.

This module uses spacy as a backend.

61.3.1. DeleteStopwords

Delete stop words(WMA)

`DeleteStopwords[`*list*`]`
    returns the words in *list* without stopwords.
`DeleteStopwords[`*string*`]`
    returns *string* without stopwords.

```
>> DeleteStopwords[{"Somewhere", "over", "the", "rainbow"}]
```
{Somewhere, over, the, rainbow}

```
>> DeleteStopwords["There was an Old Man of Apulia, whose conduct was
very peculiar"]
```
Old Man Apulia, conduct peculiar

61.3.2. TextCases

WMA link

`TextCases[`*text*, *form*`]`
    returns all elements of type *form* in *text* in order of their appearance.

```
>> TextCases["I was in London last year.", "Pronoun"]
```
{I}

```
>> TextCases["I was in London last year.", "City"]
```
{London}

```
>> TextCases["Saul, Peter and Mr Johnes say hello.", "Person",
3][[2;;3]]
```
{Peter, Johnes}

61.3.3. TextPosition

WMA link

`TextPosition[`*text*, *form*`]`
: returns the positions of elements of type *form* in *text* in order of their appearance.

```
>> TextPosition["Liverpool and London are two English cities.", "City"]
```
$\{\{1,9\},\{15,20\}\}$

61.3.4. TextSentences

Sentences in a text (WMA)

`TextSentences[`*string*`]`
: returns the sentences in *string*.

`TextSentences[`*string*, *n*`]`
: returns the first *n* sentences in *string*

```
>> TextSentences["Night and day. Day and night."]
```
{Night and day., Day and night.}

```
>> TextSentences["Night and day. Day and night.", 1]
```
{Night and day.}

```
>> TextSentences["Mr. Jones met Mrs. Jones."]
```
{Mr. Jones met Mrs. Jones.}

61.3.5. TextStructure

WMA link

`TextStructure[`*text*, *form*`]`
: returns the grammatical structure of *text* as *form*.

```
>> TextStructure["The cat sat on the mat.", "ConstituentString"]
```
{(Sentence, ((Verb Phrase, (Noun Phrase, (Determiner, The), (Noun, cat)), (Verb, sat), (Prepositional Phrase, (Preposition, on), (Noun Phrase, (Determiner, the), (Noun, mat))), (Punctuation, .))))}

61.3.6. TextWords

WMA link

`TextWords[`*string*`]`
: returns the words in *string*.

`TextWords[`*string*, *n*`]`
: returns the first *n* words in *string*

```
>> TextWords["Hickory, dickory, dock! The mouse ran up the clock."]
```
{Hickory, dickory, dock, The, mouse, ran, up, the, clock}

```
>> TextWords["Bruder Jakob, Schläfst du noch?", 2]
```
{Bruder, Jakob}

61.4. Word manipulation

This module uses pattern.en to change the form of a word.

61.4.1. Pluralize

WMA link

`Pluralize[`*word*`]`
: returns the plural form of *word*.

```
>> Pluralize["potato"]
```
potatoes

Part IV.

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Version 3, 29 June 2007

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B.5. pymimemagic

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B.6. SciPy

B.7. Three.js

Appendices

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Colophon

Mathics3 Core 9.0.0

Python
3.13.7 (main, Aug 17 2025, 17:14:11) [GCC 13.3.0]

XeTeX
XeTeX 3.141592653-2.6-0.999995 (TeX Live 2023/Debian)

Asymptote
Asymptote version 2.95 [(C) 2004 Andy Hammerlindl, John C. Bowman, Tom Prince]

mpmath 1.3.0

NumpyPy 2.3.2

SymPy 1.13.3

Ghostscript 10.02.1

cython 3.1.3

lxml 6.0.0

matplotlib 3.10.5

networkx 3.5

psutil 7.0.0

skimage 0.25.2

scipy 1.16.1

wordcloud 1.9.4