Flight tests of the path following vector field method on the formation of Crazyflie 2.1 nano quadcopters

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Flight tests of the path following vector field method on the formation of Crazyflie 2.1 nano quadcopters. The Lighthouse positioning system is used for experiments: https://www.bitcraze.io/documentation/lighthouse/

1 Basic Algorithm

1.1 Notation

(CX, CY) (or (c_e, c_n)) $-$ circle center coordinates
$k > 0$ $-$ smoothness coefficient for the path following control law
$R = const$ (or ρ) $-$ radius of the circular path
$v_f = const$ $-$ the maximum value of the additional speed component
v_{cruis} $-$ final cruising speed of a Crazyflie formation
$D_{12} = const$ $-$ the desired angular distance between the 1st and 2nd copters (we also call this value the desired phase shift)
$D_{23} = const$ $-$ the desired angular distance between the 2nd and 3rd copters
Required condition for testing on three UAVs: $D_{12} + D_{23} > 2\pi$

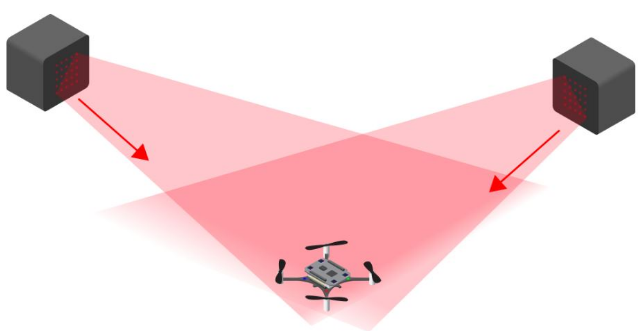


Figure 1: Lighthouse Positioning System

$k_f > 0$ – smoothness coefficient for the speed control law
T_Z – specified altitude of formation flight
v_z – vertical speed for takeoff or landing
d_i – distance from the ith Crazyflie to a circle center
phi (or φ_i) – phase angle of the ith Crazyflie
angle (or χ_i^c) – command course angle of the ith Crazyflie
vx (or v_x) – speed of Crazyflie along the x-axis in the world (global) coordinate system
vy (or v_y) – speed of Crazyflie along the y-axis in the world (global) coordinate system
vz (or v_z) – speed of Crazyflie along the z-axis in the world (global) coordinate system
px (or p_e) – coordinate of Crazyflie along the x-axis in the world (global) coordinate system
py (or p_n) – coordinate of Crazyflie along the y-axis in the world (global) coordinate system
kalman.stateX – estimation of the copter's position with the Kalman filter along the x-axis in the world (global) coordinate system
kalman.stateY – estimation of the copter's position with the Kalman filter along the y-axis in the world (global) coordinate system

1.2 Course Angle Control Law

Control law for the Crazyflie course angle:

$$\chi_i^c = \varphi_i + \lambda \left[\frac{\pi}{2} + \operatorname{atan}(k(d_i - \rho)) \right], \tag{1}$$

where $\lambda = 1$ means clockwise motion and $\lambda = -1$ means counterclockwise motion. This law is based on the one presented in the monograph [1].

1.3 Speeds Control Law

Control law for the Crazyflie speeds:

$$\begin{bmatrix} v_1^c \\ v_2^c \\ v_3^c \end{bmatrix} = \begin{bmatrix} v_{cruis} \\ v_{cruis} \\ v_{cruis} \end{bmatrix} + \begin{bmatrix} v_f\,(2/\pi)\arctan\left(k_f\,(\Delta\varphi_{12} - D_{12})\right) \\ v_f\,(2/\pi)\arctan\left(k_f\,(-\Delta\varphi_{12} + \Delta\varphi_{23} + D_{12} - D_{23})\right) \\ v_f\,(2/\pi)\arctan\left(k_f\,(-\Delta\varphi_{23} + D_{23})\right) \end{bmatrix}, \tag{2}$$

where
v_i^c - linear speed command for Crazyflie;
$v_{cruis} = const$ - final cruising speed of the formation;
$v_f = const$, $v_f \leq v_{cruis}$ - the maximum value of the additional speed component;
$k_f > 0$ - adjustable coefficient;
$\Delta\varphi_{i,j}$ (or $p_{i,j}$) - current phase shift between the i-th and j-th drone; in the code, this value is denoted as p_{12} for the phase shift between the 1st and 2nd drone

