

# Some recent advances on Approximate Bayesian Computation techniques

Jean-Michel Marin

University of Montpellier, CNRS  
Alexander Grothendieck Montpellier Institute



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- ▶ Judith, Natesh, ...

# Introduction

## Bayesian parametric paradigm



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**Extremely difficult to sample from the posterior distribution**

$$\pi(\theta|\mathbf{y}) \propto \pi(\theta)f(\mathbf{y}|\theta)$$

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If, with Christian, we work on ABC methods, we can be very grateful to our biologist colleagues!

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## Likelihood-free rejection sampler

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**Tavaré et al. (1997) Genetics**

**Pritchard et al. (1999) Mol. Biol. Evol.**

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- 5) If  $i \leq N$ , return to **2)**

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ABC target

$$\pi_{\epsilon}(\theta|\mathbf{y}) = \frac{\int \pi(\theta)f(\mathbf{z}|\theta)\mathbb{I}(\mathbf{z} \in A_{\epsilon,\mathbf{y}})d\mathbf{z}}{\int_{A_{\epsilon,\mathbf{y}} \times \Theta} \pi(\theta)f(\mathbf{z}|\theta)d\mathbf{z}d\theta}$$

$A_{\epsilon,\mathbf{y}} = \{\mathbf{z} | d(\eta(\mathbf{z}), \eta(\mathbf{y})) \leq \epsilon\}$  the acceptance set

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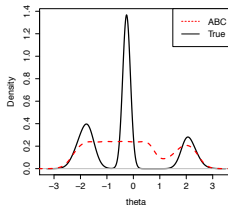
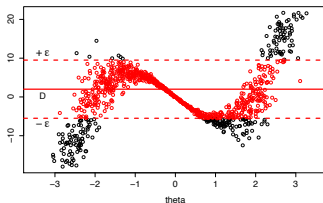
$$\theta \sim \mathcal{U}_{[-10,10]}$$

$$y = 2$$

$$d(z, y) = |z - y|$$

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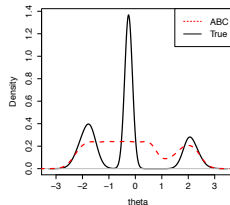
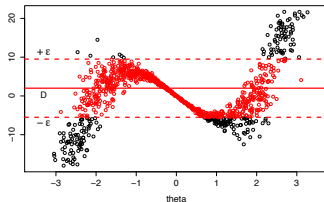
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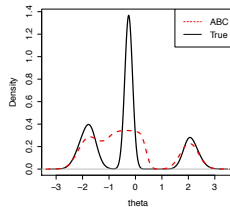
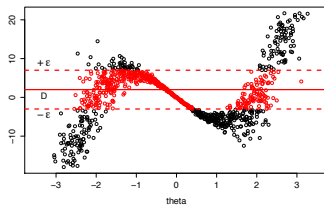
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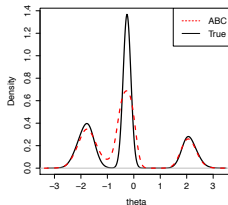
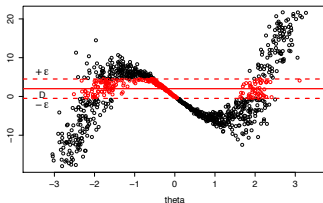
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$\epsilon = 5$

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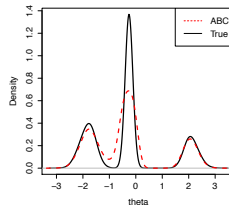
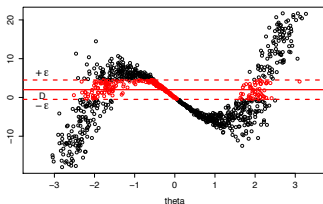
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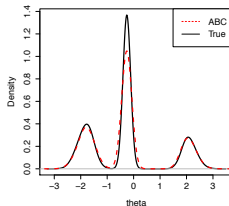
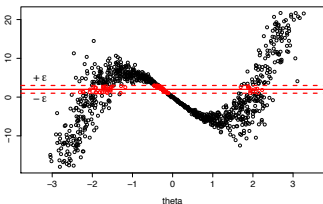
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$\epsilon$  corresponds to a quantile of the distances

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**New insights into Approximate Bayesian Computation**  
**Biau, Cérou, Guyader (2015) Annales de l'IHP**

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- intuitive

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### **New insights into Approximate Bayesian Computation**

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- ▶ intuitive
- ▶ simple to implement
- ▶ embarrassingly parallelisable
- ▶ BUT curse of dimensionality: most of the simulations are at the boundary of the space as the number of summary statistics increases

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## Two views of the ABC approximation