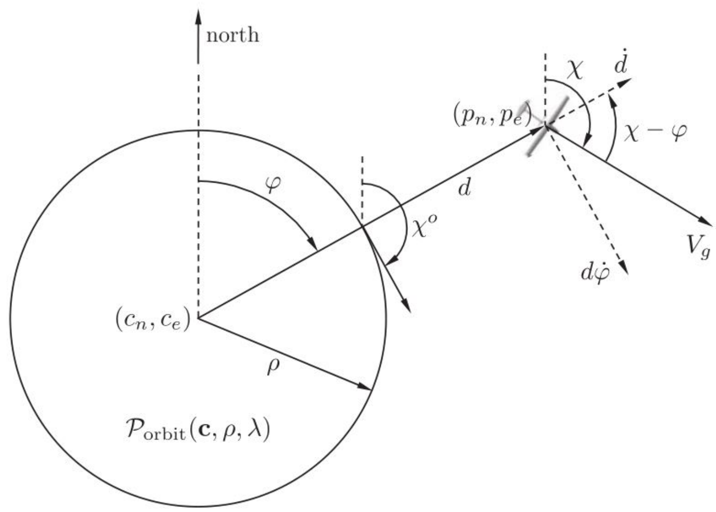


Figure 2: Notation used. Picture from [1]

and as p_{23} for the phase shift between the 2nd and 3rd drone.
$D_{i,j} = const$ - desired phase shift between the i-th and j-th drone.
Required condition for testing on three UAVs: $D_{12} + D_{23} > 2\pi$

How do we calculate the current phase shift (for example, $\Delta\varphi_{12}$)?

$\text{dot}_{product} = (p_{e_1} - c_e) \cdot (p_{e_2} - c_e) + (p_{n_1} - c_n) \cdot (p_{n_2} - c_n);$

$\text{magnitude}_1 = ((p_{e_1} - c_e)^2 + (p_{n_1} - c_n)^2)^{1/2};$

$\text{magnitude}_2 = ((p_{e_2} - c_e)^2 + (p_{n_2} - c_n)^2)^{1/2};$

$\text{triple}_{product} = (p_{e_1} - c_e) \cdot (p_{n_2} - c_n) - (p_{e_2} - c_e) \cdot (p_{n_1} - c_n);$

$\cos \Delta\varphi_{12} = \text{dot}_{product} / (\text{magnitude}_1 \cdot \text{magnitude}_2) \Rightarrow$

$\Rightarrow \Delta\varphi_{12} = \arccos(\text{dot}_{product} / (\text{magnitude}_1 \cdot \text{magnitude}_2));$

if $\text{triple}_{product} > 0$
then $\Delta\varphi_{12} := 2\pi - \Delta\varphi_{12};$ **end**
out $= [\Delta\varphi_{12}]$

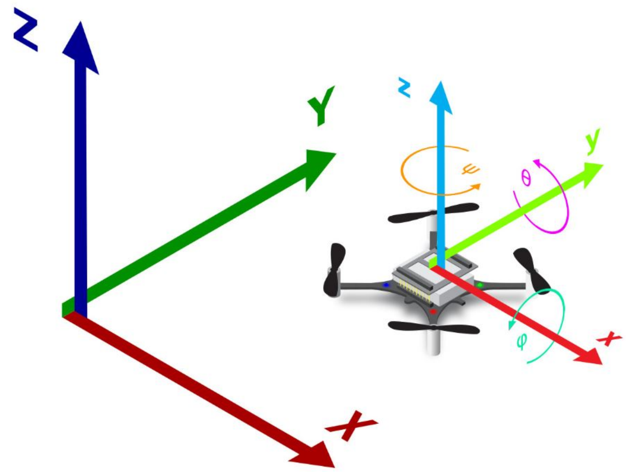


Figure 3: The Coordinate System of the Crazyflie 2.X https://www.bitcraze.io/documentation/system/platform/cf2-coordinate-system/

1.4 Crazyflie 2.1 Body Frame and Rotation Matrices

We calculate commands in the global (world) coordinate system. To get commands in the body coordinate system, we need to do a transformation with rotation matrices.

- **roll** and **yaw** are clockwise rotating around the axis looking from the origin (**right-hand-thumb**)
- **pitch** are counter-clockwise rotating around the axis looking from the origin (**left-hand-thumb**)

Formula (2.5) from [1] is modified:

$$\mathbf{R}(\varphi, \theta, \psi) = \mathbf{R}(\varphi) \cdot \mathbf{R}^T(\theta) \cdot \mathbf{R}(\psi) =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & \sin(\varphi) \\ 0 & -\sin(\varphi) & \cos(\varphi) \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \mathbf{R}(\varphi, \theta, \psi) \cdot \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix}, \tag{3}$$

where $\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$ is a vector of commands in body-frame and $\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix}$ is a vector of commands in global(world)-frame.

1.5 Velocity Control via Python API

If we use **def send hover setpoint(self, vx, vy, yawrate, zdistance)**, then the value z is taken as a *constant*. Therefore v_z from the above vector (3) does not end up being used. However, in order to correctly calculate v_x and v_y, we must use 3-by-3 matrices.

The resulting control law for implementation based on (1)-(2):

$$\begin{aligned} \dot{p}_x &= v_i^c \cdot \sin(\chi_i^c) \\ \dot{p}_y &= v_i^c \cdot \cos(\chi_i^c) \end{aligned} \tag{4}$$

Using (3):

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \mathbf{R}(\varphi, \theta, \psi) \cdot \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ 0 \end{bmatrix},$$

These values are fed to the function **def send hover setpoint(self, vx, vy, yawrate, zdistance)**

1.6 Setpoint Control via Python API

If we use **def send position setpoint(self, x, y, z, yaw)**:

$$\begin{aligned} x &= x_{t+1} = x_t + \dot{p}_x \cdot \Delta t \\ y &= y_{t+1} = y_t + \dot{p}_y \cdot \Delta t \\ z &= const \end{aligned} \tag{5}$$

where
x_t and y_t are the Crazyflie positions at the current moment in time, which can be obtained from the Kalman filter through the values of the logged variables **kalman.stateX** and **kalman.stateY**;
Δt is a fairly small time step;
$\dot{p}_x$ and $\dot{p}_y$ are the global frame speed commands from (4). These values from (5) are fed to the function **def send position setpoint(self, x, y, z, yaw)**

2 The Results of the Experiments

The following presents the experimental results for the method described in section 1.6. The code from the file **CircularMotion setpos 2 copters.py** was used for the experiments.

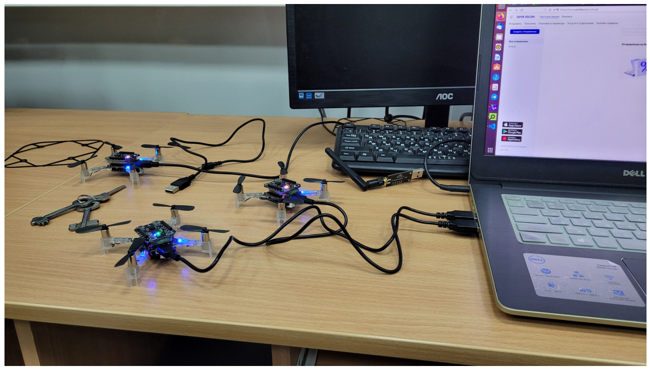
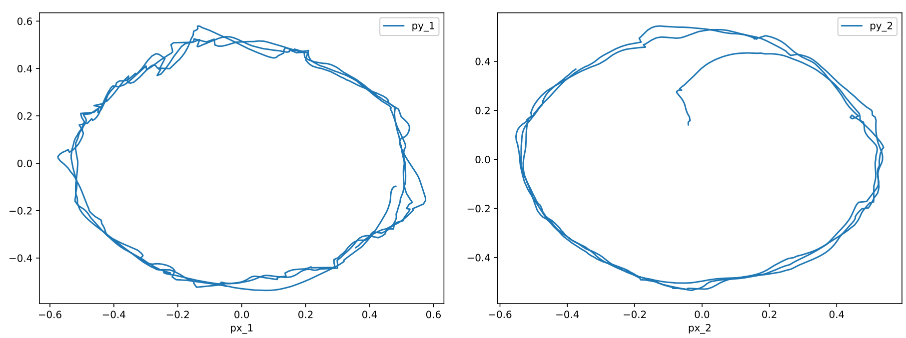