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⇒ **Blum (2010) JASA** emphasizes that ABC is a kernel smoothing approximation of the likelihood function

$$\begin{aligned}\pi_{\epsilon}(\theta|\mathbf{y}) &= \frac{\int \pi(\theta)f(\mathbf{z}|\theta)\mathbb{I}(\mathbf{z} \in A_{\epsilon,\mathbf{y}})d\mathbf{z}}{\int_{A_{\epsilon,\mathbf{y}} \times \Theta} \pi(\theta)f(\mathbf{z}|\theta)d\mathbf{z}d\theta} \\ &= \frac{\pi(\theta) \int f(\mathbf{z}|\theta)K(d(\eta(\mathbf{z}), \eta(\mathbf{y})))d\mathbf{z}}{\int \pi(\theta)f(\mathbf{z}|\theta)K(d(\eta(\mathbf{z}), \eta(\mathbf{y})))d\mathbf{z}d\theta}\end{aligned}$$

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**Sisson et al. (2007) PNAS**

**Beaumont, Cornuet, Marin and Robert (2009) Biometrika**

**Del Moral et al. (2012) Statistics and Computing**

**Marin, Pudlo and Sedki (2012) IEEE Proceedings of WSC**

**Filippi et al. (2013) SAGMB**

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## More efficient algorithms

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Likelihood free MCMC sampler **Majoram et al. (2003) PNAS**

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## Regression adjustments

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local linear regression adjustment of the parameter values

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local linear regression adjustment of the parameter values

### **Blum and Francois (2010) Statistics and Computing**

heteroscedastic models, feed-forward neural networks

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## Summary statistics

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### Best subset selection

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### Regularization techniques

- ▶ **Blum, Nunes, Prangle and Fearnhead (2013) Statistical Science** use ridge regression
- ▶ **Saulnier, Gascuel, Alizon (2017) Plos Computational Biology** use LASSO

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A k-NN approximation of the posterior probabilities

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**ABC likelihood-free methods for model choice in Gibbs random fields** **Grelaud, Robert, Marin, Rodolphe and Taly (2009) Bayesian Analysis**

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If not...

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We investigate some ABC model choice techniques that use others machine learning procedures

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**Estimation of demo-genetic model probabilities with Approximate Bayesian Computation using linear discriminant analysis on summary statistics** Estoup, Lombaert, Marin, Guillemaud, Pudlo, Robert, Cornuet (2012) *Molecular Ecology*

# Methodological aspects of ABC Softwares

**abc R package** several ABC algorithms for performing parameter estimation and model selection

**abctools R package** tuning ABC analyses

<https://journal.r-project.org/archive/2015-2/nunes-prangle.pdf>

**abcrf R package** ABC via random forests

**EasyABC R package** several algorithms for performing efficient ABC sampling schemes, including 4 sequential sampling schemes and 3 MCMC schemes

# Methodological aspects of ABC Softwares

**DIY-ABC software** performs parameter estimation and model selection for population genetics models

**ABC-SysBio python package** parameter inference and model selection for dynamical systems

**ABCtoolbox programs** various ABC algorithms including rejection sampling, MCMC without likelihood, a particle-based sampler, and ABC-GLM

**PopABC software** package for inference of the pattern of demographic divergence, coalescent simulation, bayesian model choice

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**Inferring population history with DIY ABC: a user-friendly approach Approximate Bayesian Computation** Cornuet, Santos, Beaumont, Robert, Marin, Balding, Guillemaud, Estoup (2008) Bioinformatics

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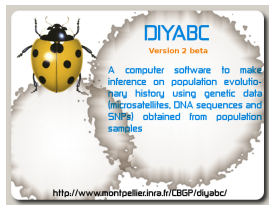
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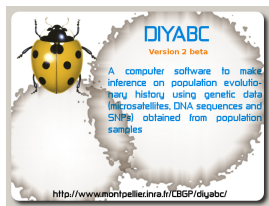




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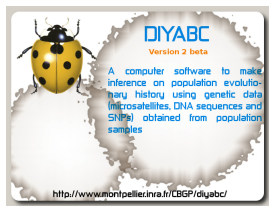


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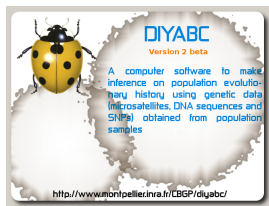


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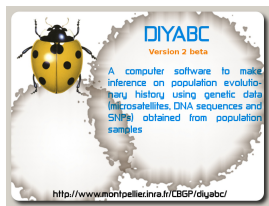


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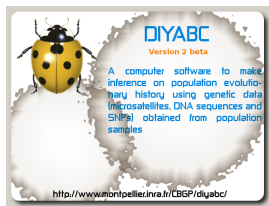