Figure 4: Crazyflie 2.1 copters used in the experiments

2.1 Experiments on two copters

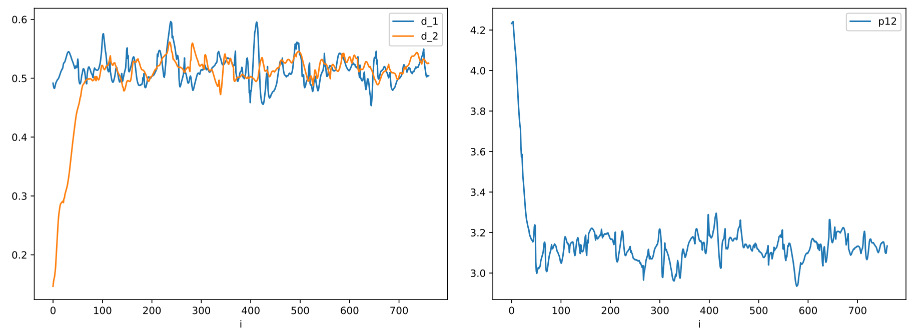
Preliminary video of the experiments is available at the link:
`https://youtu.be/Ushn_94qVtE`

2.1.1 Stationary center of the circular path



(a) (b)

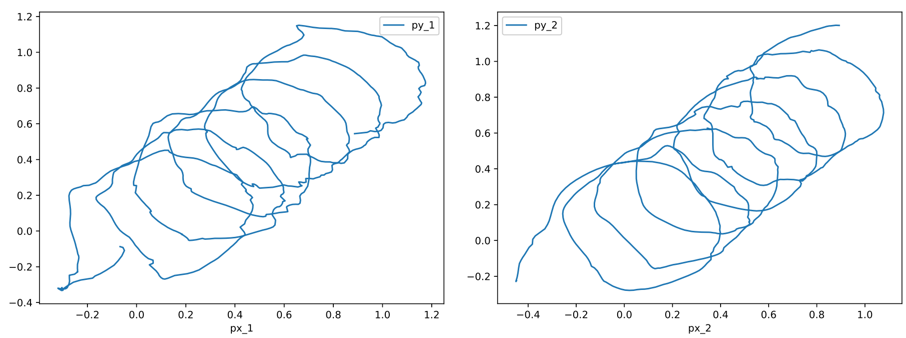
Figure 5: Two copters flight with a stationary center. (a) Trajectory of the 1st copter; (b) trajectory of the 2nd copter



(a) (b)

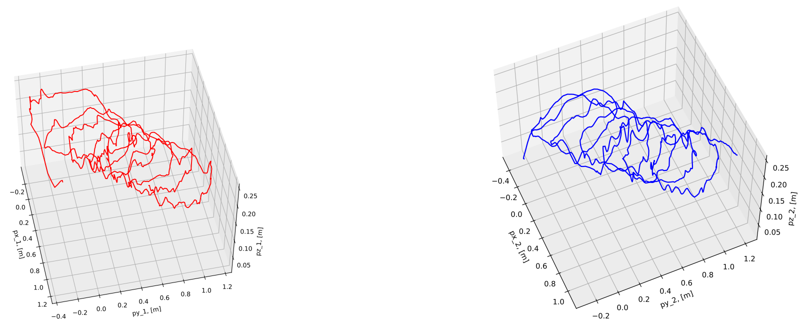
Figure 6: Two copters flight with a stationary center. (a) Distances to the circle center; (b) phase shifts

2.1.2 Moving center of the circular path



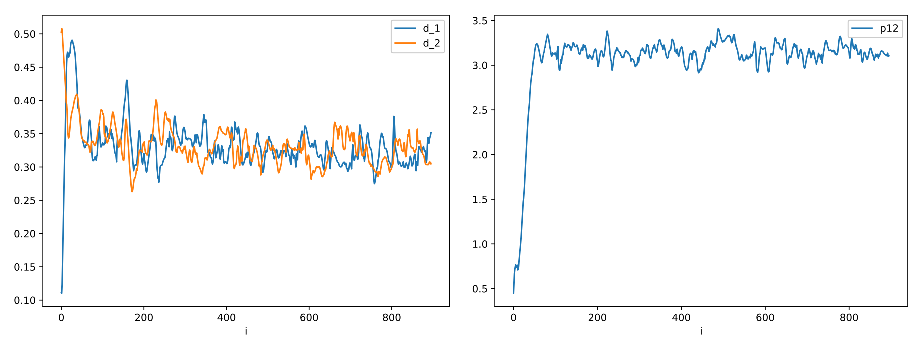
(a) (b)

Figure 7: Two copters flight with a moving center. (a) Trajectory of the 1st copter; (b) trajectory of the 2nd copter



(a) (b)

Figure 8: Two copters flight with a moving center. (a) 3D trajectory of the 1st copter; (b) 3D trajectory of the 2nd copter



(a) (b)

Figure 9: Two copters flight with a moving center. (a) Distances to the circle center; (b) phase shifts

2.2 Experiments on three copters

2.2.1 Stationary center of the circular path

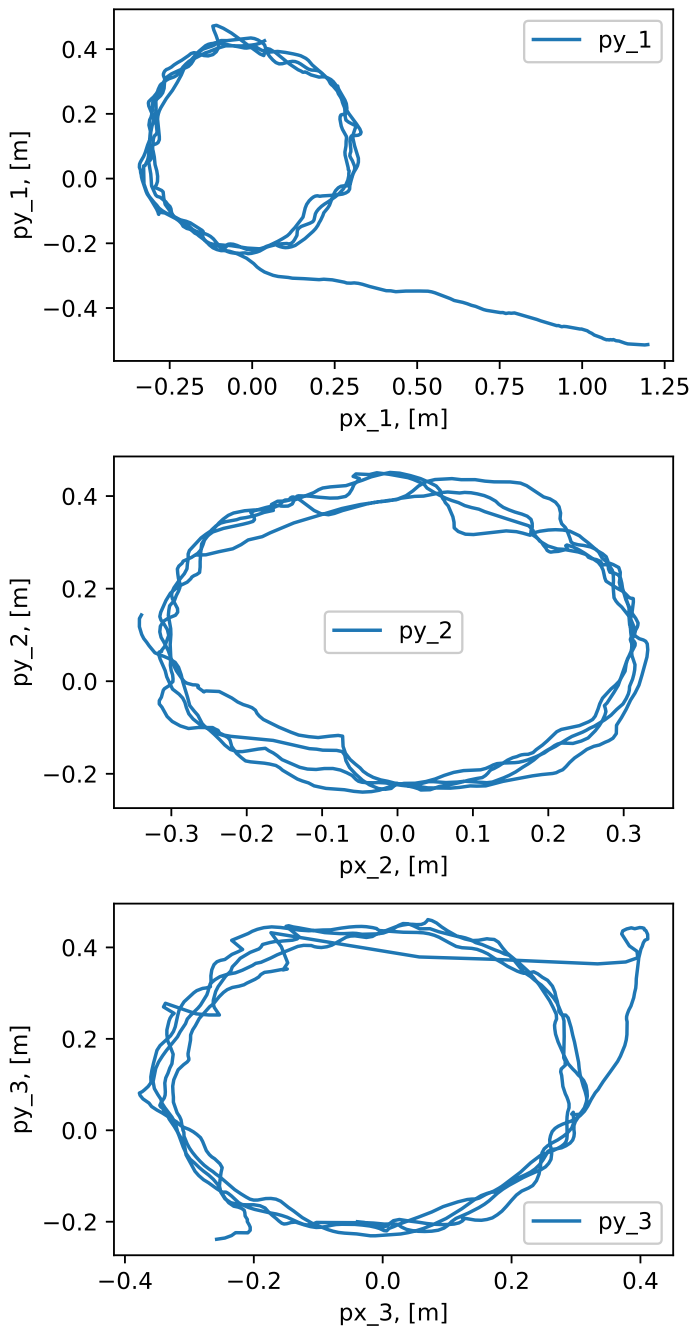
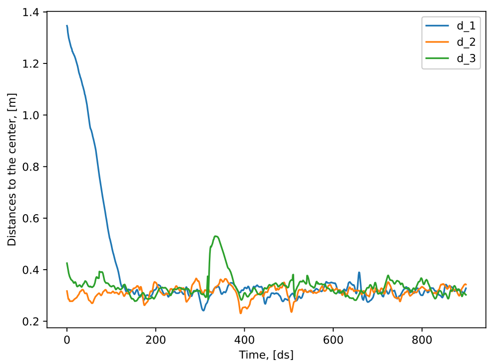
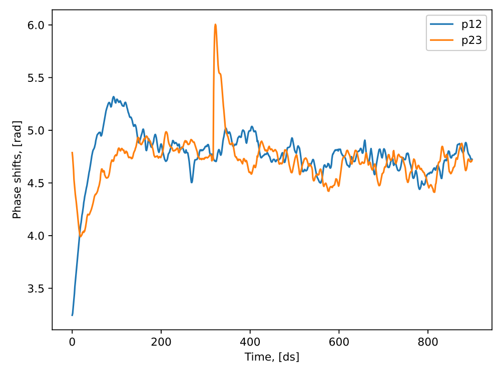