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## Frontline news from population geneticists country

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- ▶ sequential methods are difficult to calibrate and do not give reproducible results
- ▶ post hoc adjustments are crucial but they underestimate the amount of uncertainty
- ▶ available techniques to select the summary statistics do not give reproducible results

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- ▶ to ensure reliability of the method, the number of simulations should be large

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## Frontline news from population geneticists country

Despite all these works, two major difficulties

- ▶ to ensure reliability of the method, the number of simulations should be large
- ▶ choice of the summaries statistics is still a problem

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## Use modern machine learning tools



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Exploiting a large number of summary statistics is not an issue for some machine learning methods

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**Scornet, Biau, Vert (2015) The Annals of Statistics**

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As an alternative, Papamakarios and Murray (2016) propose to approximate the whole posterior distribution by using Mixture Density Networks (MDN, Bishop, 1994)

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**Fast e-free Inference of Simulation Models with Bayesian Conditional Density Estimation**

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Idea: iteratively learn an efficient proposal prior (approximating the posterior distribution), then to use this proposal to train the posterior, both steps making use of MDN

**The number of mixture components and the number of hidden layers of the networks require calibration**

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## Use modern machine learning tools

### **Deep Learning for Population Genetic Inference**

**Sheehan and Song (2016) PLOS Computational Biology**

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Deep learning makes use of multilayer neural networks to learn a feature-based function from the input (hundreds of correlated summary statistics) to the output (population genetic parameters of interest).

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Deep learning makes use of multilayer neural networks to learn a feature-based function from the input (hundreds of correlated summary statistics) to the output (population genetic parameters of interest).

**Unsupervised pretraining using autoencoders very interesting, but requires a lot of calibration**

# ABC random forests

## Model choice

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**Reliable ABC model choice via random forests** Pudlo, Marin, Estoup, Cornuet, Gauthier and Robert (2016) Bioinformatics



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For  $i = 1, \dots, M$

- a) Generate  $m_i$  from the prior  $\pi(\mathcal{M} = m)$
- b) Generate  $\theta'_{m_i}$  from the prior  $\pi_{m_i}(\cdot)$
- c) Generate  $\mathbf{z}$  from the model  $f_{m_i}(\cdot | \theta'_{m_i})$
- d) Calculate  $\mathbf{x}_i = \eta(\mathbf{z}_i)$

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**Output** a random forest classifier to infer model indexes  $m(\widehat{\eta(\mathbf{y})})$

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## Model choice

Random forest predicts a MAP model index, from the observed dataset

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**frequency of trees associated with majority model is no proper substitute to the true posterior probability**



# ABC random forests

## Model choice

Estimate of the posterior probability of the selected model

$$\mathbb{P}[\mathcal{M} = \widehat{m(\eta(\mathbf{y}))} | \eta(\mathbf{y})]$$

random comes from  $\mathcal{M}$  (bayesian)!

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$$\mathbb{P}[\mathcal{M} = \widehat{m(\eta(\mathbf{y}))} | \eta(\mathbf{y})] = 1 - \mathbb{E} \left[ \mathbb{I}(\mathcal{M} \neq \widehat{m(\eta(\mathbf{y}))}) | \eta(\mathbf{y}) \right]$$

# ABC random forests

## Model choice

**A second random forest in regression**

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## Model choice

### A second random forest in regression

- 1) compute the value of  $\mathbb{I}(\mathcal{M} \neq \widehat{m}(\widehat{\eta}(\mathbf{z})))$  for the trained random forest  $\widehat{m}$  and for all terms in the ABC reference table using the out-of-bag classifiers

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- 2) train a RF regression and get  $\widehat{\mathbb{E}} \left[ \mathbb{I}(\mathcal{M} \neq \widehat{m}(\widehat{\eta}(\mathbf{z}))) | \eta(\mathbf{z}) \right]$

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**on same reference table out-of-bag magic trick avoid overfitting!**

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## Parameter inference



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**Output** some regression RF predictors to infer posterior expectations, quantiles, variances and covariances

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## Parameter inference

**Expectations** Construct  $d$  regression RF, one per dimension

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**Covariances** new forests for which the responses variables are the products of out-of-bag errors



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We constructed forests able to estimate everywhere in the space of summary statistics but we are interested only in one point, the observed dataset

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**construct local random forest, thesis of Louis Raynal**

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**The ABC toolbox seems unable to bring anything to the PAC-Bayesian framework**

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**On the other hand**

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Convergence of the ABC posterior under model misspecification  
Use of concentration inequalities, PAC-Bayesian analysis

# End

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**Thank you very much for your attention**