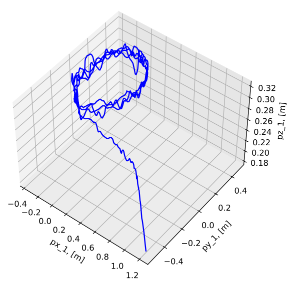
Figure 10: Three copters flight with a stationary center. Trajectories

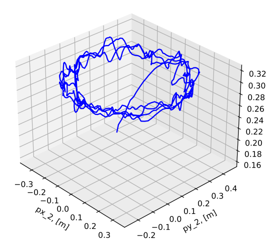




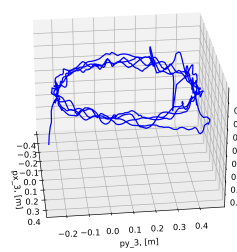
(a) (b)

Figure 11: Three copters flight with a stationary center. (a) Distances to the circle center; (b) phase shifts





(a) (b)



(c)

Figure 12: Three copters flight with a stationary center. (a) 3D trajectory of the 1st copter; (b) 3D trajectory of the 2nd copter; (c) 3D trajectory of the 3rd copter

2.2.2 Moving center of the circular path

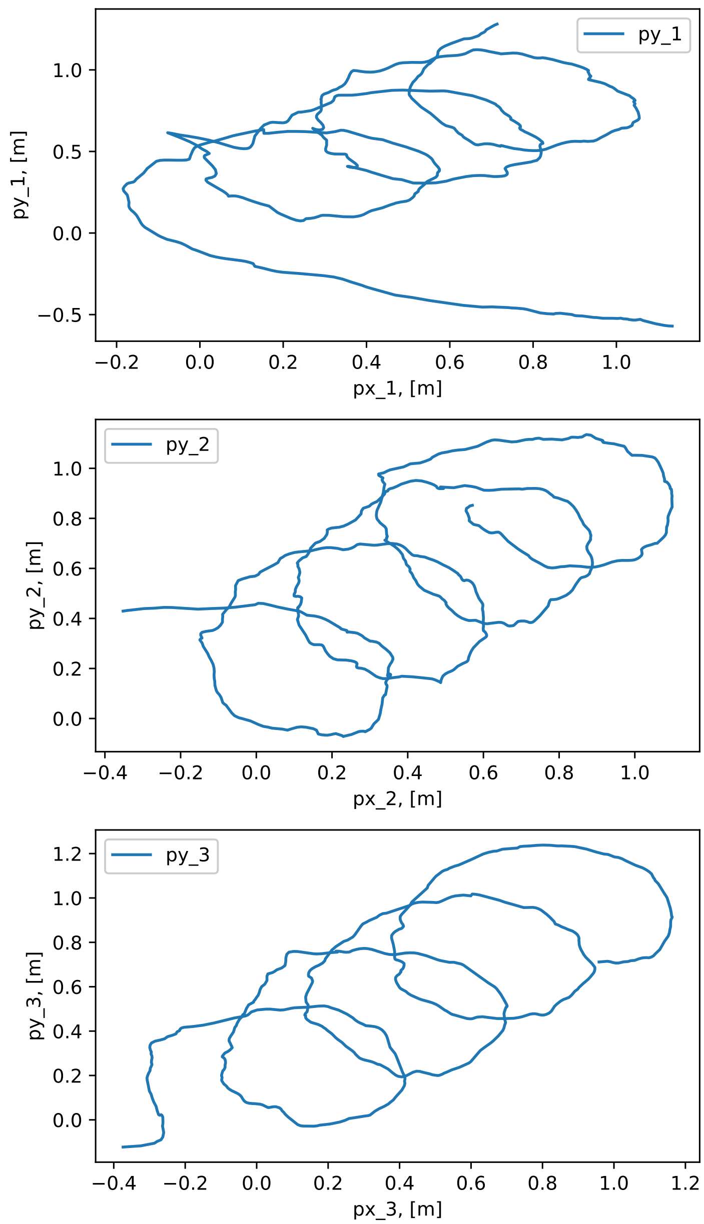
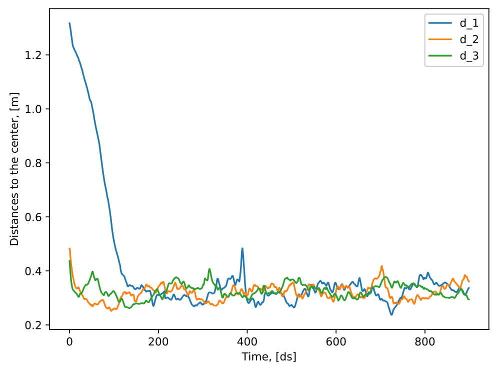
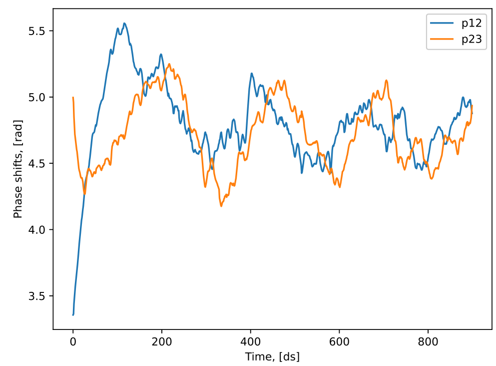


Figure 13: Three copters flight with a moving center. Trajectories

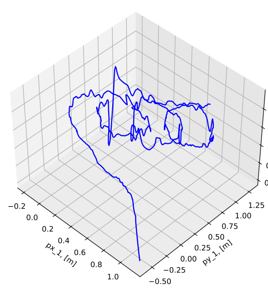


(a)

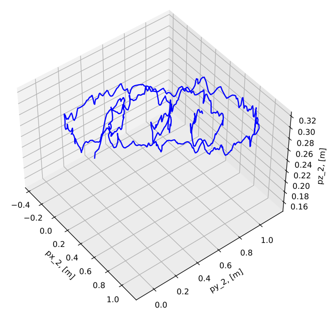


(b)

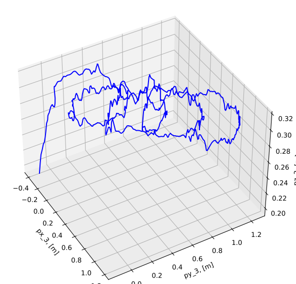
Figure 14: Three copters flight with a moving center. (a) Distances to the circle center; (b) phase shifts



(a)



(b)



(c)

Figure 15: Three copters flight with a moving center. (a) 3D trajectory of the 1st copter; (b) 3D trajectory of the 2nd copter; (c) 3D trajectory of the 3rd copter

References

[1] Beard, R. W., McLain, T. W. (2012). Small unmanned aircraft: Theory and practice. Princeton university press.